

# Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.0-a-sec<sup>m</sup>-b-trg<sup>n</sup>

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May 23, 2020

Compiled on May 23, 2020 at 1:10am

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3.164	$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b \sec(c+dx)}} dx$	709
3.165	$\int \frac{1}{\sqrt{\sec(c+dx)} \sqrt{b \sec(c+dx)}} dx$	712
3.166	$\int \frac{1}{\sec^{\frac{3}{5}}(c+dx) \sqrt{b \sec(c+dx)}} dx$	715
3.167	$\int \frac{1}{\sec^{\frac{5}{9}}(c+dx) \sqrt{b \sec(c+dx)}} dx$	719
3.168	$\int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{3/2}} dx$	723
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3.170	$\int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{3/2}} dx$	731
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3.178	$\int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{5/2}} dx$	761
3.179	$\int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{5/2}} dx$	765
3.180	$\int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{5/2}} dx$	768
3.181	$\int \frac{\sqrt{\sec(c+dx)}}{(b \sec(c+dx))^{5/2}} dx$	771
3.182	$\int \frac{1}{\sqrt{\sec(c+dx)} (b \sec(c+dx))^{5/2}} dx$	775
3.183	$\int \frac{1}{\sec^{\frac{3}{5}}(c+dx) (b \sec(c+dx))^{5/2}} dx$	779

3.184	$\int \sec^2(c + dx) \sqrt[3]{b \sec(c + dx)} dx$	783
3.185	$\int \sec(c + dx) \sqrt[3]{b \sec(c + dx)} dx$	787
3.186	$\int \sqrt[3]{b \sec(c + dx)} dx$	791
3.187	$\int \cos(c + dx) \sqrt[3]{b \sec(c + dx)} dx$	794
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3.190	$\int \sec(c + dx) (b \sec(c + dx))^{4/3} dx$	806
3.191	$\int (b \sec(c + dx))^{4/3} dx$	810
3.192	$\int \cos(c + dx) (b \sec(c + dx))^{4/3} dx$	813
3.193	$\int \cos^2(c + dx) (b \sec(c + dx))^{4/3} dx$	817
3.194	$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b \sec(c+dx)}} dx$	821
3.195	$\int \frac{\sec(c+dx)}{\sqrt[3]{b \sec(c+dx)}} dx$	825
3.196	$\int \frac{1}{\sqrt[3]{b \sec(c+dx)}} dx$	829
3.197	$\int \frac{\cos(c+dx)}{\sqrt[3]{b \sec(c+dx)}} dx$	832
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3.199	$\int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{4/3}} dx$	840
3.200	$\int \frac{\sec(c+dx)}{(b \sec(c+dx))^{4/3}} dx$	844
3.201	$\int \frac{1}{(b \sec(c+dx))^{4/3}} dx$	848
3.202	$\int \frac{\cos(c+dx)}{(b \sec(c+dx))^{4/3}} dx$	851
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3.204	$\int \sec^m(c + dx) (b \sec(c + dx))^{4/3} dx$	859
3.205	$\int \sec^m(c + dx) (b \sec(c + dx))^{2/3} dx$	863
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3.208	$\int \frac{\sec^m(c+dx)}{(b \sec(c+dx))^{2/3}} dx$	875
3.209	$\int \frac{\sec^m(c+dx)}{(b \sec(c+dx))^{4/3}} dx$	879
3.210	$\int \sec^m(c + dx) (b \sec(c + dx))^n dx$	883
3.211	$\int \sec^2(c + dx) (b \sec(c + dx))^n dx$	887
3.212	$\int \sec(c + dx) (b \sec(c + dx))^n dx$	891
3.213	$\int (b \sec(c + dx))^n dx$	895
3.214	$\int \cos(c + dx) (b \sec(c + dx))^n dx$	898
3.215	$\int \cos^2(c + dx) (b \sec(c + dx))^n dx$	902
3.216	$\int \cos^3(c + dx) (b \sec(c + dx))^n dx$	906
3.217	$\int \sec^{\frac{5}{2}}(c + dx) (b \sec(c + dx))^n dx$	910
3.218	$\int \sec^{\frac{3}{2}}(c + dx) (b \sec(c + dx))^n dx$	914



3.219	$\int \sqrt{\sec(c+dx)}(b \sec(c+dx))^n dx$	918
3.220	$\int \frac{(b \sec(c+dx))^n}{\sqrt{\sec(c+dx)}} dx$	922
3.221	$\int \frac{(b \sec(c+dx))^n}{\sec^3(c+dx)} dx$	926
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3.232	$\int (d \csc(a+bx))^{5/2} \sqrt{c \sec(a+bx)} dx$	964
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3.237	$\int \frac{\sqrt{c \sec(a+bx)}}{(d \csc(a+bx))^{5/2}} dx$	984
3.238	$\int (d \csc(a+bx))^{9/2} (c \sec(a+bx))^{3/2} dx$	990
3.239	$\int (d \csc(a+bx))^{7/2} (c \sec(a+bx))^{3/2} dx$	994
3.240	$\int (d \csc(a+bx))^{5/2} (c \sec(a+bx))^{3/2} dx$	999
3.241	$\int (d \csc(a+bx))^{3/2} (c \sec(a+bx))^{3/2} dx$	1002
3.242	$\int \sqrt{d \csc(a+bx)} (c \sec(a+bx))^{3/2} dx$	1006
3.243	$\int \frac{(c \sec(a+bx))^{3/2}}{\sqrt{d \csc(a+bx)}} dx$	1009
3.244	$\int \frac{(c \sec(a+bx))^{3/2}}{(d \csc(a+bx))^{3/2}} dx$	1013
3.245	$\int \frac{(c \sec(a+bx))^{3/2}}{(d \csc(a+bx))^{5/2}} dx$	1019
3.246	$\int (d \csc(a+bx))^{9/2} (c \sec(a+bx))^{5/2} dx$	1023
3.247	$\int (d \csc(a+bx))^{7/2} (c \sec(a+bx))^{5/2} dx$	1027
3.248	$\int (d \csc(a+bx))^{5/2} (c \sec(a+bx))^{5/2} dx$	1031
3.249	$\int (d \csc(a+bx))^{3/2} (c \sec(a+bx))^{5/2} dx$	1035
3.250	$\int \sqrt{d \csc(a+bx)} (c \sec(a+bx))^{5/2} dx$	1038
3.251	$\int \frac{(c \sec(a+bx))^{5/2}}{\sqrt{d \csc(a+bx)}} dx$	1042
3.252	$\int \frac{(c \sec(a+bx))^{5/2}}{(d \csc(a+bx))^{3/2}} dx$	1045

3.253	$\int \frac{(c \sec(a+bx))^{5/2}}{(d \csc(a+bx))^{5/2}} dx$	.1049
3.254	$\int \frac{(d \csc(a+bx))^{9/2}}{\sqrt{c \sec(a+bx)}} dx$	.1055
3.255	$\int \frac{(d \csc(a+bx))^{7/2}}{\sqrt{c \sec(a+bx)}} dx$	.1058
3.256	$\int \frac{(d \csc(a+bx))^{5/2}}{\sqrt{c \sec(a+bx)}} dx$	.1063
3.257	$\int \frac{(d \csc(a+bx))^{3/2}}{\sqrt{c \sec(a+bx)}} dx$	.1066
3.258	$\int \frac{\sqrt{d} \csc(a+bx)}{\sqrt{c \sec(a+bx)}} dx$	.1070
3.259	$\int \frac{1}{\sqrt{d} \csc(a+bx) \sqrt{c \sec(a+bx)}} dx$	.1075
3.260	$\int \frac{1}{(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} dx$	.1079
3.261	$\int \frac{1}{(d \csc(a+bx))^{5/2} \sqrt{c \sec(a+bx)}} dx$	.1085
3.262	$\int \frac{(d \csc(a+bx))^{11/2}}{(c \sec(a+bx))^{3/2}} dx$	.1089
3.263	$\int \frac{(d \csc(a+bx))^{9/2}}{(c \sec(a+bx))^{3/2}} dx$	.1093
3.264	$\int \frac{(d \csc(a+bx))^{7/2}}{(c \sec(a+bx))^{3/2}} dx$	.1098
3.265	$\int \frac{(d \csc(a+bx))^{5/2}}{(c \sec(a+bx))^{3/2}} dx$	.1101
3.266	$\int \frac{(d \csc(a+bx))^{3/2}}{(c \sec(a+bx))^{3/2}} dx$	.1105
3.267	$\int \frac{\sqrt{d} \csc(a+bx)}{(c \sec(a+bx))^{3/2}} dx$	.1111
3.268	$\int \frac{1}{\sqrt{d} \csc(a+bx) (c \sec(a+bx))^{3/2}} dx$	.1115
3.269	$\int \frac{1}{(d \csc(a+bx))^{3/2} (c \sec(a+bx))^{3/2}} dx$	.1121
3.270	$\int \frac{1}{(d \csc(a+bx))^{5/2} (c \sec(a+bx))^{3/2}} dx$	.1126
3.271	$\int \frac{(d \csc(a+bx))^{9/2}}{(c \sec(a+bx))^{5/2}} dx$	.1132
3.272	$\int \frac{(d \csc(a+bx))^{7/2}}{(c \sec(a+bx))^{5/2}} dx$	.1135
3.273	$\int \frac{(d \csc(a+bx))^{5/2}}{(c \sec(a+bx))^{5/2}} dx$	.1140
3.274	$\int \frac{(d \csc(a+bx))^{3/2}}{(c \sec(a+bx))^{5/2}} dx$	.1146
3.275	$\int \frac{\sqrt{d} \csc(a+bx)}{(c \sec(a+bx))^{5/2}} dx$	.1150
3.276	$\int \frac{1}{\sqrt{d} \csc(a+bx) (c \sec(a+bx))^{5/2}} dx$	.1156
3.277	$\int \frac{1}{(d \csc(a+bx))^{3/2} (c \sec(a+bx))^{5/2}} dx$	.1160
3.278	$\int \frac{1}{(d \csc(a+bx))^{5/2} (c \sec(a+bx))^{5/2}} dx$	.1166
3.279	$\int \frac{1}{(d \csc(a+bx))^{7/2} (c \sec(a+bx))^{5/2}} dx$	.1170
3.280	$\int \csc^n(e+fx) \sec^m(e+fx) dx$	.1176
3.281	$\int \csc^n(e+fx) (a \sec(e+fx))^m dx$	.1180

3.282	$\int (b \csc(e + fx))^n \sec^m(e + fx) dx$	.1184
3.283	$\int (b \csc(e + fx))^n (a \sec(e + fx))^m dx$	.1188
3.284	$\int (b \csc(e + fx))^n \sec^5(e + fx) dx$	.1192
3.285	$\int (b \csc(e + fx))^n \sec^3(e + fx) dx$	.1195
3.286	$\int (b \csc(e + fx))^n \sec(e + fx) dx$	.1198
3.287	$\int \cos(e + fx)(b \csc(e + fx))^n dx$	.1201
3.288	$\int \cos^3(e + fx)(b \csc(e + fx))^n dx$	.1204
3.289	$\int \cos^5(e + fx)(b \csc(e + fx))^n dx$	.1208
3.290	$\int (b \csc(e + fx))^n \sec^6(e + fx) dx$	.1212
3.291	$\int (b \csc(e + fx))^n \sec^4(e + fx) dx$	.1215
3.292	$\int (b \csc(e + fx))^n \sec^2(e + fx) dx$	.1218
3.293	$\int (b \csc(e + fx))^n dx$	.1221
3.294	$\int \cos^2(e + fx)(b \csc(e + fx))^n dx$	.1224
3.295	$\int \cos^4(e + fx)(b \csc(e + fx))^n dx$	.1227
3.296	$\int (b \csc(e + fx))^n (c \sec(e + fx))^{3/2} dx$	.1230
3.297	$\int (b \csc(e + fx))^n \sqrt{c \sec(e + fx)} dx$	.1233
3.298	$\int \frac{(b \csc(e + fx))^n}{\sqrt{c \sec(e + fx)}} dx$	.1236
3.299	$\int \frac{(b \csc(e + fx))^n}{(c \sec(e + fx))^{3/2}} dx$	.1240

#### 4 Listing of Grading functions

1245



# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 299 ]. This is test number [ 115 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. ( 299 )	% 0. ( 0 )
Mathematica	% 100. ( 299 )	% 0. ( 0 )
Maple	% 75.25 ( 225 )	% 24.75 ( 74 )
Maxima	% 31.1 ( 93 )	% 68.9 ( 206 )
Fricas	% 35.45 ( 106 )	% 64.55 ( 193 )
Sympy	% 5.35 ( 16 )	% 94.65 ( 283 )
Giac	% 12.04 ( 36 )	% 87.96 ( 263 )

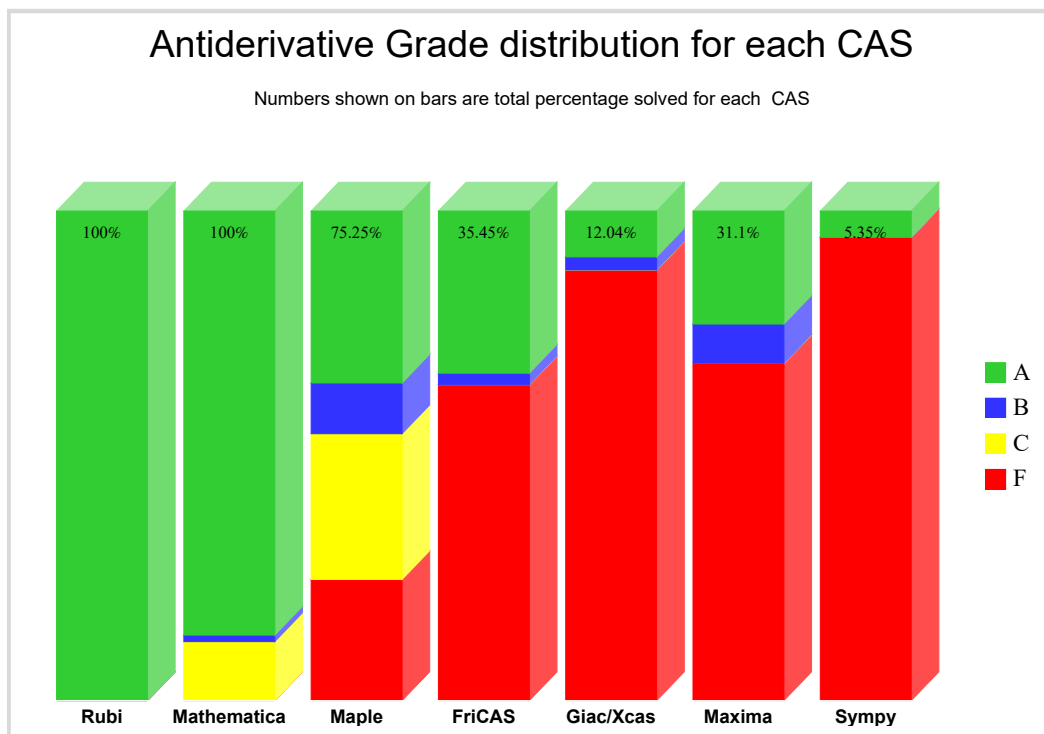
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

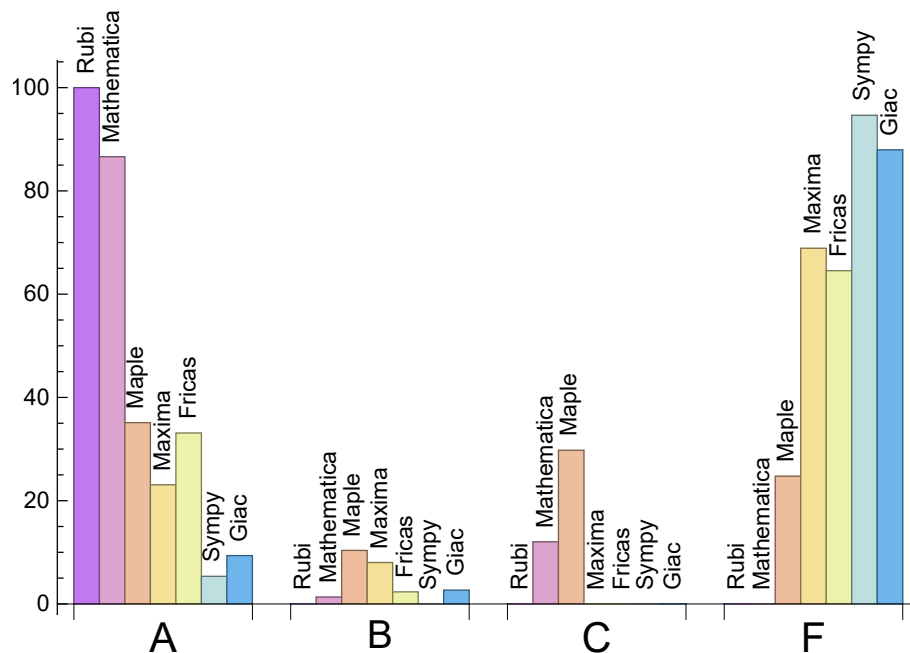
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	86.62	1.34	12.04	0.
Maple	35.12	10.37	29.77	24.75
Maxima	23.08	8.03	0.	68.9
Fricas	33.11	2.34	0.	64.55
Sympy	5.35	0.	0.	94.65
Giac	9.36	2.68	0.	87.96

The following is a Bar chart illustration of the data in the above table.





The figure below compares the CAS systems for each grade level.



### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.06	79.	1.	70.	1.
Mathematica	0.21	63.33	0.95	56.	0.87
Maple	0.19	193.93	2.33	122.	1.57
Maxima	1.94	227.58	3.3	57.	1.3
Fricas	1.68	201.01	4.18	135.	2.7
Sympy	24.87	32.56	1.	36.	1.03
Giac	1.38	57.11	1.95	55.	1.5

## 1.4 list of integrals that has no closed form antiderivative

{

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {31, 32, 33, 34, 35, 36, 38, 68, 69, 186, 187, 191, 192, 193, 196, 197, 198, 199, 201, 202, 203, 213, 214, 215, 216, 280, 281, 282, 283, 298}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via `sagemath`) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

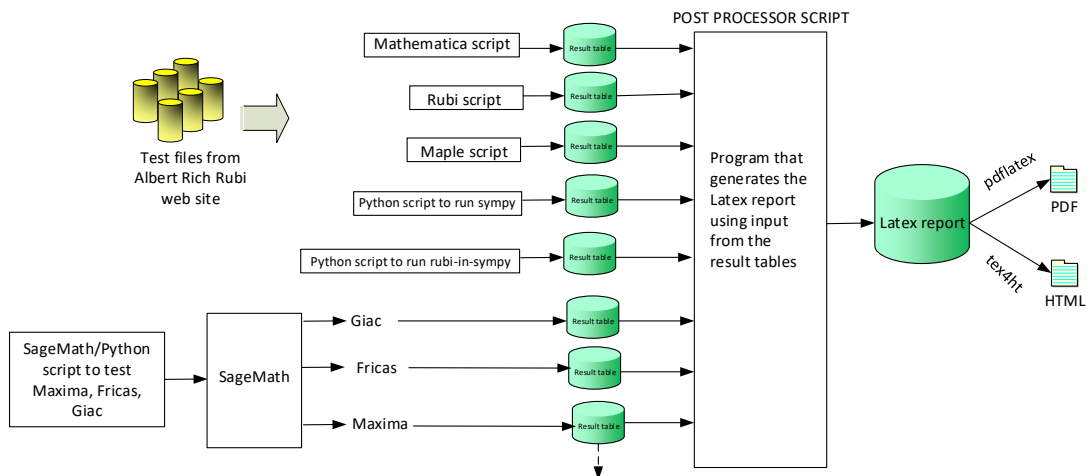
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

B grade: { }

C grade: { }

F grade: { }

## 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 233, 235, 237, 238, 240, 242, 247, 249, 251, 253, 254, 256, 262, 264, 266, 268, 270, 271, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 296, 297, 299 }

B grade: { 41, 42, 294, 295 }

C grade: { 230, 232, 234, 236, 239, 241, 243, 244, 245, 246, 248, 250, 252, 255, 257, 258, 259, 260, 261, 263, 265, 267, 269, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 298 }

F grade: { }

## 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 11, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 223, 224, 225, 226, 227, 229, 231, 233, 236, 238, 240, 242, 247, 249, 250, 251, 252, 254, 256, 262, 264, 267, 269, 271 }

B grade: { 9, 10, 12, 13, 14, 15, 16, 40, 41, 42, 228, 230, 232, 234, 239, 241, 243, 245, 246, 248, 255, 257, 259, 261, 263, 265, 272, 274, 276, 278, 287 }

C grade: { 17, 18, 19, 20, 21, 22, 23, 24, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 235, 237, 244, 253, 258, 260, 266, 268, 270, 273, 275, 277, 279 }

F grade: { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 68, 69, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }



## 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 135, 137, 138, 139, 140, 141, 145, 147, 148, 149, 150, 151, 155, 157, 158, 159, 160, 164, 165, 166, 167, 171, 172, 173, 174, 175, 179, 180, 181, 182, 183, 223, 224, 225, 226, 227, 228, 229, 287, 288, 289 }

B grade: { 47, 48, 49, 132, 133, 134, 136, 142, 143, 144, 146, 152, 153, 154, 156, 161, 162, 163, 168, 169, 170, 176, 177, 178 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

## 2.1.5 FriCAS

A grade: { 2, 4, 5, 6, 7, 8, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 223, 224, 225, 226, 227, 228, 229, 231, 233, 238, 240, 242, 247, 249, 254, 256, 262, 287, 288, 289 }

B grade: { 1, 3, 41, 42, 251, 264, 271 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 230, 232, 234, 235, 236, 237, 239, 241, 243, 244, 245, 246, 248, 250, 252, 253, 255, 257, 258, 259, 260, 261, 263, 265, 266, 267, 268, 269, 270, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

## 2.1.6 Sympy

A grade: { 1, 43, 44, 45, 51, 52, 53, 137, 138, 147, 164, 165, 166, 171, 172, 173 }

B grade: { }

C grade: { }

F grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

## 2.1.7 Giac

A grade: { 2, 3, 4, 5, 6, 7, 8, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 61, 62, 63, 64, 65, 225, 226, 228, 229 }

B grade: { 1, 40, 41, 42, 52, 223, 224, 227 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 55, 56, 57, 58, 59, 60, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	19	24	76	36	59
normalized size	1	1.	1.	1.73	2.18	6.91	3.27	5.36
time (sec)	N/A	0.004	0.002	0.003	1.104	1.526	5.915	1.312

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	42	0	14
normalized size	1	1.	1.	1.1	1.4	4.2	0.	1.4
time (sec)	N/A	0.009	0.004	0.056	1.266	1.403	0.	1.25

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	38	62	162	0	65
normalized size	1	1.	1.	1.12	1.82	4.76	0.	1.91
time (sec)	N/A	0.014	0.01	0.133	1.001	1.412	0.	1.353

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	23	24	30	81	0	30
normalized size	1	1.	0.88	0.92	1.15	3.12	0.	1.15
time (sec)	N/A	0.011	0.041	0.043	1.166	1.346	0.	1.324

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	42	57	96	200	0	85
normalized size	1	1.	0.76	1.04	1.75	3.64	0.	1.55
time (sec)	N/A	0.025	0.071	0.044	1.03	1.423	0.	1.32

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	35	34	46	108	0	46
normalized size	1	1.	0.85	0.83	1.12	2.63	0.	1.12
time (sec)	N/A	0.015	0.101	0.043	1.201	1.362	0.	1.137

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	52	76	123	231	0	99
normalized size	1	1.	0.68	1.	1.62	3.04	0.	1.3
time (sec)	N/A	0.039	0.166	0.046	1.07	1.476	0.	1.276

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	44	59	135	0	59
normalized size	1	1.	0.81	0.83	1.11	2.55	0.	1.11
time (sec)	N/A	0.017	0.219	0.043	1.075	1.393	0.	1.274

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	59	358	0	0	0	0
normalized size	1	1.	0.69	4.21	0.	0.	0.	0.
time (sec)	N/A	0.037	0.175	2.199	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	46	213	0	0	0	0
normalized size	1	1.	0.74	3.44	0.	0.	0.	0.
time (sec)	N/A	0.027	0.067	1.266	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	45	101	0	0	0	0
normalized size	1	1.	0.78	1.74	0.	0.	0.	0.
time (sec)	N/A	0.026	0.048	1.062	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	133	0	0	0	0
normalized size	1	1.	1.	3.69	0.	0.	0.	0.
time (sec)	N/A	0.017	0.029	0.883	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	133	0	0	0	0
normalized size	1	1.	1.	3.69	0.	0.	0.	0.
time (sec)	N/A	0.017	0.034	0.936	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	49	179	0	0	0	0
normalized size	1	1.	0.79	2.89	0.	0.	0.	0.
time (sec)	N/A	0.028	0.044	1.485	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	55	202	0	0	0	0
normalized size	1	1.	0.89	3.26	0.	0.	0.	0.
time (sec)	N/A	0.029	0.069	1.212	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	61	199	0	0	0	0
normalized size	1	1.	0.72	2.34	0.	0.	0.	0.
time (sec)	N/A	0.04	0.099	1.247	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	62	354	0	0	0	0
normalized size	1	1.	0.63	3.61	0.	0.	0.	0.
time (sec)	N/A	0.056	0.166	0.32	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	51	128	0	0	0	0
normalized size	1	1.	0.73	1.83	0.	0.	0.	0.
time (sec)	N/A	0.034	0.066	0.198	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	48	322	0	0	0	0
normalized size	1	1.	0.73	4.88	0.	0.	0.	0.
time (sec)	N/A	0.04	0.039	0.225	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	98	0	0	0	0
normalized size	1	1.	1.	2.58	0.	0.	0.	0.
time (sec)	N/A	0.019	0.02	0.153	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	306	0	0	0	0
normalized size	1	1.	1.	8.05	0.	0.	0.	0.
time (sec)	N/A	0.027	0.029	0.164	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	59	131	0	0	0	0
normalized size	1	1.	0.82	1.82	0.	0.	0.	0.
time (sec)	N/A	0.046	0.058	0.155	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	323	0	0	0	0
normalized size	1	1.	0.83	4.49	0.	0.	0.	0.
time (sec)	N/A	0.038	0.068	0.2	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	66	153	0	0	0	0
normalized size	1	1.	0.66	1.53	0.	0.	0.	0.
time (sec)	N/A	0.059	0.096	0.19	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	55	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.051	0.068	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	55	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.036	0.094	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.034	0.1	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.082	0.093	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	55	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.083	0.093	0.	0.	0.	0.



Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	55	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.066	0.055	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	57	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.049	0.059	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	57	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.041	0.084	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.04	0.105	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.052	0.094	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.053	0.097	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.069	0.054	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.05	0.291	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	61	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.047	0.265	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	74	72	57	162	0	80
normalized size	1	1.	1.48	1.44	1.14	3.24	0.	1.6
time (sec)	N/A	0.017	0.29	0.159	1.741	1.367	0.	1.286

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	68	64	41	139	0	72
normalized size	1	1.	1.89	1.78	1.14	3.86	0.	2.
time (sec)	N/A	0.012	0.14	0.091	1.648	1.409	0.	1.278

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	52	53	24	109	0	59
normalized size	1	1.	2.36	2.41	1.09	4.95	0.	2.68
time (sec)	N/A	0.009	0.057	0.065	1.719	1.331	0.	1.367

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	44	21	4	61	0	47
normalized size	1	1.	14.67	7.	1.33	20.33	0.	15.67
time (sec)	N/A	0.006	0.009	0.071	1.732	1.421	0.	1.339

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	15	12	10	8
normalized size	1	1.	1.	1.27	1.36	1.09	0.91	0.73
time (sec)	N/A	0.007	0.006	0.074	1.18	1.24	0.483	1.272

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	23	21	34	38	27	22
normalized size	1	1.	0.79	0.72	1.17	1.31	0.93	0.76
time (sec)	N/A	0.011	0.015	0.059	1.089	1.312	1.506	1.35

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	31	29	50	59	44	34
normalized size	1	1.	0.72	0.67	1.16	1.37	1.02	0.79
time (sec)	N/A	0.015	0.025	0.072	1.165	1.444	17.704	1.257

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	37	35	66	78	0	46
normalized size	1	1.	0.65	0.61	1.16	1.37	0.	0.81
time (sec)	N/A	0.019	0.038	0.088	1.069	1.421	0.	1.405

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	78	74	2936	186	0	107
normalized size	1	1.	0.93	0.88	34.95	2.21	0.	1.27
time (sec)	N/A	0.041	0.124	0.145	16.739	1.478	0.	1.301

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	72	66	1500	159	0	90
normalized size	1	1.	1.11	1.02	23.08	2.45	0.	1.38
time (sec)	N/A	0.031	0.121	0.087	2.673	1.57	0.	1.409

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	55	55	437	119	0	57
normalized size	1	1.	1.2	1.2	9.5	2.59	0.	1.24
time (sec)	N/A	0.022	0.051	0.061	1.946	1.471	0.	1.306

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	46	23	51	171	0	42
normalized size	1	1.	1.84	0.92	2.04	6.84	0.	1.68
time (sec)	N/A	0.015	0.008	0.056	1.936	1.464	0.	1.343

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	16	8	46	15	15
normalized size	1	1.	1.	1.23	0.62	3.54	1.15	1.15
time (sec)	N/A	0.029	0.005	0.069	1.881	1.445	0.538	1.229

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	23	19	74	37	78
normalized size	1	1.	0.75	0.64	0.53	2.06	1.03	2.17
time (sec)	N/A	0.018	0.018	0.057	1.945	1.432	1.487	1.379

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	31	30	96	60	113
normalized size	1	1.	0.65	0.56	0.55	1.75	1.09	2.05
time (sec)	N/A	0.027	0.028	0.064	1.834	1.445	17.485	1.441

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	42	37	38	115	0	149
normalized size	1	1.	0.57	0.5	0.51	1.55	0.	2.01
time (sec)	N/A	0.035	0.035	0.081	1.784	1.451	0.	1.608

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	59	223	0	0	0	0
normalized size	1	1.	0.5	1.91	0.	0.	0.	0.
time (sec)	N/A	0.052	0.093	0.368	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	43	87	0	0	0	0
normalized size	1	1.	0.66	1.34	0.	0.	0.	0.
time (sec)	N/A	0.035	0.035	0.182	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	32	191	0	0	0	0
normalized size	1	1.	0.76	4.55	0.	0.	0.	0.
time (sec)	N/A	0.026	0.017	0.205	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	31	76	0	0	0	0
normalized size	1	1.	0.7	1.73	0.	0.	0.	0.
time (sec)	N/A	0.027	0.04	0.14	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	43	198	0	0	0	0
normalized size	1	1.	0.59	2.71	0.	0.	0.	0.
time (sec)	N/A	0.037	0.094	0.156	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	59	114	0	0	0	0
normalized size	1	1.	0.5	0.97	0.	0.	0.	0.
time (sec)	N/A	0.056	0.097	0.206	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	54	53	82	228	0	90
normalized size	1	1.	0.33	0.33	0.5	1.4	0.	0.55
time (sec)	N/A	0.039	0.168	0.233	1.643	1.587	0.	1.327

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	42	41	58	165	0	66
normalized size	1	1.	0.36	0.35	0.5	1.41	0.	0.56
time (sec)	N/A	0.03	0.091	0.113	1.792	1.407	0.	1.244

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	30	29	34	101	0	30
normalized size	1	1.	0.49	0.48	0.56	1.66	0.	0.49
time (sec)	N/A	0.022	0.057	0.067	1.692	1.449	0.	1.305

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	8	43	0	8
normalized size	1	1.	1.	0.93	0.53	2.87	0.	0.53
time (sec)	N/A	0.016	0.005	0.063	1.739	1.364	0.	1.258

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	23	22	34	74	0	53
normalized size	1	1.	0.64	0.61	0.94	2.06	0.	1.47
time (sec)	N/A	0.015	0.023	0.074	1.664	1.42	0.	1.311

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	38	41	78	126	0	0
normalized size	1	1.	0.44	0.48	0.91	1.47	0.	0.
time (sec)	N/A	0.032	0.039	0.104	1.646	1.461	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	55	57	119	177	0	0
normalized size	1	1.	0.42	0.43	0.9	1.34	0.	0.
time (sec)	N/A	0.051	0.082	0.221	1.711	1.452	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	69	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.094	0.444	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	71	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.073	0.197	0.	0.	0.	0.



Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	69	152	0	0	0	0
normalized size	1	1.	0.71	1.57	0.	0.	0.	0.
time (sec)	N/A	0.06	0.212	0.224	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	69	356	0	0	0	0
normalized size	1	1.	0.73	3.75	0.	0.	0.	0.
time (sec)	N/A	0.061	0.165	0.208	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	51	130	0	0	0	0
normalized size	1	1.	0.74	1.88	0.	0.	0.	0.
time (sec)	N/A	0.042	0.084	0.155	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	47	316	0	0	0	0
normalized size	1	1.	0.75	5.02	0.	0.	0.	0.
time (sec)	N/A	0.041	0.051	0.214	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	98	0	0	0	0
normalized size	1	1.	1.	2.58	0.	0.	0.	0.
time (sec)	N/A	0.02	0.02	0.141	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	303	0	0	0	0
normalized size	1	1.	1.	7.77	0.	0.	0.	0.
time (sec)	N/A	0.03	0.035	0.149	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	51	123	0	0	0	0
normalized size	1	1.	0.76	1.84	0.	0.	0.	0.
time (sec)	N/A	0.052	0.051	0.164	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	57	315	0	0	0	0
normalized size	1	1.	0.81	4.5	0.	0.	0.	0.
time (sec)	N/A	0.051	0.059	0.208	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	63	145	0	0	0	0
normalized size	1	1.	0.66	1.53	0.	0.	0.	0.
time (sec)	N/A	0.07	0.09	0.188	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	71	325	0	0	0	0
normalized size	1	1.	0.72	3.32	0.	0.	0.	0.
time (sec)	N/A	0.069	0.244	0.181	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	64	152	0	0	0	0
normalized size	1	1.	0.67	1.6	0.	0.	0.	0.
time (sec)	N/A	0.059	0.183	0.195	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	64	356	0	0	0	0
normalized size	1	1.	0.65	3.63	0.	0.	0.	0.
time (sec)	N/A	0.059	0.175	0.19	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	49	122	0	0	0	0
normalized size	1	1.	0.73	1.82	0.	0.	0.	0.
time (sec)	N/A	0.041	0.072	0.146	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	48	320	0	0	0	0
normalized size	1	1.	0.73	4.85	0.	0.	0.	0.
time (sec)	N/A	0.034	0.037	0.199	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	98	0	0	0	0
normalized size	1	1.	1.	2.51	0.	0.	0.	0.
time (sec)	N/A	0.03	0.019	0.136	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	309	0	0	0	0
normalized size	1	1.	1.	7.54	0.	0.	0.	0.
time (sec)	N/A	0.038	0.024	0.147	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	52	129	0	0	0	0
normalized size	1	1.	0.74	1.84	0.	0.	0.	0.
time (sec)	N/A	0.054	0.049	0.16	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	58	321	0	0	0	0
normalized size	1	1.	0.81	4.46	0.	0.	0.	0.
time (sec)	N/A	0.054	0.049	0.196	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	64	151	0	0	0	0
normalized size	1	1.	0.65	1.54	0.	0.	0.	0.
time (sec)	N/A	0.073	0.078	0.184	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	72	331	0	0	0	0
normalized size	1	1.	0.72	3.31	0.	0.	0.	0.
time (sec)	N/A	0.075	0.16	0.182	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	61	152	0	0	0	0
normalized size	1	1.	0.62	1.55	0.	0.	0.	0.
time (sec)	N/A	0.059	0.18	0.191	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	61	348	0	0	0	0
normalized size	1	1.	0.63	3.59	0.	0.	0.	0.
time (sec)	N/A	0.057	0.164	0.204	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	51	128	0	0	0	0
normalized size	1	1.	0.73	1.83	0.	0.	0.	0.
time (sec)	N/A	0.032	0.017	0.159	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	324	0	0	0	0
normalized size	1	1.	0.74	4.76	0.	0.	0.	0.
time (sec)	N/A	0.045	0.031	0.218	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	98	0	0	0	0
normalized size	1	1.	1.	2.39	0.	0.	0.	0.
time (sec)	N/A	0.038	0.016	0.153	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	38	311	0	0	0	0
normalized size	1	1.	0.93	7.59	0.	0.	0.	0.
time (sec)	N/A	0.037	0.043	0.158	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	54	131	0	0	0	0
normalized size	1	1.	0.75	1.82	0.	0.	0.	0.
time (sec)	N/A	0.055	0.049	0.167	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	321	0	0	0	0
normalized size	1	1.	0.83	4.46	0.	0.	0.	0.
time (sec)	N/A	0.054	0.051	0.201	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	66	153	0	0	0	0
normalized size	1	1.	0.66	1.53	0.	0.	0.	0.
time (sec)	N/A	0.071	0.073	0.188	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	74	333	0	0	0	0
normalized size	1	1.	0.74	3.33	0.	0.	0.	0.
time (sec)	N/A	0.074	0.166	0.186	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	62	354	0	0	0	0
normalized size	1	1.	0.63	3.61	0.	0.	0.	0.
time (sec)	N/A	0.048	0.085	0.222	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	69	152	0	0	0	0
normalized size	1	1.	0.69	1.52	0.	0.	0.	0.
time (sec)	N/A	0.057	0.083	0.23	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	61	356	0	0	0	0
normalized size	1	1.	0.63	3.67	0.	0.	0.	0.
time (sec)	N/A	0.059	0.226	0.236	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	51	130	0	0	0	0
normalized size	1	1.	0.71	1.81	0.	0.	0.	0.
time (sec)	N/A	0.039	0.074	0.181	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	48	319	0	0	0	0
normalized size	1	1.	0.74	4.91	0.	0.	0.	0.
time (sec)	N/A	0.04	0.078	0.218	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	98	0	0	0	0
normalized size	1	1.	1.	2.39	0.	0.	0.	0.
time (sec)	N/A	0.021	0.016	0.158	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	306	0	0	0	0
normalized size	1	1.	1.	8.05	0.	0.	0.	0.
time (sec)	N/A	0.019	0.015	0.157	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	60	126	0	0	0	0
normalized size	1	1.	0.87	1.83	0.	0.	0.	0.
time (sec)	N/A	0.043	0.068	0.16	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	60	316	0	0	0	0
normalized size	1	1.	0.9	4.72	0.	0.	0.	0.
time (sec)	N/A	0.051	0.082	0.215	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	66	148	0	0	0	0
normalized size	1	1.	0.68	1.53	0.	0.	0.	0.
time (sec)	N/A	0.069	0.075	0.204	0.	0.	0.	0.



Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	70	328	0	0	0	0
normalized size	1	1.	0.74	3.45	0.	0.	0.	0.
time (sec)	N/A	0.069	0.189	0.202	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	69	152	0	0	0	0
normalized size	1	1.	0.69	1.52	0.	0.	0.	0.
time (sec)	N/A	0.059	0.166	0.219	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	64	356	0	0	0	0
normalized size	1	1.	0.64	3.56	0.	0.	0.	0.
time (sec)	N/A	0.058	0.053	0.223	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	56	125	0	0	0	0
normalized size	1	1.	0.78	1.74	0.	0.	0.	0.
time (sec)	N/A	0.038	0.097	0.17	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	51	322	0	0	0	0
normalized size	1	1.	0.75	4.74	0.	0.	0.	0.
time (sec)	N/A	0.04	0.057	0.224	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	98	0	0	0	0
normalized size	1	1.	1.	2.39	0.	0.	0.	0.
time (sec)	N/A	0.023	0.022	0.135	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	311	0	0	0	0
normalized size	1	1.	1.	7.59	0.	0.	0.	0.
time (sec)	N/A	0.022	0.031	0.144	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	59	131	0	0	0	0
normalized size	1	1.	0.82	1.82	0.	0.	0.	0.
time (sec)	N/A	0.033	0.049	0.155	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	60	321	0	0	0	0
normalized size	1	1.	0.87	4.65	0.	0.	0.	0.
time (sec)	N/A	0.045	0.032	0.197	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	66	153	0	0	0	0
normalized size	1	1.	0.67	1.56	0.	0.	0.	0.
time (sec)	N/A	0.072	0.091	0.184	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	73	333	0	0	0	0
normalized size	1	1.	0.75	3.43	0.	0.	0.	0.
time (sec)	N/A	0.071	0.077	0.19	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	64	152	0	0	0	0
normalized size	1	1.	0.64	1.52	0.	0.	0.	0.
time (sec)	N/A	0.057	0.132	0.217	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	64	351	0	0	0	0
normalized size	1	1.	0.64	3.51	0.	0.	0.	0.
time (sec)	N/A	0.058	0.113	0.222	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	51	130	0	0	0	0
normalized size	1	1.	0.71	1.81	0.	0.	0.	0.
time (sec)	N/A	0.038	0.066	0.174	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	51	322	0	0	0	0
normalized size	1	1.	0.75	4.74	0.	0.	0.	0.
time (sec)	N/A	0.038	0.053	0.234	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	98	0	0	0	0
normalized size	1	1.	1.	2.39	0.	0.	0.	0.
time (sec)	N/A	0.022	0.016	0.14	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	38	311	0	0	0	0
normalized size	1	1.	0.93	7.59	0.	0.	0.	0.
time (sec)	N/A	0.023	0.036	0.143	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	62	131	0	0	0	0
normalized size	1	1.	0.86	1.82	0.	0.	0.	0.
time (sec)	N/A	0.04	0.031	0.147	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	321	0	0	0	0
normalized size	1	1.	0.83	4.46	0.	0.	0.	0.
time (sec)	N/A	0.034	0.018	0.199	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	66	153	0	0	0	0
normalized size	1	1.	0.68	1.58	0.	0.	0.	0.
time (sec)	N/A	0.063	0.069	0.178	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	73	333	0	0	0	0
normalized size	1	1.	0.74	3.4	0.	0.	0.	0.
time (sec)	N/A	0.072	0.121	0.183	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	66	153	0	0	0	0
normalized size	1	1.	0.66	1.53	0.	0.	0.	0.
time (sec)	N/A	0.051	0.02	0.187	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	64	131	2236	608	0	0
normalized size	1	1.	0.6	1.22	20.9	5.68	0.	0.
time (sec)	N/A	0.033	0.141	0.178	2.489	1.608	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	45	52	397	115	0	0
normalized size	1	1.	0.64	0.74	5.67	1.64	0.	0.
time (sec)	N/A	0.017	0.087	0.129	2.08	1.412	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	50	112	892	528	0	0
normalized size	1	1.	0.69	1.56	12.39	7.33	0.	0.
time (sec)	N/A	0.019	0.052	0.135	2.113	1.548	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	39	73	78	0	0
normalized size	1	1.	1.	1.22	2.28	2.44	0.	0.
time (sec)	N/A	0.012	0.018	0.13	2.147	1.367	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	52	88	290	0	0
normalized size	1	1.	1.	1.58	2.67	8.79	0.	0.
time (sec)	N/A	0.007	0.011	0.116	2.064	1.496	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	35	266	5	0
normalized size	1	1.	1.	1.33	1.46	11.08	0.21	0.
time (sec)	N/A	0.002	0.014	0.108	1.786	1.638	1.55	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	41	18	76	36	0
normalized size	1	1.	1.	1.28	0.56	2.38	1.12	0.
time (sec)	N/A	0.007	0.036	0.127	2.057	1.402	20.9	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	45	54	34	428	0	0
normalized size	1	1.	0.71	0.86	0.54	6.79	0.	0.
time (sec)	N/A	0.014	0.064	0.14	2.019	1.649	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	45	52	57	130	0	0
normalized size	1	1.	0.64	0.74	0.81	1.86	0.	0.
time (sec)	N/A	0.016	0.109	0.135	2.072	1.383	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	55	74	66	537	0	0
normalized size	1	1.	0.56	0.76	0.67	5.48	0.	0.
time (sec)	N/A	0.025	0.116	0.157	2.132	1.79	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	64	131	2352	621	0	0
normalized size	1	1.	0.58	1.19	21.38	5.65	0.	0.
time (sec)	N/A	0.035	0.111	0.139	2.631	1.585	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	45	52	404	117	0	0
normalized size	1	1.	0.62	0.72	5.61	1.62	0.	0.
time (sec)	N/A	0.017	0.088	0.118	2.103	1.354	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	50	112	933	536	0	0
normalized size	1	1.	0.68	1.51	12.61	7.24	0.	0.
time (sec)	N/A	0.021	0.058	0.117	2.191	1.648	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	39	73	81	0	0
normalized size	1	1.	0.97	1.18	2.21	2.45	0.	0.
time (sec)	N/A	0.011	0.02	0.113	2.036	1.416	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	52	92	293	0	0
normalized size	1	1.	0.97	1.53	2.71	8.62	0.	0.
time (sec)	N/A	0.007	0.021	0.117	2.061	1.783	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	32	35	269	5	0
normalized size	1	1.	0.96	1.28	1.4	10.76	0.2	0.
time (sec)	N/A	0.003	0.025	0.089	1.787	1.857	38.559	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	41	18	78	0	0
normalized size	1	1.	0.97	1.24	0.55	2.36	0.	0.
time (sec)	N/A	0.007	0.044	0.114	2.002	1.578	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	54	38	436	0	0
normalized size	1	1.	0.69	0.83	0.58	6.71	0.	0.
time (sec)	N/A	0.015	0.079	0.124	2.038	1.967	0.	0.



Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	45	52	61	135	0	0
normalized size	1	1.	0.62	0.72	0.85	1.88	0.	0.
time (sec)	N/A	0.017	0.138	0.109	2.083	1.668	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	55	74	72	551	0	0
normalized size	1	1.	0.54	0.73	0.71	5.46	0.	0.
time (sec)	N/A	0.028	0.154	0.123	2.153	2.016	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	57	62	952	158	0	0
normalized size	1	1.	0.49	0.53	8.21	1.36	0.	0.
time (sec)	N/A	0.024	0.217	0.119	2.537	1.728	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	52	420	123	0	0
normalized size	1	1.	0.59	0.68	5.53	1.62	0.	0.
time (sec)	N/A	0.017	0.079	0.106	1.978	1.693	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	50	112	1008	544	0	0
normalized size	1	1.	0.64	1.44	12.92	6.97	0.	0.
time (sec)	N/A	0.02	0.077	0.132	2.191	1.98	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	39	73	84	0	0
normalized size	1	1.	0.91	1.11	2.09	2.4	0.	0.
time (sec)	N/A	0.012	0.031	0.127	2.06	1.636	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	52	97	296	0	0
normalized size	1	1.	0.92	1.44	2.69	8.22	0.	0.
time (sec)	N/A	0.008	0.034	0.097	1.916	1.845	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	32	35	271	0	0
normalized size	1	1.	0.89	1.19	1.3	10.04	0.	0.
time (sec)	N/A	0.003	0.019	0.091	1.825	1.973	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	41	18	81	0	0
normalized size	1	1.	0.91	1.17	0.51	2.31	0.	0.
time (sec)	N/A	0.007	0.065	0.105	1.879	1.662	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	45	54	43	444	0	0
normalized size	1	1.	0.65	0.78	0.62	6.43	0.	0.
time (sec)	N/A	0.015	0.113	0.105	2.128	2.04	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	52	66	140	0	0
normalized size	1	1.	0.59	0.68	0.87	1.84	0.	0.
time (sec)	N/A	0.017	0.163	0.114	2.	1.643	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	50	112	892	533	0	0
normalized size	1	1.	0.69	1.56	12.39	7.4	0.	0.
time (sec)	N/A	0.019	0.057	0.137	2.025	1.964	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	39	80	81	0	0
normalized size	1	1.	1.	1.22	2.5	2.53	0.	0.
time (sec)	N/A	0.011	0.026	0.119	2.039	1.642	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	52	88	298	0	0
normalized size	1	1.	1.	1.58	2.67	9.03	0.	0.
time (sec)	N/A	0.007	0.016	0.12	1.982	1.9	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	35	274	5	0
normalized size	1	1.	1.	1.33	1.46	11.42	0.21	0.
time (sec)	N/A	0.002	0.015	0.092	1.58	1.972	19.077	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	41	18	81	36	0
normalized size	1	1.	1.	1.28	0.56	2.53	1.12	0.
time (sec)	N/A	0.007	0.036	0.154	2.093	1.748	25.96	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	45	54	34	437	82	0
normalized size	1	1.	0.71	0.86	0.54	6.94	1.3	0.
time (sec)	N/A	0.014	0.077	0.135	2.054	2.005	44.261	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	45	52	57	132	0	0
normalized size	1	1.	0.64	0.74	0.81	1.89	0.	0.
time (sec)	N/A	0.016	0.094	0.124	2.122	1.685	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	50	112	905	539	0	0
normalized size	1	1.	0.64	1.44	11.6	6.91	0.	0.
time (sec)	N/A	0.02	0.057	0.117	2.15	1.955	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	39	90	84	0	0
normalized size	1	1.	0.91	1.11	2.57	2.4	0.	0.
time (sec)	N/A	0.012	0.035	0.101	2.174	1.672	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	52	88	301	0	0
normalized size	1	1.	0.92	1.44	2.44	8.36	0.	0.
time (sec)	N/A	0.007	0.026	0.092	2.053	1.886	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	32	35	277	5	0
normalized size	1	1.	0.89	1.19	1.3	10.26	0.19	0.
time (sec)	N/A	0.003	0.025	0.083	1.643	1.937	133.827	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	41	18	84	36	0
normalized size	1	1.	0.91	1.17	0.51	2.4	1.03	0.
time (sec)	N/A	0.008	0.041	0.105	1.93	1.658	19.552	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	45	54	34	443	82	0
normalized size	1	1.	0.65	0.78	0.49	6.42	1.19	0.
time (sec)	N/A	0.014	0.066	0.123	2.044	1.977	49.173	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	52	57	135	0	0
normalized size	1	1.	0.59	0.68	0.75	1.78	0.	0.
time (sec)	N/A	0.017	0.085	0.107	2.049	1.718	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	55	74	66	552	0	0
normalized size	1	1.	0.51	0.69	0.62	5.16	0.	0.
time (sec)	N/A	0.027	0.121	0.139	1.871	2.154	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	53	112	929	539	0	0
normalized size	1	1.	0.68	1.44	11.91	6.91	0.	0.
time (sec)	N/A	0.02	0.051	0.117	2.12	1.957	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	39	90	84	0	0
normalized size	1	1.	0.91	1.11	2.57	2.4	0.	0.
time (sec)	N/A	0.012	0.033	0.106	1.856	1.638	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	52	88	301	0	0
normalized size	1	1.	0.92	1.44	2.44	8.36	0.	0.
time (sec)	N/A	0.008	0.022	0.088	1.949	1.904	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	32	35	277	0	0
normalized size	1	1.	0.89	1.19	1.3	10.26	0.	0.
time (sec)	N/A	0.003	0.026	0.077	1.772	1.956	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	41	18	84	0	0
normalized size	1	1.	1.	1.17	0.51	2.4	0.	0.
time (sec)	N/A	0.007	0.025	0.094	1.942	1.646	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	48	54	34	443	0	0
normalized size	1	1.	0.7	0.78	0.49	6.42	0.	0.
time (sec)	N/A	0.015	0.052	0.127	2.174	1.984	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	48	52	57	135	0	0
normalized size	1	1.	0.63	0.68	0.75	1.78	0.	0.
time (sec)	N/A	0.017	0.072	0.134	2.161	1.698	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	58	74	66	552	0	0
normalized size	1	1.	0.54	0.69	0.62	5.16	0.	0.
time (sec)	N/A	0.028	0.058	0.127	1.976	2.463	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.075	0.067	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	60	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.047	0.063	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.042	0.131	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.067	0.11	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	59	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.143	0.166	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.08	0.068	0.	0.	0.	0.



Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	60	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.049	0.064	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	57	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.008	0.055	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.009	0.106	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.054	0.194	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.06	0.065	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	60	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.047	0.092	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.054	0.091	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.115	0.068	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.07	0.142	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	60	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.043	0.087	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	58	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.015	0.085	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.004	0.059	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.017	0.069	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.099	0.138	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	83	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.092	0.102	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	83	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.138	0.094	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	83	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.141	0.086	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	83	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.141	0.082	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	83	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.13	0.091	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	83	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.183	0.082	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	76	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.065	0.594	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.047	0.328	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	65	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.04	0.396	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	61	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.042	0.243	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	68	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.094	0.859	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	71	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.095	0.968	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	73	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.108	1.35	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.104	0.12	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.092	0.117	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.101	0.128	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.117	0.128	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.139	0.12	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.18	0.123	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	31	66	0	45
normalized size	1	1.	1.	0.85	1.55	3.3	0.	2.25
time (sec)	N/A	0.035	0.055	0.021	1.127	1.633	0.	1.274

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	31	63	0	45
normalized size	1	1.	1.	0.85	1.55	3.15	0.	2.25
time (sec)	N/A	0.036	0.041	0.017	1.145	1.622	0.	1.279

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	31	38	0	34
normalized size	1	1.	1.	0.94	1.72	2.11	0.	1.89
time (sec)	N/A	0.036	0.033	0.018	1.158	1.663	0.	1.339

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	31	54	0	30
normalized size	1	1.	1.	0.94	1.72	3.	0.	1.67
time (sec)	N/A	0.033	0.038	0.025	1.058	1.658	0.	1.325

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	31	65	0	47
normalized size	1	1.	1.	0.85	1.55	3.25	0.	2.35
time (sec)	N/A	0.033	0.053	0.025	1.185	1.709	0.	3.622

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	32	357	49	97	0	66
normalized size	1	1.	0.78	8.71	1.2	2.37	0.	1.61
time (sec)	N/A	0.049	0.22	0.199	1.097	1.658	0.	1.189

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	36	51	105	0	66
normalized size	1	1.	0.98	0.84	1.19	2.44	0.	1.53
time (sec)	N/A	0.049	0.103	0.149	1.205	1.695	0.	1.343



Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	122	542	0	0	0	0
normalized size	1	1.	0.95	4.23	0.	0.	0.	0.
time (sec)	N/A	0.204	1.487	0.253	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	56	54	0	155	0	0
normalized size	1	1.	0.81	0.78	0.	2.25	0.	0.
time (sec)	N/A	0.097	0.116	0.191	0.	1.826	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	109	285	0	0	0	0
normalized size	1	1.	1.17	3.06	0.	0.	0.	0.
time (sec)	N/A	0.143	0.858	0.2	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	42	0	85	0	0
normalized size	1	1.	1.	1.35	0.	2.74	0.	0.
time (sec)	N/A	0.049	0.063	0.164	0.	1.709	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	68	157	0	0	0	0
normalized size	1	1.	1.28	2.96	0.	0.	0.	0.
time (sec)	N/A	0.093	0.671	0.196	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	171	276	0	0	0	0
normalized size	1	1.	0.63	1.02	0.	0.	0.	0.
time (sec)	N/A	0.14	1.326	0.148	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	80	188	0	0	0	0
normalized size	1	1.	0.86	2.02	0.	0.	0.	0.
time (sec)	N/A	0.147	0.673	0.186	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	222	522	0	0	0	0
normalized size	1	1.	0.69	1.62	0.	0.	0.	0.
time (sec)	N/A	0.212	1.789	0.225	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	57	64	0	203	0	0
normalized size	1	1.	0.55	0.62	0.	1.95	0.	0.
time (sec)	N/A	0.162	0.298	0.196	0.	2.447	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	114	996	0	0	0	0
normalized size	1	1.	0.69	6.	0.	0.	0.	0.
time (sec)	N/A	0.268	1.12	0.207	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	45	54	0	135	0	0
normalized size	1	1.	0.65	0.78	0.	1.96	0.	0.
time (sec)	N/A	0.103	0.135	0.162	0.	2.062	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	99	503	0	0	0	0
normalized size	1	1.	0.79	4.02	0.	0.	0.	0.
time (sec)	N/A	0.205	0.512	0.187	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	42	0	84	0	0
normalized size	1	1.	1.	1.35	0.	2.71	0.	0.
time (sec)	N/A	0.048	0.061	0.169	0.	1.658	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	66	505	0	0	0	0
normalized size	1	1.	0.74	5.67	0.	0.	0.	0.
time (sec)	N/A	0.143	0.367	0.191	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	64	656	0	0	0	0
normalized size	1	1.	0.2	2.01	0.	0.	0.	0.
time (sec)	N/A	0.219	0.242	0.171	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	69	520	0	0	0	0
normalized size	1	1.	0.73	5.53	0.	0.	0.	0.
time (sec)	N/A	0.153	0.407	0.17	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	92	563	0	0	0	0
normalized size	1	1.	0.55	3.39	0.	0.	0.	0.
time (sec)	N/A	0.265	1.423	0.232	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	57	64	0	207	0	0
normalized size	1	1.	0.54	0.6	0.	1.95	0.	0.
time (sec)	N/A	0.162	0.193	0.165	0.	1.871	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	87	304	0	0	0	0
normalized size	1	1.	0.66	2.32	0.	0.	0.	0.
time (sec)	N/A	0.204	0.652	0.195	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	45	54	0	132	0	0
normalized size	1	1.	0.65	0.78	0.	1.91	0.	0.
time (sec)	N/A	0.102	0.207	0.161	0.	1.55	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	68	190	0	0	0	0
normalized size	1	1.	0.73	2.04	0.	0.	0.	0.
time (sec)	N/A	0.141	0.579	0.206	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	0	127	0	0
normalized size	1	1.	1.	1.27	0.	3.85	0.	0.
time (sec)	N/A	0.049	0.112	0.149	0.	1.363	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	70	192	0	0	0	0
normalized size	1	1.	0.71	1.96	0.	0.	0.	0.
time (sec)	N/A	0.15	0.528	0.176	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	280	544	0	0	0	0
normalized size	1	1.	0.85	1.65	0.	0.	0.	0.
time (sec)	N/A	0.217	1.73	0.172	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	45	54	0	185	0	0
normalized size	1	1.	0.65	0.78	0.	2.68	0.	0.
time (sec)	N/A	0.1	0.258	0.164	0.	2.328	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	104	992	0	0	0	0
normalized size	1	1.	0.81	7.75	0.	0.	0.	0.
time (sec)	N/A	0.193	1.048	0.207	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	0	116	0	0
normalized size	1	1.	1.	1.27	0.	3.52	0.	0.
time (sec)	N/A	0.048	0.112	0.153	0.	2.066	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	80	502	0	0	0	0
normalized size	1	1.	0.9	5.64	0.	0.	0.	0.
time (sec)	N/A	0.139	0.466	0.185	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	55	322	0	0	0	0
normalized size	1	1.	0.2	1.19	0.	0.	0.	0.
time (sec)	N/A	0.131	0.125	0.184	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	66	517	0	0	0	0
normalized size	1	1.	1.25	9.75	0.	0.	0.	0.
time (sec)	N/A	0.088	0.218	0.172	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	66	668	0	0	0	0
normalized size	1	1.	0.2	2.07	0.	0.	0.	0.
time (sec)	N/A	0.204	0.256	0.17	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	84	531	0	0	0	0
normalized size	1	1.	0.88	5.59	0.	0.	0.	0.
time (sec)	N/A	0.141	0.354	0.213	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	57	54	0	203	0	0
normalized size	1	1.	0.52	0.49	0.	1.85	0.	0.
time (sec)	N/A	0.154	0.281	0.158	0.	2.212	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	119	550	0	0	0	0
normalized size	1	1.	0.88	4.07	0.	0.	0.	0.
time (sec)	N/A	0.2	1.422	0.193	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	45	42	0	131	0	0
normalized size	1	1.	1.36	1.27	0.	3.97	0.	0.
time (sec)	N/A	0.053	0.133	0.142	0.	1.818	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	105	290	0	0	0	0
normalized size	1	1.	1.07	2.96	0.	0.	0.	0.
time (sec)	N/A	0.148	0.798	0.173	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	199	975	0	0	0	0
normalized size	1	1.	0.61	2.98	0.	0.	0.	0.
time (sec)	N/A	0.215	0.635	0.178	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	195	0	0	0	0
normalized size	1	1.	0.91	2.12	0.	0.	0.	0.
time (sec)	N/A	0.143	0.641	0.19	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	223	526	0	0	0	0
normalized size	1	1.	0.69	1.63	0.	0.	0.	0.
time (sec)	N/A	0.213	1.973	0.19	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	89	222	0	0	0	0
normalized size	1	1.	0.66	1.64	0.	0.	0.	0.
time (sec)	N/A	0.207	0.555	0.181	0.	0.	0.	0.



Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	246	556	0	0	0	0
normalized size	1	1.	0.66	1.5	0.	0.	0.	0.
time (sec)	N/A	0.297	2.304	0.174	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	45	42	0	151	0	0
normalized size	1	1.	1.36	1.27	0.	4.58	0.	0.
time (sec)	N/A	0.054	0.158	0.138	0.	2.309	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	101	993	0	0	0	0
normalized size	1	1.	0.75	7.36	0.	0.	0.	0.
time (sec)	N/A	0.202	1.802	0.196	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	55	1259	0	0	0	0
normalized size	1	1.	0.17	3.83	0.	0.	0.	0.
time (sec)	N/A	0.214	0.2	0.179	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	80	515	0	0	0	0
normalized size	1	1.	0.85	5.48	0.	0.	0.	0.
time (sec)	N/A	0.147	0.661	0.169	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	70	668	0	0	0	0
normalized size	1	1.	0.22	2.07	0.	0.	0.	0.
time (sec)	N/A	0.203	0.221	0.175	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	79	530	0	0	0	0
normalized size	1	1.	0.83	5.58	0.	0.	0.	0.
time (sec)	N/A	0.142	0.363	0.192	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	81	696	0	0	0	0
normalized size	1	1.	0.22	1.88	0.	0.	0.	0.
time (sec)	N/A	0.291	0.379	0.183	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	90	544	0	0	0	0
normalized size	1	1.	0.67	4.03	0.	0.	0.	0.
time (sec)	N/A	0.206	0.658	0.165	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	406	95	722	0	0	0	0
normalized size	1	1.	0.23	1.78	0.	0.	0.	0.
time (sec)	N/A	0.39	0.411	0.221	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	278	0	0	0	0	0
normalized size	1	1.	3.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	2.249	0.565	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	280	0	0	0	0	0
normalized size	1	1.	3.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.549	0.509	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	281	0	0	0	0	0
normalized size	1	1.	3.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.312	0.529	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	283	0	0	0	0	0
normalized size	1	1.	3.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.188	0.558	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.051	0.338	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	51	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.043	0.323	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.033	0.31	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	23	66	39	59	0	0
normalized size	1	1.	0.96	2.75	1.62	2.46	0.	0.
time (sec)	N/A	0.034	0.025	0.07	1.149	1.699	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	45	0	78	115	0	0
normalized size	1	1.	0.87	0.	1.5	2.21	0.	0.
time (sec)	N/A	0.052	0.126	1.153	1.155	1.747	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	81	0	116	178	0	0
normalized size	1	1.	1.04	0.	1.49	2.28	0.	0.
time (sec)	N/A	0.064	0.554	1.199	1.143	1.763	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	77	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.577	0.352	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	77	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.509	0.351	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	75	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	0.443	0.323	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	65	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.1	0.471	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	165	0	0	0	0	0
normalized size	1	1.	2.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.457	0.946	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	246	0	0	0	0	0
normalized size	1	1.	3.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.62	1.065	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	92	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	1.827	0.159	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	90	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	1.61	0.152	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	326	0	0	0	0	0
normalized size	1	1.	4.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	3.101	0.144	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	115	0	0	0	0	0
normalized size	1	1.	1.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	1.006	0.136	0.	0.	0.	0.

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [270] had the largest ratio of [ 0.44 ]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	6	0.167
2	A	2	2	1.	8	0.25
3	A	2	2	1.	8	0.25
4	A	2	1	1.	8	0.125
5	A	3	2	1.	8	0.25
6	A	2	1	1.	8	0.125
7	A	4	2	1.	8	0.25
8	A	2	1	1.	8	0.125
9	A	4	3	1.	10	0.3
10	A	3	3	1.	10	0.3
11	A	3	3	1.	10	0.3
12	A	2	2	1.	10	0.2
13	A	2	2	1.	10	0.2
14	A	3	3	1.	10	0.3
15	A	3	3	1.	10	0.3
16	A	4	3	1.	10	0.3
17	A	4	3	1.	12	0.25
18	A	3	3	1.	12	0.25
19	A	3	3	1.	12	0.25
20	A	2	2	1.	12	0.167
21	A	2	2	1.	12	0.167
22	A	3	3	1.	12	0.25
23	A	3	3	1.	12	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	4	3	1.	12	0.25
25	A	2	2	1.	10	0.2
26	A	2	2	1.	10	0.2
27	A	2	2	1.	10	0.2
28	A	2	2	1.	10	0.2
29	A	2	2	1.	10	0.2
30	A	2	2	1.	10	0.2
31	A	2	2	1.	12	0.167
32	A	2	2	1.	12	0.167
33	A	2	2	1.	12	0.167
34	A	2	2	1.	12	0.167
35	A	2	2	1.	12	0.167
36	A	2	2	1.	12	0.167
37	A	2	2	1.	8	0.25
38	A	2	2	1.	10	0.2
39	A	5	3	1.	8	0.375
40	A	4	3	1.	8	0.375
41	A	3	3	1.	8	0.375
42	A	2	2	1.	8	0.25
43	A	2	2	1.	8	0.25
44	A	3	3	1.	8	0.375
45	A	4	3	1.	8	0.375
46	A	5	3	1.	8	0.375
47	A	6	4	1.	10	0.4
48	A	5	4	1.	10	0.4
49	A	4	4	1.	10	0.4
50	A	3	3	1.	10	0.3
51	A	2	2	1.	10	0.2
52	A	3	3	1.	10	0.3
53	A	4	3	1.	10	0.3
54	A	5	3	1.	10	0.3
55	A	7	4	1.	10	0.4

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	5	4	1.	10	0.4
57	A	4	4	1.	10	0.4
58	A	4	4	1.	10	0.4
59	A	5	4	1.	10	0.4
60	A	7	4	1.	10	0.4
61	A	3	2	1.	10	0.2
62	A	3	2	1.	10	0.2
63	A	3	2	1.	10	0.2
64	A	3	3	1.	10	0.3
65	A	3	3	1.	10	0.3
66	A	5	3	1.	10	0.3
67	A	7	3	1.	10	0.3
68	A	3	3	1.	12	0.25
69	A	3	3	1.	14	0.214
70	A	5	4	1.	21	0.19
71	A	5	4	1.	21	0.19
72	A	4	4	1.	21	0.19
73	A	4	4	1.	19	0.21
74	A	2	2	1.	12	0.167
75	A	3	3	1.	19	0.158
76	A	4	4	1.	21	0.19
77	A	4	4	1.	21	0.19
78	A	5	4	1.	21	0.19
79	A	5	4	1.	21	0.19
80	A	5	4	1.	21	0.19
81	A	5	4	1.	21	0.19
82	A	4	4	1.	19	0.21
83	A	3	3	1.	12	0.25
84	A	3	3	1.	19	0.158
85	A	3	3	1.	21	0.143
86	A	4	4	1.	21	0.19
87	A	4	4	1.	21	0.19

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	5	4	1.	21	0.19
89	A	5	4	1.	21	0.19
90	A	5	4	1.	21	0.19
91	A	5	4	1.	19	0.21
92	A	3	3	1.	12	0.25
93	A	4	4	1.	19	0.21
94	A	3	3	1.	21	0.143
95	A	3	3	1.	21	0.143
96	A	4	4	1.	21	0.19
97	A	4	4	1.	21	0.19
98	A	5	4	1.	21	0.19
99	A	5	4	1.	21	0.19
100	A	4	3	1.	12	0.25
101	A	5	4	1.	21	0.19
102	A	5	4	1.	21	0.19
103	A	4	4	1.	21	0.19
104	A	4	4	1.	21	0.19
105	A	3	3	1.	19	0.158
106	A	2	2	1.	12	0.167
107	A	4	4	1.	19	0.21
108	A	4	4	1.	21	0.19
109	A	5	4	1.	21	0.19
110	A	5	4	1.	21	0.19
111	A	5	4	1.	21	0.19
112	A	5	4	1.	21	0.19
113	A	4	4	1.	21	0.19
114	A	4	4	1.	21	0.19
115	A	3	3	1.	21	0.143
116	A	3	3	1.	19	0.158
117	A	3	3	1.	12	0.25
118	A	4	4	1.	19	0.21
119	A	5	4	1.	21	0.19

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	5	4	1.	21	0.19
121	A	5	4	1.	21	0.19
122	A	5	4	1.	21	0.19
123	A	4	4	1.	21	0.19
124	A	4	4	1.	21	0.19
125	A	3	3	1.	21	0.143
126	A	3	3	1.	21	0.143
127	A	4	4	1.	19	0.21
128	A	3	3	1.	12	0.25
129	A	5	4	1.	19	0.21
130	A	5	4	1.	21	0.19
131	A	4	3	1.	12	0.25
132	A	4	3	1.	23	0.13
133	A	3	2	1.	23	0.087
134	A	3	3	1.	23	0.13
135	A	3	3	1.	23	0.13
136	A	2	2	1.	23	0.087
137	A	2	2	1.	23	0.087
138	A	2	2	1.	23	0.087
139	A	3	3	1.	23	0.13
140	A	3	2	1.	23	0.087
141	A	4	3	1.	23	0.13
142	A	4	3	1.	23	0.13
143	A	3	2	1.	23	0.087
144	A	3	3	1.	23	0.13
145	A	3	3	1.	23	0.13
146	A	2	2	1.	23	0.087
147	A	2	2	1.	23	0.087
148	A	2	2	1.	23	0.087
149	A	3	3	1.	23	0.13
150	A	3	2	1.	23	0.087
151	A	4	3	1.	23	0.13

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	3	2	1.	23	0.087
153	A	3	2	1.	23	0.087
154	A	3	3	1.	23	0.13
155	A	3	3	1.	23	0.13
156	A	2	2	1.	23	0.087
157	A	2	2	1.	23	0.087
158	A	2	2	1.	23	0.087
159	A	3	3	1.	23	0.13
160	A	3	2	1.	23	0.087
161	A	3	3	1.	23	0.13
162	A	3	3	1.	23	0.13
163	A	2	2	1.	23	0.087
164	A	2	2	1.	23	0.087
165	A	2	2	1.	23	0.087
166	A	3	3	1.	23	0.13
167	A	3	2	1.	23	0.087
168	A	3	3	1.	23	0.13
169	A	3	3	1.	23	0.13
170	A	2	2	1.	23	0.087
171	A	2	2	1.	23	0.087
172	A	2	2	1.	23	0.087
173	A	3	3	1.	23	0.13
174	A	3	2	1.	23	0.087
175	A	4	3	1.	23	0.13
176	A	3	3	1.	23	0.13
177	A	3	3	1.	23	0.13
178	A	2	2	1.	23	0.087
179	A	2	2	1.	23	0.087
180	A	2	2	1.	23	0.087
181	A	3	3	1.	23	0.13
182	A	3	2	1.	23	0.087
183	A	4	3	1.	23	0.13

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	3	3	1.	21	0.143
185	A	3	3	1.	19	0.158
186	A	2	2	1.	12	0.167
187	A	3	3	1.	19	0.158
188	A	3	3	1.	21	0.143
189	A	3	3	1.	21	0.143
190	A	3	3	1.	19	0.158
191	A	2	2	1.	12	0.167
192	A	3	3	1.	19	0.158
193	A	3	3	1.	21	0.143
194	A	3	3	1.	21	0.143
195	A	3	3	1.	19	0.158
196	A	2	2	1.	12	0.167
197	A	3	3	1.	19	0.158
198	A	3	3	1.	21	0.143
199	A	3	3	1.	21	0.143
200	A	3	3	1.	19	0.158
201	A	2	2	1.	12	0.167
202	A	3	3	1.	19	0.158
203	A	3	3	1.	21	0.143
204	A	3	3	1.	21	0.143
205	A	3	3	1.	21	0.143
206	A	3	3	1.	21	0.143
207	A	3	3	1.	21	0.143
208	A	3	3	1.	21	0.143
209	A	3	3	1.	21	0.143
210	A	3	3	1.	19	0.158
211	A	3	3	1.	19	0.158
212	A	3	3	1.	17	0.176
213	A	2	2	1.	10	0.2
214	A	3	3	1.	17	0.176
215	A	3	3	1.	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	3	3	1.	19	0.158
217	A	3	3	1.	21	0.143
218	A	3	3	1.	21	0.143
219	A	3	3	1.	21	0.143
220	A	3	3	1.	21	0.143
221	A	3	3	1.	21	0.143
222	A	3	3	1.	21	0.143
223	A	2	2	1.	19	0.105
224	A	2	2	1.	19	0.105
225	A	2	2	1.	19	0.105
226	A	2	2	1.	19	0.105
227	A	2	2	1.	19	0.105
228	A	3	2	1.	21	0.095
229	A	3	2	1.	21	0.095
230	A	5	4	1.	25	0.16
231	A	2	2	1.	25	0.08
232	A	4	4	1.	25	0.16
233	A	1	1	1.	25	0.04
234	A	3	3	1.	25	0.12
235	A	12	9	1.	25	0.36
236	A	4	4	1.	25	0.16
237	A	13	10	1.	25	0.4
238	A	3	2	1.	25	0.08
239	A	6	5	1.	25	0.2
240	A	2	2	1.	25	0.08
241	A	5	5	1.	25	0.2
242	A	1	1	1.	25	0.04
243	A	4	4	1.	25	0.16
244	A	13	10	1.	25	0.4
245	A	4	4	1.	25	0.16
246	A	6	5	1.	25	0.2
247	A	3	3	1.	25	0.12

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
248	A	5	5	1.	25	0.2
249	A	2	2	1.	25	0.08
250	A	4	4	1.	25	0.16
251	A	1	1	1.	25	0.04
252	A	4	4	1.	25	0.16
253	A	13	10	1.	25	0.4
254	A	2	2	1.	25	0.08
255	A	5	4	1.	25	0.16
256	A	1	1	1.	25	0.04
257	A	4	4	1.	25	0.16
258	A	12	9	1.	25	0.36
259	A	3	3	1.	25	0.12
260	A	13	10	1.	25	0.4
261	A	4	4	1.	25	0.16
262	A	3	3	1.	25	0.12
263	A	5	5	1.	25	0.2
264	A	1	1	1.	25	0.04
265	A	4	4	1.	25	0.16
266	A	13	10	1.	25	0.4
267	A	4	4	1.	25	0.16
268	A	13	10	1.	25	0.4
269	A	5	5	1.	25	0.2
270	A	14	11	1.	25	0.44
271	A	1	1	1.	25	0.04
272	A	5	5	1.	25	0.2
273	A	13	10	1.	25	0.4
274	A	4	4	1.	25	0.16
275	A	13	10	1.	25	0.4
276	A	4	4	1.	25	0.16
277	A	14	11	1.	25	0.44
278	A	5	5	1.	25	0.2
279	A	15	11	1.	25	0.44

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	2	2	1.	17	0.118
281	A	2	2	1.	19	0.105
282	A	2	2	1.	19	0.105
283	A	2	2	1.	21	0.095
284	A	2	2	1.	19	0.105
285	A	2	2	1.	19	0.105
286	A	2	2	1.	17	0.118
287	A	2	2	1.	17	0.118
288	A	3	2	1.	19	0.105
289	A	3	2	1.	19	0.105
290	A	2	2	1.	19	0.105
291	A	2	2	1.	19	0.105
292	A	2	2	1.	19	0.105
293	A	2	2	1.	10	0.2
294	A	2	2	1.	19	0.105
295	A	2	2	1.	19	0.105
296	A	2	2	1.	23	0.087
297	A	2	2	1.	23	0.087
298	A	2	2	1.	23	0.087
299	A	2	2	1.	23	0.087



# Chapter 3

## Listing of integrals

### 3.1 $\int \sec(a + bx) dx$

Optimal. Leaf size=11

$$\frac{\tanh^{-1}(\sin(a + bx))}{b}$$

[Out] ArcTanh[Sin[a + b\*x]]/b

---

**Rubi [A]** time = 0.0042149, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3770}

$$\frac{\tanh^{-1}(\sin(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x], x]

[Out] ArcTanh[Sin[a + b\*x]]/b

#### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x]  
/; FreeQ[{c, d}, x]

Rubi steps

$$\int \sec(a + bx) dx = \frac{\tanh^{-1}(\sin(a + bx))}{b}$$

**Mathematica [A]** time = 0.0022212, size = 11, normalized size = 1.

$$\frac{\tanh^{-1}(\sin(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*x], x]

[Out] ArcTanh[Sin[a + b\*x]]/b

**Maple [A]** time = 0.003, size = 19, normalized size = 1.7

$$\frac{\ln(\sec(bx + a) + \tan(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b\*x+a), x)

[Out] 1/b\*ln(sec(b\*x+a)+tan(b\*x+a))

**Maxima [A]** time = 1.10381, size = 24, normalized size = 2.18

$$\frac{\log(\sec(bx + a) + \tan(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a), x, algorithm="maxima")

[Out] log(sec(b\*x + a) + tan(b\*x + a))/b

---

**Fricas [B]** time = 1.52565, size = 76, normalized size = 6.91

$$\frac{\log(\sin(bx + a) + 1) - \log(-\sin(bx + a) + 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a),x, algorithm="fricas")

[Out] 1/2\*(log(sin(b\*x + a) + 1) - log(-sin(b\*x + a) + 1))/b

---

**Sympy [A]** time = 5.91471, size = 36, normalized size = 3.27

$$\begin{cases} \frac{\log(\tan(a+bx)+\sec(a+bx))}{b} & \text{for } b \neq 0 \\ x \frac{\tan(a)\sec(a)+\sec^2(a)}{\tan(a)+\sec(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a),x)

[Out] Piecewise((log(tan(a + b\*x) + sec(a + b\*x))/b, Ne(b, 0)), (x\*(tan(a)\*sec(a) + sec(a)\*\*2)/(tan(a) + sec(a)), True))

---

**Giac [B]** time = 1.31224, size = 59, normalized size = 5.36

$$\frac{\log\left(\left|\frac{1}{\sin(bx+a)} + \sin(bx+a) + 2\right|\right) - \log\left(\left|\frac{1}{\sin(bx+a)} + \sin(bx+a) - 2\right|\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a),x, algorithm="giac")

[Out] 1/4\*(log(abs(1/sin(b\*x + a) + sin(b\*x + a) + 2)) - log(abs(1/sin(b\*x + a) + sin(b\*x + a) - 2)))/b

## 3.2 $\int \sec^2(a + bx) dx$

**Optimal.** Leaf size=10

$$\frac{\tan(a + bx)}{b}$$

[Out] Tan[a + b\*x]/b

**Rubi [A]** time = 0.0087791, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3767, 8}

$$\frac{\tan(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^2,x]

[Out] Tan[a + b\*x]/b

### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned} \int \sec^2(a + bx) dx &= -\frac{\text{Subst}(\int 1 dx, x, -\tan(a + bx))}{b} \\ &= \frac{\tan(a + bx)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.0035802, size = 10, normalized size = 1.

$$\frac{\tan(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*x]^2,x]

[Out] Tan[a + b\*x]/b

---

**Maple [A]** time = 0.056, size = 11, normalized size = 1.1

$$\frac{\tan(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b\*x+a)^2,x)

[Out] tan(b\*x+a)/b

---

**Maxima [A]** time = 1.26571, size = 14, normalized size = 1.4

$$\frac{\tan(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^2,x, algorithm="maxima")

[Out] tan(b\*x + a)/b

---

**Fricas [A]** time = 1.40265, size = 42, normalized size = 4.2

$$\frac{\sin(bx + a)}{b \cos(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sin(b*x + a)/(b*cos(b*x + a))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2,x)
```

```
[Out] Integral(sec(a + b*x)**2, x)
```

---

**Giac [A]** time = 1.24984, size = 14, normalized size = 1.4

$$\frac{\tan(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] tan(b*x + a)/b
```

### 3.3 $\int \sec^3(a + bx) dx$

**Optimal.** Leaf size=34

$$\frac{\tanh^{-1}(\sin(a + bx))}{2b} + \frac{\tan(a + bx) \sec(a + bx)}{2b}$$

[Out] ArcTanh[Sin[a + b\*x]]/(2\*b) + (Sec[a + b\*x]\*Tan[a + b\*x])/(2\*b)

**Rubi [A]** time = 0.0143485, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3768, 3770}

$$\frac{\tanh^{-1}(\sin(a + bx))}{2b} + \frac{\tan(a + bx) \sec(a + bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^3, x]

[Out] ArcTanh[Sin[a + b\*x]]/(2\*b) + (Sec[a + b\*x]\*Tan[a + b\*x])/(2\*b)

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> -Simp[(b\*Csc[c + d\*x] \* (b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \sec^3(a + bx) dx &= \frac{\sec(a + bx) \tan(a + bx)}{2b} + \frac{1}{2} \int \sec(a + bx) dx \\ &= \frac{\tanh^{-1}(\sin(a + bx))}{2b} + \frac{\sec(a + bx) \tan(a + bx)}{2b} \end{aligned}$$

**Mathematica [A]** time = 0.0098694, size = 34, normalized size = 1.

$$\frac{\tanh^{-1}(\sin(a + bx))}{2b} + \frac{\tan(a + bx) \sec(a + bx)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*x]^3,x]

[Out] ArcTanh[Sin[a + b\*x]]/(2\*b) + (Sec[a + b\*x]\*Tan[a + b\*x])/(2\*b)

**Maple [A]** time = 0.133, size = 38, normalized size = 1.1

$$\frac{\sec(bx + a) \tan(bx + a)}{2b} + \frac{\ln(\sec(bx + a) + \tan(bx + a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b\*x+a)^3,x)

[Out] 1/2\*sec(b\*x+a)\*tan(b\*x+a)/b+1/2/b\*ln(sec(b\*x+a)+tan(b\*x+a))

**Maxima [A]** time = 1.00103, size = 62, normalized size = 1.82

$$\frac{\frac{2 \sin(bx+a)}{\sin(bx+a)^2-1} - \log(\sin(bx+a) + 1) + \log(\sin(bx+a) - 1)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^3,x, algorithm="maxima")

[Out] -1/4\*(2\*sin(b\*x + a)/(sin(b\*x + a)^2 - 1) - log(sin(b\*x + a) + 1) + log(sin(b\*x + a) - 1))/b

**Fricas [B]** time = 1.41226, size = 162, normalized size = 4.76

$$\frac{\cos(bx + a)^2 \log(\sin(bx + a) + 1) - \cos(bx + a)^2 \log(-\sin(bx + a) + 1) + 2 \sin(bx + a)}{4b \cos(bx + a)^2}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(cos(b*x + a)^2*log(sin(b*x + a) + 1) - cos(b*x + a)^2*log(-sin(b*x + a) + 1) + 2*sin(b*x + a))/(b*cos(b*x + a)^2)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3,x)
```

```
[Out] Integral(sec(a + b*x)**3, x)
```

**Giac [A]** time = 1.35288, size = 65, normalized size = 1.91

$$\frac{\frac{2 \sin(bx+a)}{\sin(bx+a)^2-1} - \log(|\sin(bx+a)+1|) + \log(|\sin(bx+a)-1|)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(abs(sin(b*x + a) + 1)) + log(abs(sin(b*x + a) - 1)))/b
```

### 3.4 $\int \sec^4(a + bx) dx$

**Optimal.** Leaf size=26

$$\frac{\tan^3(a + bx)}{3b} + \frac{\tan(a + bx)}{b}$$

[Out] Tan[a + b\*x]/b + Tan[a + b\*x]^3/(3\*b)

**Rubi [A]** time = 0.0112607, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {3767}

$$\frac{\tan^3(a + bx)}{3b} + \frac{\tan(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^4,x]

[Out] Tan[a + b\*x]/b + Tan[a + b\*x]^3/(3\*b)

#### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

#### Rubi steps

$$\begin{aligned} \int \sec^4(a + bx) dx &= -\frac{\text{Subst}\left(\int (1 + x^2) dx, x, -\tan(a + bx)\right)}{b} \\ &= \frac{\tan(a + bx)}{b} + \frac{\tan^3(a + bx)}{3b} \end{aligned}$$

**Mathematica [A]** time = 0.0409817, size = 23, normalized size = 0.88

$$\frac{\frac{1}{3} \tan^3(a + bx) + \tan(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*x]^4,x]

[Out] (Tan[a + b\*x] + Tan[a + b\*x]^3/3)/b

**Maple [A]** time = 0.043, size = 24, normalized size = 0.9

$$-\frac{\tan(bx+a)}{b} \left( -\frac{2}{3} - \frac{(\sec(bx+a))^2}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b\*x+a)^4,x)

[Out] -1/b\*(-2/3-1/3\*sec(b\*x+a)^2)\*tan(b\*x+a)

**Maxima [A]** time = 1.16608, size = 30, normalized size = 1.15

$$\frac{\tan(bx+a)^3 + 3 \tan(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^4,x, algorithm="maxima")

[Out] 1/3\*(tan(b\*x + a)^3 + 3\*tan(b\*x + a))/b

**Fricas [A]** time = 1.34596, size = 81, normalized size = 3.12

$$\frac{(2 \cos(bx+a)^2 + 1) \sin(bx+a)}{3b \cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^4,x, algorithm="fricas")

[Out]  $1/3*(2*\cos(b*x + a)^2 + 1)*\sin(b*x + a)/(b*\cos(b*x + a)^3)$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec^4(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(b*x+a)**4,x)`

[Out] `Integral(sec(a + b*x)**4, x)`

---

**Giac [A]** time = 1.32413, size = 30, normalized size = 1.15

$$\frac{\tan(bx + a)^3 + 3 \tan(bx + a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(b*x+a)^4,x, algorithm="giac")`

[Out]  $1/3*(\tan(b*x + a)^3 + 3*\tan(b*x + a))/b$

### 3.5 $\int \sec^5(a + bx) dx$

**Optimal.** Leaf size=55

$$\frac{3 \tanh^{-1}(\sin(a + bx))}{8b} + \frac{\tan(a + bx) \sec^3(a + bx)}{4b} + \frac{3 \tan(a + bx) \sec(a + bx)}{8b}$$

[Out] (3\*ArcTanh[Sin[a + b\*x]])/(8\*b) + (3\*Sec[a + b\*x]\*Tan[a + b\*x])/(8\*b) + (Sec[a + b\*x]^3\*Tan[a + b\*x])/(4\*b)

**Rubi [A]** time = 0.0250803, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3768, 3770}

$$\frac{3 \tanh^{-1}(\sin(a + bx))}{8b} + \frac{\tan(a + bx) \sec^3(a + bx)}{4b} + \frac{3 \tan(a + bx) \sec(a + bx)}{8b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^5, x]

[Out] (3\*ArcTanh[Sin[a + b\*x]])/(8\*b) + (3\*Sec[a + b\*x]\*Tan[a + b\*x])/(8\*b) + (Sec[a + b\*x]^3\*Tan[a + b\*x])/(4\*b)

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x])\* (b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned}
\int \sec^5(a + bx) dx &= \frac{\sec^3(a + bx) \tan(a + bx)}{4b} + \frac{3}{4} \int \sec^3(a + bx) dx \\
&= \frac{3 \sec(a + bx) \tan(a + bx)}{8b} + \frac{\sec^3(a + bx) \tan(a + bx)}{4b} + \frac{3}{8} \int \sec(a + bx) dx \\
&= \frac{3 \tanh^{-1}(\sin(a + bx))}{8b} + \frac{3 \sec(a + bx) \tan(a + bx)}{8b} + \frac{\sec^3(a + bx) \tan(a + bx)}{4b}
\end{aligned}$$

**Mathematica [A]** time = 0.0714815, size = 42, normalized size = 0.76

$$\frac{3 \tanh^{-1}(\sin(a + bx)) + \tan(a + bx) \sec(a + bx) (2 \sec^2(a + bx) + 3)}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*x]^5,x]

[Out] (3\*ArcTanh[Sin[a + b\*x]] + Sec[a + b\*x]\*(3 + 2\*Sec[a + b\*x]^2)\*Tan[a + b\*x])/ (8\*b)

**Maple [A]** time = 0.044, size = 57, normalized size = 1.

$$\frac{(\sec(bx + a))^3 \tan(bx + a)}{4b} + \frac{3 \sec(bx + a) \tan(bx + a)}{8b} + \frac{3 \ln(\sec(bx + a) + \tan(bx + a))}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b\*x+a)^5,x)

[Out] 1/4\*sec(b\*x+a)^3\*tan(b\*x+a)/b+3/8\*sec(b\*x+a)\*tan(b\*x+a)/b+3/8/b\*ln(sec(b\*x+a)+tan(b\*x+a))

**Maxima [A]** time = 1.03009, size = 96, normalized size = 1.75

$$\frac{2(3 \sin(bx+a)^3 - 5 \sin(bx+a))}{\sin(bx+a)^4 - 2 \sin(bx+a)^2 + 1} - 3 \log(\sin(bx + a) + 1) + 3 \log(\sin(bx + a) - 1)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^5,x, algorithm="maxima")

[Out] 
$$-1/16*(2*(3*\sin(b*x + a)^3 - 5*\sin(b*x + a))/(\sin(b*x + a)^4 - 2*\sin(b*x + a)^2 + 1) - 3*\log(\sin(b*x + a) + 1) + 3*\log(\sin(b*x + a) - 1))/b$$

**Fricas [A]** time = 1.42324, size = 200, normalized size = 3.64

$$\frac{3 \cos(bx + a)^4 \log(\sin(bx + a) + 1) - 3 \cos(bx + a)^4 \log(-\sin(bx + a) + 1) + 2(3 \cos(bx + a)^2 + 2) \sin(bx + a)}{16 b \cos(bx + a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^5,x, algorithm="fricas")

[Out] 
$$1/16*(3*\cos(b*x + a)^4*\log(\sin(b*x + a) + 1) - 3*\cos(b*x + a)^4*\log(-\sin(b*x + a) + 1) + 2*(3*\cos(b*x + a)^2 + 2)*\sin(b*x + a))/(b*\cos(b*x + a)^4)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec^5(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)\*\*5,x)

[Out] Integral(sec(a + b\*x)\*\*5, x)

**Giac [A]** time = 1.31957, size = 85, normalized size = 1.55

$$\frac{2(3 \sin(bx+a)^3 - 5 \sin(bx+a))}{(\sin(bx+a)^2 - 1)^2} - 3 \log(|\sin(bx + a) + 1|) + 3 \log(|\sin(bx + a) - 1|)$$


---


$$16 b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/16*(2*(3*sin(b*x + a)^3 - 5*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 - 3*log  
(abs(sin(b*x + a) + 1)) + 3*log(abs(sin(b*x + a) - 1)))/b
```



### 3.6 $\int \sec^6(a + bx) dx$

**Optimal.** Leaf size=41

$$\frac{\tan^5(a + bx)}{5b} + \frac{2 \tan^3(a + bx)}{3b} + \frac{\tan(a + bx)}{b}$$

[Out]  $\text{Tan}[a + b*x]/b + (2*\text{Tan}[a + b*x]^3)/(3*b) + \text{Tan}[a + b*x]^5/(5*b)$

**Rubi [A]** time = 0.0148022, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {3767}

$$\frac{\tan^5(a + bx)}{5b} + \frac{2 \tan^3(a + bx)}{3b} + \frac{\tan(a + bx)}{b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sec}[a + b*x]^6, x]$

[Out]  $\text{Tan}[a + b*x]/b + (2*\text{Tan}[a + b*x]^3)/(3*b) + \text{Tan}[a + b*x]^5/(5*b)$

#### Rule 3767

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[n/2, 0]$

#### Rubi steps

$$\begin{aligned} \int \sec^6(a + bx) dx &= -\frac{\text{Subst}\left(\int (1 + 2x^2 + x^4) dx, x, -\tan(a + bx)\right)}{b} \\ &= \frac{\tan(a + bx)}{b} + \frac{2 \tan^3(a + bx)}{3b} + \frac{\tan^5(a + bx)}{5b} \end{aligned}$$

**Mathematica [A]** time = 0.100569, size = 35, normalized size = 0.85

$$\frac{\frac{1}{5} \tan^5(a + bx) + \frac{2}{3} \tan^3(a + bx) + \tan(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*x]^6,x]

[Out] (Tan[a + b\*x] + (2\*Tan[a + b\*x]^3)/3 + Tan[a + b\*x]^5/5)/b

**Maple [A]** time = 0.043, size = 34, normalized size = 0.8

$$-\frac{\tan (bx+a)}{b}\left(-\frac{8}{15}-\frac{(\sec (bx+a))^4}{5}-\frac{4(\sec (bx+a))^2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b\*x+a)^6,x)

[Out] -1/b\*(-8/15-1/5\*sec(b\*x+a)^4-4/15\*sec(b\*x+a)^2)\*tan(b\*x+a)

**Maxima [A]** time = 1.20084, size = 46, normalized size = 1.12

$$\frac{3 \tan (bx+a)^5+10 \tan (bx+a)^3+15 \tan (bx+a)}{15 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^6,x, algorithm="maxima")

[Out] 1/15\*(3\*tan(b\*x + a)^5 + 10\*tan(b\*x + a)^3 + 15\*tan(b\*x + a))/b

**Fricas [A]** time = 1.36232, size = 108, normalized size = 2.63

$$\frac{(8 \cos (bx+a)^4+4 \cos (bx+a)^2+3) \sin (bx+a)}{15 b \cos (bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^6,x, algorithm="fricas")

[Out]  $1/15*(8*\cos(b*x + a)^4 + 4*\cos(b*x + a)^2 + 3)*\sin(b*x + a)/(b*\cos(b*x + a)^5)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec^6(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(b*x+a)**6,x)`

[Out] `Integral(sec(a + b*x)**6, x)`

**Giac [A]** time = 1.13672, size = 46, normalized size = 1.12

$$\frac{3 \tan(bx + a)^5 + 10 \tan(bx + a)^3 + 15 \tan(bx + a)}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(b*x+a)^6,x, algorithm="giac")`

[Out]  $1/15*(3*\tan(b*x + a)^5 + 10*\tan(b*x + a)^3 + 15*\tan(b*x + a))/b$

### 3.7 $\int \sec^7(a + bx) dx$

**Optimal.** Leaf size=76

$$\frac{5 \tanh^{-1}(\sin(a + bx))}{16b} + \frac{\tan(a + bx) \sec^5(a + bx)}{6b} + \frac{5 \tan(a + bx) \sec^3(a + bx)}{24b} + \frac{5 \tan(a + bx) \sec(a + bx)}{16b}$$

[Out] (5\*ArcTanh[Sin[a + b\*x]])/(16\*b) + (5\*Sec[a + b\*x]\*Tan[a + b\*x])/(16\*b) + (5\*Sec[a + b\*x]^3\*Tan[a + b\*x])/(24\*b) + (Sec[a + b\*x]^5\*Tan[a + b\*x])/(6\*b)

**Rubi [A]** time = 0.0386378, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3768, 3770}

$$\frac{5 \tanh^{-1}(\sin(a + bx))}{16b} + \frac{\tan(a + bx) \sec^5(a + bx)}{6b} + \frac{5 \tan(a + bx) \sec^3(a + bx)}{24b} + \frac{5 \tan(a + bx) \sec(a + bx)}{16b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^7,x]

[Out] (5\*ArcTanh[Sin[a + b\*x]])/(16\*b) + (5\*Sec[a + b\*x]\*Tan[a + b\*x])/(16\*b) + (5\*Sec[a + b\*x]^3\*Tan[a + b\*x])/(24\*b) + (Sec[a + b\*x]^5\*Tan[a + b\*x])/(6\*b)

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^n], x\_Symbol] := -Simp[(b\*Csc[c + d\*x] \* (b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned}
\int \sec^7(a + bx) dx &= \frac{\sec^5(a + bx) \tan(a + bx)}{6b} + \frac{5}{6} \int \sec^5(a + bx) dx \\
&= \frac{5 \sec^3(a + bx) \tan(a + bx)}{24b} + \frac{\sec^5(a + bx) \tan(a + bx)}{6b} + \frac{5}{8} \int \sec^3(a + bx) dx \\
&= \frac{5 \sec(a + bx) \tan(a + bx)}{16b} + \frac{5 \sec^3(a + bx) \tan(a + bx)}{24b} + \frac{\sec^5(a + bx) \tan(a + bx)}{6b} + \frac{5}{16} \int \sec(a + bx) dx \\
&= \frac{5 \tanh^{-1}(\sin(a + bx))}{16b} + \frac{5 \sec(a + bx) \tan(a + bx)}{16b} + \frac{5 \sec^3(a + bx) \tan(a + bx)}{24b} + \frac{\sec^5(a + bx) \tan(a + bx)}{6b}
\end{aligned}$$

**Mathematica [A]** time = 0.166078, size = 52, normalized size = 0.68

$$\frac{15 \tanh^{-1}(\sin(a + bx)) + \tan(a + bx) \sec(a + bx) (8 \sec^4(a + bx) + 10 \sec^2(a + bx) + 15)}{48b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*x]^7, x]

[Out] (15\*ArcTanh[Sin[a + b\*x]] + Sec[a + b\*x]\*(15 + 10\*Sec[a + b\*x]^2 + 8\*Sec[a + b\*x]^4)\*Tan[a + b\*x])/(48\*b)

**Maple [A]** time = 0.046, size = 76, normalized size = 1.

$$\frac{(\sec(bx + a))^5 \tan(bx + a)}{6b} + \frac{5 (\sec(bx + a))^3 \tan(bx + a)}{24b} + \frac{5 \sec(bx + a) \tan(bx + a)}{16b} + \frac{5 \ln(\sec(bx + a) + \tan(bx + a))}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b\*x+a)^7, x)

[Out] 1/6\*sec(b\*x+a)^5\*tan(b\*x+a)/b+5/24\*sec(b\*x+a)^3\*tan(b\*x+a)/b+5/16\*sec(b\*x+a)\*tan(b\*x+a)/b+5/16/b\*ln(sec(b\*x+a)+tan(b\*x+a))

**Maxima [A]** time = 1.07036, size = 123, normalized size = 1.62

$$\frac{2(15 \sin(bx+a)^5 - 40 \sin(bx+a)^3 + 33 \sin(bx+a))}{\sin(bx+a)^6 - 3 \sin(bx+a)^4 + 3 \sin(bx+a)^2 - 1} - 15 \log(\sin(bx + a) + 1) + 15 \log(\sin(bx + a) - 1)$$

96 b

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^7,x, algorithm="maxima")

[Out] 
$$\frac{-1/96*(2*(15*\sin(b*x + a)^5 - 40*\sin(b*x + a)^3 + 33*\sin(b*x + a)))/(\sin(b*x + a)^6 - 3*\sin(b*x + a)^4 + 3*\sin(b*x + a)^2 - 1) - 15*\log(\sin(b*x + a) + 1) + 15*\log(\sin(b*x + a) - 1))/b$$

**Fricas [A]** time = 1.47555, size = 231, normalized size = 3.04

$$\frac{15 \cos(bx + a)^6 \log(\sin(bx + a) + 1) - 15 \cos(bx + a)^6 \log(-\sin(bx + a) + 1) + 2(15 \cos(bx + a)^4 + 10 \cos(bx + a)^2)}{96 b \cos(bx + a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^7,x, algorithm="fricas")

[Out] 
$$\frac{1/96*(15*\cos(b*x + a)^6*\log(\sin(b*x + a) + 1) - 15*\cos(b*x + a)^6*\log(-\sin(b*x + a) + 1) + 2*(15*\cos(b*x + a)^4 + 10*\cos(b*x + a)^2 + 8)*\sin(b*x + a))}{(b*\cos(b*x + a)^6)}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec^7(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)\*\*7,x)

[Out] Integral(sec(a + b\*x)\*\*7, x)

**Giac [A]** time = 1.27607, size = 99, normalized size = 1.3

$$\frac{2(15 \sin(bx+a)^5 - 40 \sin(bx+a)^3 + 33 \sin(bx+a))}{(\sin(bx+a)^2 - 1)^3} - 15 \log(|\sin(bx + a) + 1|) + 15 \log(|\sin(bx + a) - 1|)$$

96 b

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/96*(2*(15*sin(b*x + a)^5 - 40*sin(b*x + a)^3 + 33*sin(b*x + a))/(sin(b*x + a)^2 - 1)^3 - 15*log(abs(sin(b*x + a) + 1)) + 15*log(abs(sin(b*x + a) - 1)))/b
```

### 3.8 $\int \sec^8(a + bx) dx$

**Optimal.** Leaf size=53

$$\frac{\tan^7(a + bx)}{7b} + \frac{3 \tan^5(a + bx)}{5b} + \frac{\tan^3(a + bx)}{b} + \frac{\tan(a + bx)}{b}$$

[Out] Tan[a + b\*x]/b + Tan[a + b\*x]^3/b + (3\*Tan[a + b\*x]^5)/(5\*b) + Tan[a + b\*x]^7/(7\*b)

**Rubi [A]** time = 0.0167995, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {3767}

$$\frac{\tan^7(a + bx)}{7b} + \frac{3 \tan^5(a + bx)}{5b} + \frac{\tan^3(a + bx)}{b} + \frac{\tan(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^8,x]

[Out] Tan[a + b\*x]/b + Tan[a + b\*x]^3/b + (3\*Tan[a + b\*x]^5)/(5\*b) + Tan[a + b\*x]^7/(7\*b)

#### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

#### Rubi steps

$$\begin{aligned} \int \sec^8(a + bx) dx &= -\frac{\text{Subst}\left(\int (1 + 3x^2 + 3x^4 + x^6) dx, x, -\tan(a + bx)\right)}{b} \\ &= \frac{\tan(a + bx)}{b} + \frac{\tan^3(a + bx)}{b} + \frac{3 \tan^5(a + bx)}{5b} + \frac{\tan^7(a + bx)}{7b} \end{aligned}$$

**Mathematica [A]** time = 0.218945, size = 43, normalized size = 0.81

$$\frac{\frac{1}{7} \tan^7(a + bx) + \frac{3}{5} \tan^5(a + bx) + \tan^3(a + bx) + \tan(a + bx)}{b}$$



Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*x]^8,x]

[Out] (Tan[a + b\*x] + Tan[a + b\*x]^3 + (3\*Tan[a + b\*x]^5)/5 + Tan[a + b\*x]^7/7)/b

**Maple [A]** time = 0.043, size = 44, normalized size = 0.8

$$-\frac{\tan(bx+a)}{b} \left( -\frac{16}{35} - \frac{(\sec(bx+a))^6}{7} - \frac{6(\sec(bx+a))^4}{35} - \frac{8(\sec(bx+a))^2}{35} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b\*x+a)^8,x)

[Out] -1/b\*(-16/35-1/7\*sec(b\*x+a)^6-6/35\*sec(b\*x+a)^4-8/35\*sec(b\*x+a)^2)\*tan(b\*x+a)

**Maxima [A]** time = 1.07473, size = 59, normalized size = 1.11

$$\frac{5 \tan(bx+a)^7 + 21 \tan(bx+a)^5 + 35 \tan(bx+a)^3 + 35 \tan(bx+a)}{35b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^8,x, algorithm="maxima")

[Out] 1/35\*(5\*tan(b\*x + a)^7 + 21\*tan(b\*x + a)^5 + 35\*tan(b\*x + a)^3 + 35\*tan(b\*x + a))/b

**Fricas [A]** time = 1.39288, size = 135, normalized size = 2.55

$$\frac{(16 \cos(bx+a)^6 + 8 \cos(bx+a)^4 + 6 \cos(bx+a)^2 + 5) \sin(bx+a)}{35b \cos(bx+a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] 1/35*(16*cos(b*x + a)^6 + 8*cos(b*x + a)^4 + 6*cos(b*x + a)^2 + 5)*sin(b*x + a)/(b*cos(b*x + a)^7)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec^8(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**8,x)
```

```
[Out] Integral(sec(a + b*x)**8, x)
```

**Giac [A]** time = 1.27431, size = 59, normalized size = 1.11

$$\frac{5 \tan(bx + a)^7 + 21 \tan(bx + a)^5 + 35 \tan(bx + a)^3 + 35 \tan(bx + a)}{35b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8,x, algorithm="giac")
```

```
[Out] 1/35*(5*tan(b*x + a)^7 + 21*tan(b*x + a)^5 + 35*tan(b*x + a)^3 + 35*tan(b*x + a))/b
```

### 3.9 $\int \sec^{\frac{7}{2}}(a + bx) dx$

**Optimal.** Leaf size=85

$$\frac{2 \sin(a + bx) \sec^{\frac{5}{2}}(a + bx)}{5b} + \frac{6 \sin(a + bx) \sqrt{\sec(a + bx)}}{5b} - \frac{6 \sqrt{\cos(a + bx)} \sqrt{\sec(a + bx)} E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{5b}$$

[Out]  $(-6 \sqrt{\cos[a + b*x]} * \text{EllipticE}[(a + b*x)/2, 2] * \sqrt{\sec[a + b*x]}) / (5*b) + (6 \sqrt{\sec[a + b*x]} * \sin[a + b*x]) / (5*b) + (2 * \sec[a + b*x]^{(5/2)} * \sin[a + b*x]) / (5*b)$

**Rubi [A]** time = 0.0370816, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {3768, 3771, 2639}

$$\frac{2 \sin(a + bx) \sec^{\frac{5}{2}}(a + bx)}{5b} + \frac{6 \sin(a + bx) \sqrt{\sec(a + bx)}}{5b} - \frac{6 \sqrt{\cos(a + bx)} \sqrt{\sec(a + bx)} E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{5b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\sec[a + b*x]^{(7/2)}, x]$

[Out]  $(-6 \sqrt{\cos[a + b*x]} * \text{EllipticE}[(a + b*x)/2, 2] * \sqrt{\sec[a + b*x]}) / (5*b) + (6 \sqrt{\sec[a + b*x]} * \sin[a + b*x]) / (5*b) + (2 * \sec[a + b*x]^{(5/2)} * \sin[a + b*x]) / (5*b)$

#### Rule 3768

$\text{Int}[(\text{csc}[(c_.) + (d_.)(x_.)]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow -\text{Simp}[(b * \cos[c + d*x] * (b * \text{Csc}[c + d*x])^{(n-1)}) / (d * (n-1)), x] + \text{Dist}[(b^2 * (n-2)) / (n-1), \text{Int}[(b * \text{Csc}[c + d*x])^{(n-2)}, x], x] /;$  FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

$\text{Int}[(\text{csc}[(c_.) + (d_.)(x_.)]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[(b * \text{Csc}[c + d*x])^{(n)} * \sin[c + d*x]^n, \text{Int}[1/\sin[c + d*x]^n, x], x] /;$  FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
 \int \sec^{\frac{7}{2}}(a + bx) dx &= \frac{2 \sec^{\frac{5}{2}}(a + bx) \sin(a + bx)}{5b} + \frac{3}{5} \int \sec^{\frac{3}{2}}(a + bx) dx \\
 &= \frac{6\sqrt{\sec(a + bx)} \sin(a + bx)}{5b} + \frac{2 \sec^{\frac{5}{2}}(a + bx) \sin(a + bx)}{5b} - \frac{3}{5} \int \frac{1}{\sqrt{\sec(a + bx)}} dx \\
 &= \frac{6\sqrt{\sec(a + bx)} \sin(a + bx)}{5b} + \frac{2 \sec^{\frac{5}{2}}(a + bx) \sin(a + bx)}{5b} - \frac{1}{5} (3\sqrt{\cos(a + bx)}\sqrt{\sec(a + bx)}) \int \sqrt{\cos} \\
 &= -\frac{6\sqrt{\cos(a + bx)}E\left(\frac{1}{2}(a + bx) \middle| 2\right) \sqrt{\sec(a + bx)}}{5b} + \frac{6\sqrt{\sec(a + bx)} \sin(a + bx)}{5b} + \frac{2 \sec^{\frac{5}{2}}(a + bx) \sin(a + bx)}{5b}
 \end{aligned}$$

**Mathematica [A]** time = 0.174773, size = 59, normalized size = 0.69

$$\frac{\sec^{\frac{5}{2}}(a + bx) \left( 7 \sin(a + bx) + 3 \sin(3(a + bx)) - 12 \cos^{\frac{5}{2}}(a + bx) E\left(\frac{1}{2}(a + bx) \middle| 2\right) \right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(7/2), x]
```

```
[Out] (Sec[a + b*x]^(5/2)*(-12*Cos[a + b*x]^(5/2)*EllipticE[(a + b*x)/2, 2] + 7*Sin[a + b*x] + 3*Sin[3*(a + b*x)])/(10*b)
```

**Maple [B]** time = 2.199, size = 358, normalized size = 4.2

$$\frac{2}{5b} \sqrt{-(-2(\cos(1/2 bx + a/2))^2 + 1) \left(\sin\left(\frac{bx}{2} + \frac{a}{2}\right)\right)^2} \left( 12 \sqrt{2(\sin(1/2 bx + a/2))^2 - 1} \sqrt{(\sin(1/2 bx + a/2))^2} \text{EllipticE}\left(\frac{1}{2}(a + bx) \middle| 2\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(7/2), x)
```

```
[Out] 2/5*(-(-2*cos(1/2*b*x+1/2*a)^2+1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)/sin(1/2*b*x+1/2*a)^3*(12*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^4-24*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)-12*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2+4*sin(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2*a)+3*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))-8*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(bx + a)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(7/2), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec(bx + a)^{\frac{7}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(7/2), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec (bx + a)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(7/2), x)
```

### 3.10 $\int \sec^{\frac{5}{2}}(a + bx) dx$

**Optimal.** Leaf size=62

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right)}{3b} + \frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}$$

[Out] (2\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[Sec[a + b\*x]])/(3\*b) + (2\*Sec[a + b\*x]^(3/2)\*Sin[a + b\*x])/(3\*b)

**Rubi [A]** time = 0.0269803, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {3768, 3771, 2641}

$$\frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b} + \frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^(5/2), x]

[Out] (2\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[Sec[a + b\*x]])/(3\*b) + (2\*Sec[a + b\*x]^(3/2)\*Sin[a + b\*x])/(3\*b)

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Csc[c + d\*x])\*(b\*Csc[c + d\*x])^(n - 1)/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \sec^{\frac{5}{2}}(a + bx) dx &= \frac{2 \sec^{\frac{3}{2}}(a + bx) \sin(a + bx)}{3b} + \frac{1}{3} \int \sqrt{\sec(a + bx)} dx \\ &= \frac{2 \sec^{\frac{3}{2}}(a + bx) \sin(a + bx)}{3b} + \frac{1}{3} (\sqrt{\cos(a + bx)} \sqrt{\sec(a + bx)}) \int \frac{1}{\sqrt{\cos(a + bx)}} dx \\ &= \frac{2 \sqrt{\cos(a + bx)} F\left(\frac{1}{2}(a + bx) \middle| 2\right) \sqrt{\sec(a + bx)}}{3b} + \frac{2 \sec^{\frac{3}{2}}(a + bx) \sin(a + bx)}{3b} \end{aligned}$$

**Mathematica [A]** time = 0.0674676, size = 46, normalized size = 0.74

$$\frac{2 \sec^{\frac{3}{2}}(a + bx) \left( \cos^{\frac{3}{2}}(a + bx) \text{EllipticF}\left(\frac{1}{2}(a + bx), 2\right) + \sin(a + bx) \right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(5/2), x]
```

```
[Out] (2*Sec[a + b*x]^(3/2)*(Cos[a + b*x]^(3/2)*EllipticF[(a + b*x)/2, 2] + Sin[a
+ b*x]))/(3*b)
```

**Maple [B]** time = 1.266, size = 213, normalized size = 3.4

$$-\frac{2}{3b} \left( -2 \sqrt{(\sin(1/2 bx + a/2))^2} \sqrt{2 (\sin(1/2 bx + a/2))^2 - 1} \text{EllipticF}\left(\cos(1/2 bx + a/2), \sqrt{2}\right) (\sin(1/2 bx + a/2))^2 + \sqrt{\left(\sin(1/2 bx + a/2)\right)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(5/2), x)
```

```
[Out] -2/3*(-2*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a), 2^(1/2))*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*a)^2)^(1/2)
```



$$2)^{(1/2)} * (2 * \sin(1/2 * b * x + 1/2 * a)^2 - 1)^{(1/2)} * \text{EllipticF}(\cos(1/2 * b * x + 1/2 * a), 2^{(1/2)}) - 2 * \sin(1/2 * b * x + 1/2 * a)^2 * \cos(1/2 * b * x + 1/2 * a) * ((2 * \cos(1/2 * b * x + 1/2 * a)^2 - 1) * \sin(1/2 * b * x + 1/2 * a)^2)^{(1/2)} / (-2 * \sin(1/2 * b * x + 1/2 * a)^4 + \sin(1/2 * b * x + 1/2 * a)^2)^{(1/2)} / (2 * \cos(1/2 * b * x + 1/2 * a)^2 - 1)^{(3/2)} / \sin(1/2 * b * x + 1/2 * a) / b$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(bx + a)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(5/2),x, algorithm="maxima")

[Out] integrate(sec(b\*x + a)^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec(bx + a)^{5/2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(5/2),x, algorithm="fricas")

[Out] integral(sec(b\*x + a)^(5/2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)\*\*(5/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec (bx + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(5/2),x, algorithm="giac")

[Out] integrate(sec(b\*x + a)^(5/2), x)

### 3.11 $\int \sec^{\frac{3}{2}}(a + bx) dx$

**Optimal.** Leaf size=58

$$\frac{2 \sin(a + bx) \sqrt{\sec(a + bx)}}{b} - \frac{2 \sqrt{\cos(a + bx)} \sqrt{\sec(a + bx)} E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{b}$$

[Out]  $(-2 \sqrt{\cos[a + b*x]} \text{EllipticE}[(a + b*x)/2, 2] \sqrt{\sec[a + b*x]})/b + (2 \sqrt{\sec[a + b*x]} \sin[a + b*x])/b$

**Rubi [A]** time = 0.0255371, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {3768, 3771, 2639}

$$\frac{2 \sin(a + bx) \sqrt{\sec(a + bx)}}{b} - \frac{2 \sqrt{\cos(a + bx)} \sqrt{\sec(a + bx)} E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sec}[a + b*x]^{(3/2)}, x]$

[Out]  $(-2 \sqrt{\cos[a + b*x]} \text{EllipticE}[(a + b*x)/2, 2] \sqrt{\sec[a + b*x]})/b + (2 \sqrt{\sec[a + b*x]} \sin[a + b*x])/b$

#### Rule 3768

$\text{Int}[(\text{csc}[(c_.) + (d_.)(x_)]*(b_.))^{(n_.)}, x\_Symbol] \text{ :> } -\text{Simp}[(b*\text{Cos}[c + d*x] * (b*\text{Csc}[c + d*x])^{(n - 1)})/(d*(n - 1)), x] + \text{Dist}[(b^2*(n - 2))/(n - 1), \text{Int}[(b*\text{Csc}[c + d*x])^{(n - 2)}, x], x] \text{ /; FreeQ}\{b, c, d, x\} \ \&\& \ \text{GtQ}\{n, 1\} \ \&\& \ \text{IntegerQ}\{2*n\}$

#### Rule 3771

$\text{Int}[(\text{csc}[(c_.) + (d_.)(x_)]*(b_.))^{(n_.)}, x\_Symbol] \text{ :> } \text{Dist}[(b*\text{Csc}[c + d*x])^{(n)}*\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] \text{ /; FreeQ}\{b, c, d, x\} \ \&\& \ \text{EqQ}\{n^2, 1/4\}$

#### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \sec^{\frac{3}{2}}(a + bx) dx &= \frac{2\sqrt{\sec(a + bx)} \sin(a + bx)}{b} - \int \frac{1}{\sqrt{\sec(a + bx)}} dx \\ &= \frac{2\sqrt{\sec(a + bx)} \sin(a + bx)}{b} - \left(\sqrt{\cos(a + bx)}\sqrt{\sec(a + bx)}\right) \int \sqrt{\cos(a + bx)} dx \\ &= -\frac{2\sqrt{\cos(a + bx)}E\left(\frac{1}{2}(a + bx) \middle| 2\right) \sqrt{\sec(a + bx)}}{b} + \frac{2\sqrt{\sec(a + bx)} \sin(a + bx)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.0475868, size = 45, normalized size = 0.78

$$\frac{2\sqrt{\sec(a + bx)} \left( \sin(a + bx) - \sqrt{\cos(a + bx)} E\left(\frac{1}{2}(a + bx) \middle| 2\right) \right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(3/2), x]
```

```
[Out] (2*Sqrt[Sec[a + b*x]]*(-(Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]) + Si
n[a + b*x]))/b
```

**Maple [A]** time = 1.062, size = 101, normalized size = 1.7

$$-2 \frac{\sqrt{2} (\sin(1/2 bx + a/2))^2 - 1 \sqrt{(\sin(1/2 bx + a/2))^2 \text{EllipticE}(\cos(1/2 bx + a/2), \sqrt{2})} - 2 (\sin(1/2 bx + a/2))^2 \cos(1/2)}{\sin(1/2 bx + a/2) \sqrt{2 (\cos(1/2 bx + a/2))^2 - 1b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(3/2), x)
```

```
[Out] -2*((2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE
(cos(1/2*b*x+1/2*a), 2^(1/2))-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/sin
```

$$(1/2*b*x+1/2*a)/(2*\cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b$$


---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(bx + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(3/2),x, algorithm="maxima")

[Out] integrate(sec(b\*x + a)^(3/2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec(bx + a)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(3/2),x, algorithm="fricas")

[Out] integral(sec(b\*x + a)^(3/2), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec^{\frac{3}{2}}(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)\*\*(3/2),x)

[Out] Integral(sec(a + b\*x)\*\*(3/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(bx + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(3/2), x)
```

### 3.12 $\int \sqrt{\sec(a + bx)} dx$

**Optimal.** Leaf size=36

$$\frac{2\sqrt{\cos(a + bx)}\sqrt{\sec(a + bx)}\text{EllipticF}\left(\frac{1}{2}(a + bx), 2\right)}{b}$$

[Out] (2\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[Sec[a + b\*x]])/b

**Rubi [A]** time = 0.0167242, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {3771, 2641}

$$\frac{2\sqrt{\cos(a + bx)}\sqrt{\sec(a + bx)}F\left(\frac{1}{2}(a + bx) \middle| 2\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[a + b\*x]], x]

[Out] (2\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[Sec[a + b\*x]])/b

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \sqrt{\sec(a + bx)} dx &= \left(\sqrt{\cos(a + bx)}\sqrt{\sec(a + bx)}\right) \int \frac{1}{\sqrt{\cos(a + bx)}} dx \\ &= \frac{2\sqrt{\cos(a + bx)}F\left(\frac{1}{2}(a + bx) \middle| 2\right)\sqrt{\sec(a + bx)}}{b} \end{aligned}$$

**Mathematica [A]** time = 0.028913, size = 36, normalized size = 1.

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[a + b\*x]], x]

[Out] (2\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[Sec[a + b\*x]])/b

**Maple [B]** time = 0.883, size = 133, normalized size = 3.7

$$-2 \frac{\sqrt{(2(\cos(1/2bx + a/2))^2 - 1)(\sin(1/2bx + a/2))^2} \sqrt{(\sin(1/2bx + a/2))^2} \sqrt{-2(\cos(1/2bx + a/2))^2 + 1} \text{EllipticF}(\cos(1/2bx + a/2), 2)}{\sqrt{-2(\sin(1/2bx + a/2))^4 + (\sin(1/2bx + a/2))^2 \sin(1/2bx + a/2)} \sqrt{2(\cos(1/2bx + a/2))^2 - 1} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b\*x+a)^(1/2), x)

[Out] -2\*((2\*cos(1/2\*b\*x+1/2\*a)^2-1)\*sin(1/2\*b\*x+1/2\*a)^2)^(1/2)\*(sin(1/2\*b\*x+1/2\*a)^2)^(1/2)\*(-2\*cos(1/2\*b\*x+1/2\*a)^2+1)^(1/2)/(-2\*sin(1/2\*b\*x+1/2\*a)^4+sin(1/2\*b\*x+1/2\*a)^2)^(1/2)\*EllipticF(cos(1/2\*b\*x+1/2\*a), 2^(1/2))/sin(1/2\*b\*x+1/2\*a)/(2\*cos(1/2\*b\*x+1/2\*a)^2-1)^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sec(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(sec(b\*x + a)), x)



**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{\sec(bx + a)}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(sec(b\*x + a)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sec(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)\*\*(1/2),x)

[Out] Integral(sqrt(sec(a + b\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sec(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(sec(b\*x + a)), x)

### 3.13 $\int \frac{1}{\sqrt{\sec(a+bx)}} dx$

**Optimal.** Leaf size=36

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

[Out] (2\*Sqrt[Cos[a + b\*x]]\*EllipticE[(a + b\*x)/2, 2]\*Sqrt[Sec[a + b\*x]])/b

**Rubi [A]** time = 0.0171722, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {3771, 2639}

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[Sec[a + b\*x]],x]

[Out] (2\*Sqrt[Cos[a + b\*x]]\*EllipticE[(a + b\*x)/2, 2]\*Sqrt[Sec[a + b\*x]])/b

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\int \frac{1}{\sqrt{\sec(a+bx)}} dx = (\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}) \int \sqrt{\cos(a+bx)} dx$$

$$= \frac{2\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b}$$

**Mathematica [A]** time = 0.0344761, size = 36, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[Sec[a + b\*x]],x]

[Out] (2\*EllipticE[(a + b\*x)/2, 2])/(b\*Sqrt[Cos[a + b\*x]]\*Sqrt[Sec[a + b\*x]])

**Maple [B]** time = 0.936, size = 133, normalized size = 3.7

$$2 \frac{\sqrt{(2(\cos(1/2 bx + a/2))^2 - 1)(\sin(1/2 bx + a/2))^2} \sqrt{(\sin(1/2 bx + a/2))^2} \sqrt{-2(\cos(1/2 bx + a/2))^2 + 1} \text{EllipticE}(\cos(1/2 bx + a/2), 2)}{\sqrt{-2(\sin(1/2 bx + a/2))^4 + (\sin(1/2 bx + a/2))^2 \sin(1/2 bx + a/2)} \sqrt{2(\cos(1/2 bx + a/2))^2 - 1} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(b\*x+a)^(1/2),x)

[Out] 2\*((2\*cos(1/2\*b\*x+1/2\*a)^2-1)\*sin(1/2\*b\*x+1/2\*a)^2)^(1/2)\*(sin(1/2\*b\*x+1/2\*a)^2)^(1/2)\*(-2\*cos(1/2\*b\*x+1/2\*a)^2+1)^(1/2)\*EllipticE(cos(1/2\*b\*x+1/2\*a), 2^(1/2))/(-2\*sin(1/2\*b\*x+1/2\*a)^4+sin(1/2\*b\*x+1/2\*a)^2)^(1/2)/sin(1/2\*b\*x+1/2\*a)/(2\*cos(1/2\*b\*x+1/2\*a)^2-1)^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sec(bx+a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(b\*x+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(sec(b\*x + a)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{\sec(bx + a)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(b\*x+a)^(1/2),x, algorithm="fricas")

[Out] integral(1/sqrt(sec(b\*x + a)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sec(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(b\*x+a)\*\*(1/2),x)

[Out] Integral(1/sqrt(sec(a + b\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(b\*x+a)^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(sec(b\*x + a)), x)

$$3.14 \quad \int \frac{1}{\sec^{\frac{3}{2}}(a+bx)} dx$$

**Optimal.** Leaf size=62

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right)}{3b} + \frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}$$

[Out] (2\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[Sec[a + b\*x]])/(3\*b) + (2\*Sin[a + b\*x])/(3\*b\*Sqrt[Sec[a + b\*x]])

**Rubi [A]** time = 0.0280829, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {3769, 3771, 2641}

$$\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}} + \frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^(-3/2), x]

[Out] (2\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[Sec[a + b\*x]])/(3\*b) + (2\*Sin[a + b\*x])/(3\*b\*Sqrt[Sec[a + b\*x]])

#### Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d^n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

#### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sec^{\frac{3}{2}}(a+bx)} dx &= \frac{2 \sin(a+bx)}{3b\sqrt{\sec(a+bx)}} + \frac{1}{3} \int \sqrt{\sec(a+bx)} dx \\ &= \frac{2 \sin(a+bx)}{3b\sqrt{\sec(a+bx)}} + \frac{1}{3} \left( \sqrt{\cos(a+bx)} \sqrt{\sec(a+bx)} \right) \int \frac{1}{\sqrt{\cos(a+bx)}} dx \\ &= \frac{2\sqrt{\cos(a+bx)} F\left(\frac{1}{2}(a+bx) \middle| 2\right) \sqrt{\sec(a+bx)}}{3b} + \frac{2 \sin(a+bx)}{3b\sqrt{\sec(a+bx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0442057, size = 49, normalized size = 0.79

$$\frac{\sqrt{\sec(a+bx)} \left( 2\sqrt{\cos(a+bx)} \text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right) + \sin(2(a+bx)) \right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(-3/2), x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + Sin[2
*(a + b*x)]))/(3*b)
```

**Maple [B]** time = 1.485, size = 179, normalized size = 2.9

$$-\frac{2}{3b} \sqrt{\left(2 \left(\cos\left(\frac{1}{2}bx + \frac{a}{2}\right)\right)^2 - 1\right) \left(\sin\left(\frac{bx}{2} + \frac{a}{2}\right)\right)^2} \left( 4 \left(\sin\left(\frac{1}{2}bx + \frac{a}{2}\right)\right)^4 \cos\left(\frac{1}{2}bx + \frac{a}{2}\right) + \sqrt{\left(\sin\left(\frac{bx}{2} + \frac{a}{2}\right)\right)^2} \sqrt{2} \left(\sin\left(\frac{1}{2}bx + \frac{a}{2}\right)\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(b*x+a)^(3/2), x)
```

```
[Out] -2/3*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(4*sin(1/2*b*x
+1/2*a)^4*cos(1/2*b*x+1/2*a)+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/
```

$2*a)^{2-1})^{1/2} * \text{EllipticF}(\cos(1/2*b*x+1/2*a), 2^{1/2}) - 2*\sin(1/2*b*x+1/2*a)^{2*\cos(1/2*b*x+1/2*a)} / (-2*\sin(1/2*b*x+1/2*a)^4 + \sin(1/2*b*x+1/2*a)^2)^{1/2} / \sin(1/2*b*x+1/2*a) / (2*\cos(1/2*b*x+1/2*a)^{2-1})^{1/2} / b$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(bx + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(b\*x+a)^(3/2),x, algorithm="maxima")

[Out] integrate(sec(b\*x + a)^(-3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sec(bx + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(b\*x+a)^(3/2),x, algorithm="fricas")

[Out] integral(sec(b\*x + a)^(-3/2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{3}{2}}(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(b\*x+a)\*\*(3/2),x)

[Out] Integral(sec(a + b\*x)\*\*(-3/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(bx + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(b\*x+a)^(3/2),x, algorithm="giac")

[Out] integrate(sec(b\*x + a)^(-3/2), x)



$$3.15 \quad \int \frac{1}{5 \sec^2(a+bx)} dx$$

**Optimal.** Leaf size=62

$$\frac{2 \sin(a+bx)}{5b \sec^{\frac{3}{2}}(a+bx)} + \frac{6\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}$$

[Out] (6\*Sqrt[Cos[a + b\*x]]\*EllipticE[(a + b\*x)/2, 2]\*Sqrt[Sec[a + b\*x]])/(5\*b) + (2\*Sin[a + b\*x])/(5\*b\*Sec[a + b\*x]^(3/2))

**Rubi [A]** time = 0.0286673, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {3769, 3771, 2639}

$$\frac{2 \sin(a+bx)}{5b \sec^{\frac{3}{2}}(a+bx)} + \frac{6\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^(-5/2), x]

[Out] (6\*Sqrt[Cos[a + b\*x]]\*EllipticE[(a + b\*x)/2, 2]\*Sqrt[Sec[a + b\*x]])/(5\*b) + (2\*Sin[a + b\*x])/(5\*b\*Sec[a + b\*x]^(3/2))

#### Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

#### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sec^{\frac{5}{2}}(a+bx)} dx &= \frac{2 \sin(a+bx)}{5b \sec^{\frac{3}{2}}(a+bx)} + \frac{3}{5} \int \frac{1}{\sqrt{\sec(a+bx)}} dx \\ &= \frac{2 \sin(a+bx)}{5b \sec^{\frac{3}{2}}(a+bx)} + \frac{1}{5} (3\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}) \int \sqrt{\cos(a+bx)} dx \\ &= \frac{6\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{5b} + \frac{2 \sin(a+bx)}{5b \sec^{\frac{3}{2}}(a+bx)} \end{aligned}$$

**Mathematica [A]** time = 0.0693045, size = 55, normalized size = 0.89

$$\frac{\sqrt{\sec(a+bx)}\left(\sin(a+bx) + \sin(3(a+bx)) + 12\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(-5/2), x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2] + Sin[
a + b*x] + Sin[3*(a + b*x)]))/(10*b)
```

**Maple [B]** time = 1.212, size = 202, normalized size = 3.3

$$-\frac{2}{5b} \sqrt{(2(\cos(1/2bx + a/2))^2 - 1) \left(\sin\left(\frac{bx}{2} + \frac{a}{2}\right)\right)^2} \left(-8(\sin(1/2bx + a/2))^6 \cos(1/2bx + a/2) + 8(\sin(1/2bx + a/2))^4 \cos(1/2bx + a/2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(b*x+a)^(5/2), x)
```

```
[Out] -2/5*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(-8*sin(1/2*b*
x+1/2*a)^6*cos(1/2*b*x+1/2*a)+8*sin(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2*a)-3*(
```

$2*\sin(1/2*b*x+1/2*a)^{2-1}^{(1/2)}*(\sin(1/2*b*x+1/2*a)^2)^{(1/2)}*EllipticE(\cos(1/2*b*x+1/2*a),2^{(1/2)})-2*\sin(1/2*b*x+1/2*a)^2*\cos(1/2*b*x+1/2*a))/(-2*\sin(1/2*b*x+1/2*a)^4+\sin(1/2*b*x+1/2*a)^2)^{(1/2)}/\sin(1/2*b*x+1/2*a)/(2*\cos(1/2*b*x+1/2*a)^{2-1}^{(1/2)})/b$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{5}{2}}(bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(b\*x+a)^(5/2),x, algorithm="maxima")

[Out] integrate(sec(b\*x + a)^(-5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sec^{\frac{5}{2}}(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(b\*x+a)^(5/2),x, algorithm="fricas")

[Out] integral(sec(b\*x + a)^(-5/2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(b\*x+a)\*\*(5/2),x)

[Out] Integral(sec(a + b\*x)\*\*(-5/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(bx + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(b\*x+a)^(5/2),x, algorithm="giac")

[Out] integrate(sec(b\*x + a)^(-5/2), x)

$$3.16 \quad \int \frac{1}{7 \sec^2(a+bx)} dx$$

**Optimal.** Leaf size=85

$$\frac{10\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right)}{21b} + \frac{2 \sin(a+bx)}{7b \sec^{\frac{5}{2}}(a+bx)} + \frac{10 \sin(a+bx)}{21b\sqrt{\sec(a+bx)}}$$

[Out] (10\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[Sec[a + b\*x]])/(21\*b) + (2\*Sin[a + b\*x])/(7\*b\*Sec[a + b\*x]^(5/2)) + (10\*Sin[a + b\*x])/(21\*b\*Sqrt[Sec[a + b\*x]])

**Rubi [A]** time = 0.0403677, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {3769, 3771, 2641}

$$\frac{2 \sin(a+bx)}{7b \sec^{\frac{5}{2}}(a+bx)} + \frac{10 \sin(a+bx)}{21b\sqrt{\sec(a+bx)}} + \frac{10\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}F\left(\frac{1}{2}(a+bx) \middle| 2\right)}{21b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^(-7/2), x]

[Out] (10\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[Sec[a + b\*x]])/(21\*b) + (2\*Sin[a + b\*x])/(7\*b\*Sec[a + b\*x]^(5/2)) + (10\*Sin[a + b\*x])/(21\*b\*Sqrt[Sec[a + b\*x]])

#### Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]
```

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sec^2(a+bx)} dx &= \frac{2 \sin(a+bx)}{7b \sec^{\frac{5}{2}}(a+bx)} + \frac{5}{7} \int \frac{1}{\sec^2(a+bx)} dx \\
 &= \frac{2 \sin(a+bx)}{7b \sec^{\frac{5}{2}}(a+bx)} + \frac{10 \sin(a+bx)}{21b \sqrt{\sec(a+bx)}} + \frac{5}{21} \int \sqrt{\sec(a+bx)} dx \\
 &= \frac{2 \sin(a+bx)}{7b \sec^{\frac{5}{2}}(a+bx)} + \frac{10 \sin(a+bx)}{21b \sqrt{\sec(a+bx)}} + \frac{1}{21} (5\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}) \int \frac{1}{\sqrt{\cos(a+bx)}} dx \\
 &= \frac{10\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx) \middle| 2\right) \sqrt{\sec(a+bx)}}{21b} + \frac{2 \sin(a+bx)}{7b \sec^{\frac{5}{2}}(a+bx)} + \frac{10 \sin(a+bx)}{21b \sqrt{\sec(a+bx)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0986761, size = 61, normalized size = 0.72

$$\frac{\sqrt{\sec(a+bx)} \left( 40\sqrt{\cos(a+bx)} \text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right) + 26 \sin(2(a+bx)) + 3 \sin(4(a+bx)) \right)}{84b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(-7/2), x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(40*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 26*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)])/(84*b)
```

**Maple [B]** time = 1.247, size = 199, normalized size = 2.3

$$-\frac{2}{21b} \sqrt{\left(2 (\cos(1/2 bx + a/2))^2 - 1\right) \left(\sin\left(\frac{bx}{2} + \frac{a}{2}\right)\right)^2 \left(48 (\cos(1/2 bx + a/2))^9 - 120 (\cos(1/2 bx + a/2))^7 + 128 (\cos(1/2 bx + a/2))^5 - 64 (\cos(1/2 bx + a/2))^3 + 16 (\cos(1/2 bx + a/2))\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/sec(b*x+a)^(7/2),x)`

[Out] 
$$-2/21*((2*\cos(1/2*b*x+1/2*a)^2-1)*\sin(1/2*b*x+1/2*a)^2)^{(1/2)}*(48*\cos(1/2*b*x+1/2*a)^9-120*\cos(1/2*b*x+1/2*a)^7+128*\cos(1/2*b*x+1/2*a)^5-72*\cos(1/2*b*x+1/2*a)^3+5*(\sin(1/2*b*x+1/2*a)^2)^{(1/2)}*(-2*\cos(1/2*b*x+1/2*a)^2+1)^{(1/2)}*EllipticF(\cos(1/2*b*x+1/2*a),2^{(1/2)})+16*\cos(1/2*b*x+1/2*a))/(-2*\sin(1/2*b*x+1/2*a)^4+\sin(1/2*b*x+1/2*a)^2)^{(1/2)}/\sin(1/2*b*x+1/2*a)/(2*\cos(1/2*b*x+1/2*a)^2-1)^{(1/2)}/b$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(bx+a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(b*x+a)^(7/2),x, algorithm="maxima")`

[Out] `integrate(sec(b*x + a)^(-7/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sec(bx+a)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(b*x+a)^(7/2),x, algorithm="fricas")`

[Out] `integral(sec(b*x + a)^(-7/2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(bx + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(-7/2), x)
```



### 3.17 $\int (c \sec(a + bx))^{7/2} dx$

**Optimal.** Leaf size=98

$$\frac{6c^3 \sin(a + bx) \sqrt{c \sec(a + bx)}}{5b} - \frac{6c^4 E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{5b \sqrt{\cos(a + bx)} \sqrt{c \sec(a + bx)}} + \frac{2c \sin(a + bx) (c \sec(a + bx))^{5/2}}{5b}$$

[Out]  $(-6*c^4*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*x]]) + (6*c^3*Sqrt[c*Sec[a + b*x]]*Sin[a + b*x])/(5*b) + (2*c*(c*Sec[a + b*x])^(5/2)*Sin[a + b*x])/(5*b)$

**Rubi [A]** time = 0.0558185, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3768, 3771, 2639}

$$\frac{6c^3 \sin(a + bx) \sqrt{c \sec(a + bx)}}{5b} - \frac{6c^4 E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{5b \sqrt{\cos(a + bx)} \sqrt{c \sec(a + bx)}} + \frac{2c \sin(a + bx) (c \sec(a + bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*\text{Sec}[a + b*x])^{(7/2)}, x]$

[Out]  $(-6*c^4*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*x]]) + (6*c^3*Sqrt[c*Sec[a + b*x]]*Sin[a + b*x])/(5*b) + (2*c*(c*Sec[a + b*x])^(5/2)*Sin[a + b*x])/(5*b)$

#### Rule 3768

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Csc}[c + d*x])^{(n-1)})/(d*(n-1)), x] + \text{Dist}[(b^2*(n-2))/(n-1), \text{Int}[(b*\text{Csc}[c + d*x])^{(n-2)}, x], x] /;$  FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^{n-1}*\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /;$  FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int (c \sec(a + bx))^{7/2} dx &= \frac{2c(c \sec(a + bx))^{5/2} \sin(a + bx)}{5b} + \frac{1}{5} (3c^2) \int (c \sec(a + bx))^{3/2} dx \\
&= \frac{6c^3 \sqrt{c \sec(a + bx)} \sin(a + bx)}{5b} + \frac{2c(c \sec(a + bx))^{5/2} \sin(a + bx)}{5b} - \frac{1}{5} (3c^4) \int \frac{1}{\sqrt{c \sec(a + bx)}} dx \\
&= \frac{6c^3 \sqrt{c \sec(a + bx)} \sin(a + bx)}{5b} + \frac{2c(c \sec(a + bx))^{5/2} \sin(a + bx)}{5b} - \frac{(3c^4) \int \sqrt{\cos(a + bx)} dx}{5\sqrt{\cos(a + bx)}\sqrt{c \sec(a + bx)}} \\
&= -\frac{6c^4 E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{5b\sqrt{\cos(a + bx)}\sqrt{c \sec(a + bx)}} + \frac{6c^3 \sqrt{c \sec(a + bx)} \sin(a + bx)}{5b} + \frac{2c(c \sec(a + bx))^{5/2} \sin(a + bx)}{5b}
\end{aligned}$$

**Mathematica [A]** time = 0.165577, size = 62, normalized size = 0.63

$$\frac{c(c \sec(a + bx))^{5/2} \left(7 \sin(a + bx) + 3 \sin(3(a + bx)) - 12 \cos^{\frac{5}{2}}(a + bx) E\left(\frac{1}{2}(a + bx) \middle| 2\right)\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(7/2), x]
```

```
[Out] (c*(c*Sec[a + b*x])^(5/2)*(-12*Cos[a + b*x]^(5/2)*EllipticE[(a + b*x)/2, 2]
+ 7*Sin[a + b*x] + 3*Sin[3*(a + b*x)]))/(10*b)
```

**Maple [C]** time = 0.32, size = 354, normalized size = 3.6

$$\frac{2(-1 + \cos(bx + a))^2 \cos(bx + a) (\cos(bx + a) + 1)^2}{5b(\sin(bx + a))^5} \left( 3i(\cos(bx + a))^3 \text{EllipticE}\left(\frac{i(-1 + \cos(bx + a))}{\sin(bx + a)}, i\right) \sqrt{(\cos(bx + a) + 1)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(7/2), x)
```

```
[Out] 2/5/b*(-1+cos(b*x+a))^2*(3*I*cos(b*x+a)^3*EllipticE(I*(-1+cos(b*x+a))/sin(b
*x+a),I)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*sin(b*x
+a)-3*I*cos(b*x+a)^3*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*(1/(cos(b*x+
a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*sin(b*x+a)+3*I*(1/(cos(b*x+a
)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*cos(b*x+a)^2*EllipticE(I*(-1+
cos(b*x+a))/sin(b*x+a),I)*sin(b*x+a)-3*I*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+
a)/(cos(b*x+a)+1))^(1/2)*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*cos(b*x+
a)^2*sin(b*x+a)-3*cos(b*x+a)^3+2*cos(b*x+a)^2+1)*cos(b*x+a)*(cos(b*x+a)+1)^
2*(c/cos(b*x+a))^(7/2)/sin(b*x+a)^5
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec(bx + a))^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(7/2), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{c \sec(bx + a)} c^3 \sec(bx + a)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sec(b*x + a))*c^3*sec(b*x + a)^3, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(7/2),x)
```

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec (bx + a))^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(7/2),x, algorithm="giac")

[Out] integrate((c\*sec(b\*x + a))^(7/2), x)

### 3.18 $\int (c \sec(a + bx))^{5/2} dx$

**Optimal.** Leaf size=70

$$\frac{2c^2 \sqrt{\cos(a + bx)} \operatorname{EllipticF}\left(\frac{1}{2}(a + bx), 2\right) \sqrt{c \sec(a + bx)}}{3b} + \frac{2c \sin(a + bx) (c \sec(a + bx))^{3/2}}{3b}$$

[Out] (2\*c^2\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[c\*Sec[a + b\*x]])/(3\*b) + (2\*c\*(c\*Sec[a + b\*x])^(3/2)\*Sin[a + b\*x])/(3\*b)

**Rubi [A]** time = 0.0336493, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3768, 3771, 2641}

$$\frac{2c^2 \sqrt{\cos(a + bx)} F\left(\frac{1}{2}(a + bx) \middle| 2\right) \sqrt{c \sec(a + bx)}}{3b} + \frac{2c \sin(a + bx) (c \sec(a + bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In] Int[(c\*Sec[a + b\*x])^(5/2), x]

[Out] (2\*c^2\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[c\*Sec[a + b\*x]])/(3\*b) + (2\*c\*(c\*Sec[a + b\*x])^(3/2)\*Sin[a + b\*x])/(3\*b)

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Csc[c + d\*x])\*(b\*Csc[c + d\*x])^(n - 1)/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n \* Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (c \sec(a + bx))^{5/2} dx &= \frac{2c(c \sec(a + bx))^{3/2} \sin(a + bx)}{3b} + \frac{1}{3}c^2 \int \sqrt{c \sec(a + bx)} dx \\
&= \frac{2c(c \sec(a + bx))^{3/2} \sin(a + bx)}{3b} + \frac{1}{3} \left( c^2 \sqrt{\cos(a + bx)} \sqrt{c \sec(a + bx)} \right) \int \frac{1}{\sqrt{\cos(a + bx)}} dx \\
&= \frac{2c^2 \sqrt{\cos(a + bx)} F\left(\frac{1}{2}(a + bx) \middle| 2\right) \sqrt{c \sec(a + bx)}}{3b} + \frac{2c(c \sec(a + bx))^{3/2} \sin(a + bx)}{3b}
\end{aligned}$$

**Mathematica [A]** time = 0.0658061, size = 51, normalized size = 0.73

$$\frac{2c^2 \sqrt{c \sec(a + bx)} \left( \sqrt{\cos(a + bx)} \text{EllipticF}\left(\frac{1}{2}(a + bx), 2\right) + \tan(a + bx) \right)}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*Sec[a + b\*x])^(5/2), x]

[Out] (2\*c^2\*Sqrt[c\*Sec[a + b\*x]]\*(Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2] + Tan[a + b\*x]))/(3\*b)

**Maple [C]** time = 0.198, size = 128, normalized size = 1.8

$$\frac{(-2 + 2 \cos(bx + a)) \cos(bx + a) (\cos(bx + a) + 1)^2}{3b (\sin(bx + a))^3} \left( i \sqrt{(\cos(bx + a) + 1)^{-1}} \sqrt{\frac{\cos(bx + a)}{\cos(bx + a) + 1}} \text{EllipticF}\left(\frac{i(-1 + \cos(bx + a))}{\sin(bx + a)}, 2\right) + \tan(bx + a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*sec(b\*x+a))^(5/2), x)

[Out] -2/3/b\*(-1+cos(b\*x+a))\*(I\*(1/(cos(b\*x+a)+1))^(1/2)\*(cos(b\*x+a)/(cos(b\*x+a)+1))^(1/2)\*EllipticF(I\*(-1+cos(b\*x+a))/sin(b\*x+a), I)\*cos(b\*x+a)\*sin(b\*x+a)-cos(b\*x+a)+1)\*cos(b\*x+a)\*(cos(b\*x+a)+1)^2\*(c/cos(b\*x+a))^(5/2)/sin(b\*x+a)^3

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec (bx + a))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(5/2),x, algorithm="maxima")

[Out] integrate((c\*sec(b\*x + a))^(5/2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{c \sec (bx + a)} c^2 \sec (bx + a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(c\*sec(b\*x + a))\*c^2\*sec(b\*x + a)^2, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))\*\*(5/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec (bx + a))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(5/2), x)
```



### 3.19 $\int (c \sec(a + bx))^{3/2} dx$

**Optimal.** Leaf size=66

$$\frac{2c \sin(a + bx) \sqrt{c \sec(a + bx)}}{b} - \frac{2c^2 E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{b \sqrt{\cos(a + bx)} \sqrt{c \sec(a + bx)}}$$

[Out]  $(-2*c^2*EllipticE[(a + b*x)/2, 2])/(b*sqrt[Cos[a + b*x]]*sqrt[c*Sec[a + b*x]]) + (2*c*sqrt[c*Sec[a + b*x]]*Sin[a + b*x])/b$

**Rubi [A]** time = 0.0395353, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3768, 3771, 2639}

$$\frac{2c \sin(a + bx) \sqrt{c \sec(a + bx)}}{b} - \frac{2c^2 E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{b \sqrt{\cos(a + bx)} \sqrt{c \sec(a + bx)}}$$

Antiderivative was successfully verified.

[In] Int[(c\*Sec[a + b\*x])^(3/2), x]

[Out]  $(-2*c^2*EllipticE[(a + b*x)/2, 2])/(b*sqrt[Cos[a + b*x]]*sqrt[c*Sec[a + b*x]]) + (2*c*sqrt[c*Sec[a + b*x]]*Sin[a + b*x])/b$

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x])\*(b\*Csc[c + d\*x])^(n - 1)/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int (c \sec(a + bx))^{3/2} dx &= \frac{2c\sqrt{c \sec(a + bx)} \sin(a + bx)}{b} - c^2 \int \frac{1}{\sqrt{c \sec(a + bx)}} dx \\ &= \frac{2c\sqrt{c \sec(a + bx)} \sin(a + bx)}{b} - \frac{c^2 \int \sqrt{\cos(a + bx)} dx}{\sqrt{\cos(a + bx)}\sqrt{c \sec(a + bx)}} \\ &= -\frac{2c^2 E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{b\sqrt{\cos(a + bx)}\sqrt{c \sec(a + bx)}} + \frac{2c\sqrt{c \sec(a + bx)} \sin(a + bx)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.0391133, size = 48, normalized size = 0.73

$$\frac{2c\sqrt{c \sec(a + bx)} \left( \sin(a + bx) - \sqrt{\cos(a + bx)} E\left(\frac{1}{2}(a + bx) \middle| 2\right) \right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(3/2), x]
```

```
[Out] (2*c*Sqrt[c*Sec[a + b*x]]*(-(Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])
+ Sin[a + b*x]))/b
```

**Maple [C]** time = 0.225, size = 322, normalized size = 4.9

$$2 \frac{(\cos(bx + a) + 1)^2 (-1 + \cos(bx + a))^2 \cos(bx + a)}{b (\sin(bx + a))^5} \left( i \operatorname{EllipticE}\left(\frac{i(-1 + \cos(bx + a))}{\sin(bx + a)}, i\right) \cos(bx + a) \sin(bx + a) \sqrt{\cos(bx + a)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(3/2), x)
```

```
[Out] 2/b*(cos(b*x+a)+1)^2*(-1+cos(b*x+a))^2*(I*EllipticE(I*(-1+cos(b*x+a))/sin(b
*x+a), I)*cos(b*x+a)*sin(b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*
x+a)+1))^(1/2)-I*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)
```

```
*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*cos(b*x+a)*sin(b*x+a)+I*Elliptic
E(I*(-1+cos(b*x+a))/sin(b*x+a),I)*sin(b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(cos(
b*x+a)/(cos(b*x+a)+1))^(1/2)-I*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*si
n(b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)-cos(b*x
+a)+1)*cos(b*x+a)*(c/cos(b*x+a))^(3/2)/sin(b*x+a)^5
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec (bx + a))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{c \sec (bx + a)} c \sec (bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sec(b*x + a))*c*sec(b*x + a), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec (a + bx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(3/2),x)
```

```
[Out] Integral((c*sec(a + b*x))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec (bx + a))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2), x)
```

### 3.20 $\int \sqrt{c \sec(a + bx)} dx$

**Optimal.** Leaf size=38

$$\frac{2\sqrt{\cos(a + bx)}\text{EllipticF}\left(\frac{1}{2}(a + bx), 2\right)\sqrt{c \sec(a + bx)}}{b}$$

[Out] (2\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[c\*Sec[a + b\*x]])/b

**Rubi [A]** time = 0.0192504, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3771, 2641}

$$\frac{2\sqrt{\cos(a + bx)}F\left(\frac{1}{2}(a + bx) \middle| 2\right)\sqrt{c \sec(a + bx)}}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c\*Sec[a + b\*x]],x]

[Out] (2\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[c\*Sec[a + b\*x]])/b

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] :> Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \sqrt{c \sec(a + bx)} dx &= \left(\sqrt{\cos(a + bx)}\sqrt{c \sec(a + bx)}\right) \int \frac{1}{\sqrt{\cos(a + bx)}} dx \\ &= \frac{2\sqrt{\cos(a + bx)}F\left(\frac{1}{2}(a + bx) \middle| 2\right)\sqrt{c \sec(a + bx)}}{b} \end{aligned}$$

**Mathematica [A]** time = 0.0197674, size = 38, normalized size = 1.

$$\frac{2\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right)\sqrt{c\sec(a+bx)}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c\*Sec[a + b\*x]], x]

[Out] (2\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[c\*Sec[a + b\*x]])/b

**Maple [C]** time = 0.153, size = 98, normalized size = 2.6

$$\frac{-2i(-1 + \cos(bx + a))(\cos(bx + a) + 1)^2}{b(\sin(bx + a))^2} \sqrt{\frac{c}{\cos(bx + a)}} \sqrt{(\cos(bx + a) + 1)^{-1}} \sqrt{\frac{\cos(bx + a)}{\cos(bx + a) + 1}} \text{EllipticF}\left(\frac{i(-1 + \cos(bx + a))}{\sin(bx + a)}, 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*sec(b\*x+a))^(1/2), x)

[Out] -2\*I/b\*(c/cos(b\*x+a))^(1/2)\*(-1+cos(b\*x+a))\*(1/(cos(b\*x+a)+1))^(1/2)\*(cos(b\*x+a)/(cos(b\*x+a)+1))^(1/2)\*EllipticF(I\*(-1+cos(b\*x+a))/sin(b\*x+a), I)\*(cos(b\*x+a)+1)^2/sin(b\*x+a)^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \sec(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(c\*sec(b\*x + a)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{c \sec(bx + a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*sec(b*x+a))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(c*sec(b*x + a)), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \sec(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*sec(b*x+a))**(1/2),x)`

[Out] `Integral(sqrt(c*sec(a + b*x)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \sec(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*sec(b*x+a))^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(c*sec(b*x + a)), x)`

$$3.21 \quad \int \frac{1}{\sqrt{c \sec(a+bx)}} dx$$

**Optimal.** Leaf size=38

$$\frac{2E\left(\frac{1}{2}(a+bx) \middle| 2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c \sec(a+bx)}}$$

[Out] (2\*EllipticE[(a + b\*x)/2, 2])/(b\*Sqrt[Cos[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]])

**Rubi [A]** time = 0.0272559, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3771, 2639}

$$\frac{2E\left(\frac{1}{2}(a+bx) \middle| 2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c \sec(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[c\*Sec[a + b\*x]], x]

[Out] (2\*EllipticE[(a + b\*x)/2, 2])/(b\*Sqrt[Cos[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]])

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps



$$\int \frac{1}{\sqrt{c \sec(a + bx)}} dx = \frac{\int \sqrt{\cos(a + bx)} dx}{\sqrt{\cos(a + bx)} \sqrt{c \sec(a + bx)}} \\ = \frac{2E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{b\sqrt{\cos(a + bx)} \sqrt{c \sec(a + bx)}}$$

**Mathematica [A]** time = 0.0290382, size = 38, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{b\sqrt{\cos(a + bx)} \sqrt{c \sec(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[c\*Sec[a + b\*x]],x]

[Out] (2\*EllipticE[(a + b\*x)/2, 2])/(b\*Sqrt[Cos[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]])

**Maple [C]** time = 0.164, size = 306, normalized size = 8.1

$$2 \frac{1}{b \sin(bx + a) c} \left( i \sqrt{(\cos(bx + a) + 1)^{-1}} \sqrt{\frac{\cos(bx + a)}{\cos(bx + a) + 1}} \text{EllipticF} \left( \frac{i(-1 + \cos(bx + a))}{\sin(bx + a)}, i \right) \cos(bx + a) \sin(bx + a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*sec(b\*x+a))^(1/2),x)

[Out] 2/b\*(I\*(1/(cos(b\*x+a)+1))^(1/2)\*(cos(b\*x+a)/(cos(b\*x+a)+1))^(1/2)\*EllipticF(I\*(-1+cos(b\*x+a))/sin(b\*x+a),I)\*cos(b\*x+a)\*sin(b\*x+a)-I\*EllipticE(I\*(-1+cos(b\*x+a))/sin(b\*x+a),I)\*cos(b\*x+a)\*sin(b\*x+a)\*(1/(cos(b\*x+a)+1))^(1/2)\*(cos(b\*x+a)/(cos(b\*x+a)+1))^(1/2)+I\*EllipticF(I\*(-1+cos(b\*x+a))/sin(b\*x+a),I)\*(1/(cos(b\*x+a)+1))^(1/2)\*(cos(b\*x+a)/(cos(b\*x+a)+1))^(1/2)\*sin(b\*x+a)-I\*EllipticE(I\*(-1+cos(b\*x+a))/sin(b\*x+a),I)\*sin(b\*x+a)\*(1/(cos(b\*x+a)+1))^(1/2)\*(cos(b\*x+a)/(cos(b\*x+a)+1))^(1/2)-cos(b\*x+a)^2+cos(b\*x+a)\*(c/cos(b\*x+a))^(1/2)/sin(b\*x+a)/c

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c \sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(c\*sec(b\*x + a)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c \sec(bx + a)}}{c \sec(bx + a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c\*sec(b\*x + a))/(c\*sec(b\*x + a)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c \sec(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))\*\*(1/2),x)

[Out] Integral(1/sqrt(c\*sec(a + b\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c \sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*sec(b*x + a)), x)
```

$$3.22 \quad \int \frac{1}{(c \sec(a+bx))^{3/2}} dx$$

**Optimal.** Leaf size=72

$$\frac{2\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right)\sqrt{c \sec(a+bx)}}{3bc^2} + \frac{2 \sin(a+bx)}{3bc\sqrt{c \sec(a+bx)}}$$

[Out] (2\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[c\*Sec[a + b\*x]])/(3\*b\*c^2) + (2\*Sin[a + b\*x])/(3\*b\*c\*Sqrt[c\*Sec[a + b\*x]])

**Rubi [A]** time = 0.0461409, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3769, 3771, 2641}

$$\frac{2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c \sec(a+bx)}}{3bc^2} + \frac{2 \sin(a+bx)}{3bc\sqrt{c \sec(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[(c\*Sec[a + b\*x])^(-3/2), x]

[Out] (2\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[c\*Sec[a + b\*x]])/(3\*b\*c^2) + (2\*Sin[a + b\*x])/(3\*b\*c\*Sqrt[c\*Sec[a + b\*x]])

#### Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d^n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

#### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{(c \sec(a + bx))^{3/2}} dx &= \frac{2 \sin(a + bx)}{3bc \sqrt{c \sec(a + bx)}} + \frac{\int \sqrt{c \sec(a + bx)} dx}{3c^2} \\ &= \frac{2 \sin(a + bx)}{3bc \sqrt{c \sec(a + bx)}} + \frac{(\sqrt{\cos(a + bx)} \sqrt{c \sec(a + bx)}) \int \frac{1}{\sqrt{\cos(a + bx)}} dx}{3c^2} \\ &= \frac{2 \sqrt{\cos(a + bx)} F\left(\frac{1}{2}(a + bx) \middle| 2\right) \sqrt{c \sec(a + bx)}}{3bc^2} + \frac{2 \sin(a + bx)}{3bc \sqrt{c \sec(a + bx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0579416, size = 59, normalized size = 0.82

$$\frac{\sec^2(a + bx) \left( 2 \sqrt{\cos(a + bx)} \text{EllipticF}\left(\frac{1}{2}(a + bx), 2\right) + \sin(2(a + bx)) \right)}{3b(c \sec(a + bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(-3/2), x]
```

```
[Out] (Sec[a + b*x]^2*(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + Sin[2*(a + b*x)]))/(3*b*(c*Sec[a + b*x])^(3/2))
```

**Maple [C]** time = 0.155, size = 131, normalized size = 1.8

$$\frac{2 (\cos(bx + a) + 1)^2 (-1 + \cos(bx + a))}{3b (\cos(bx + a))^2 (\sin(bx + a))^3} \left( i \text{EllipticF}\left(\frac{i(-1 + \cos(bx + a))}{\sin(bx + a)}, i\right) \sqrt{(\cos(bx + a) + 1)^{-1}} \sqrt{\frac{\cos(bx + a)}{\cos(bx + a) + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sec(b*x+a))^(3/2), x)
```

```
[Out] -2/3/b*(cos(b*x+a)+1)^2*(-1+cos(b*x+a))*(I*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a), I)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*sin(b*
```

$$\frac{x+a - \cos(b*x+a)^2 + \cos(b*x+a)}{(c/\cos(b*x+a))^{3/2} / \cos(b*x+a)^2 / \sin(b*x+a)^3}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c \sec(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate((c\*sec(b\*x + a))^(-3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c \sec(bx + a)}}{c^2 \sec(bx + a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c\*sec(b\*x + a))/(c^2\*sec(b\*x + a)^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c \sec(a + bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a)\*\*(3/2),x)

[Out] Integral((c\*sec(a + b\*x))\*\*(-3/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c \sec(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))^(3/2),x, algorithm="giac")

[Out] integrate((c\*sec(b\*x + a))^(-3/2), x)

$$3.23 \quad \int \frac{1}{(c \sec(a+bx))^{5/2}} dx$$

**Optimal.** Leaf size=72

$$\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bc^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}} + \frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}}$$

[Out] (6\*EllipticE[(a + b\*x)/2, 2])/(5\*b\*c^2\*Sqrt[Cos[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]) + (2\*Sin[a + b\*x])/(5\*b\*c\*(c\*Sec[a + b\*x])^(3/2))

**Rubi [A]** time = 0.0378077, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3769, 3771, 2639}

$$\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bc^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}} + \frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*Sec[a + b\*x])^(-5/2), x]

[Out] (6\*EllipticE[(a + b\*x)/2, 2])/(5\*b\*c^2\*Sqrt[Cos[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]) + (2\*Sin[a + b\*x])/(5\*b\*c\*(c\*Sec[a + b\*x])^(3/2))

#### Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

#### Rule 2639



```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{(c \sec(a + bx))^{5/2}} dx &= \frac{2 \sin(a + bx)}{5bc(c \sec(a + bx))^{3/2}} + \frac{3 \int \frac{1}{\sqrt{c \sec(a + bx)}} dx}{5c^2} \\ &= \frac{2 \sin(a + bx)}{5bc(c \sec(a + bx))^{3/2}} + \frac{3 \int \sqrt{\cos(a + bx)} dx}{5c^2 \sqrt{\cos(a + bx)} \sqrt{c \sec(a + bx)}} \\ &= \frac{6E\left(\frac{1}{2}(a + bx) \middle| 2\right)}{5bc^2 \sqrt{\cos(a + bx)} \sqrt{c \sec(a + bx)}} + \frac{2 \sin(a + bx)}{5bc(c \sec(a + bx))^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.0679626, size = 60, normalized size = 0.83

$$\frac{\sqrt{c \sec(a + bx)} \left( \sin(a + bx) + \sin(3(a + bx)) + 12 \sqrt{\cos(a + bx)} E\left(\frac{1}{2}(a + bx) \middle| 2\right) \right)}{10bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(-5/2),x]
```

```
[Out] (Sqrt[c*Sec[a + b*x]]*(12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2] + Sin[a + b*x] + Sin[3*(a + b*x)]))/(10*b*c^3)
```

**Maple [C]** time = 0.2, size = 323, normalized size = 4.5

$$\frac{2}{5b(\cos(bx + a))^3 \sin(bx + a)} \left( 3i \sqrt{(\cos(bx + a) + 1)^{-1}} \sqrt{\frac{\cos(bx + a)}{\cos(bx + a) + 1}} \text{EllipticF}\left(\frac{i(-1 + \cos(bx + a))}{\sin(bx + a)}, i\right) \cos(bx + a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sec(b*x+a))^(5/2),x)
```

```
[Out] 2/5/b*(3*I*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*cos(b*x+a)*sin(b*x+a)-3*I*EllipticE(I*
```

$(-1+\cos(b*x+a))/\sin(b*x+a), I) * \cos(b*x+a) * \sin(b*x+a) * (1/(\cos(b*x+a)+1))^{1/2} * (\cos(b*x+a)/(\cos(b*x+a)+1))^{1/2} + 3*I * \text{EllipticF}(I * (-1+\cos(b*x+a))/\sin(b*x+a), I) * (1/(\cos(b*x+a)+1))^{1/2} * (\cos(b*x+a)/(\cos(b*x+a)+1))^{1/2} * \sin(b*x+a) - 3*I * \text{EllipticE}(I * (-1+\cos(b*x+a))/\sin(b*x+a), I) * \sin(b*x+a) * (1/(\cos(b*x+a)+1))^{1/2} * (\cos(b*x+a)/(\cos(b*x+a)+1))^{1/2} - \cos(b*x+a)^4 - 2 * \cos(b*x+a)^2 + 3 * \cos(b*x+a) / (c/\cos(b*x+a))^{5/2} / \cos(b*x+a)^3 / \sin(b*x+a)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c \sec(bx + a))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))^(5/2),x, algorithm="maxima")

[Out] integrate((c\*sec(b\*x + a))^(-5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c \sec(bx + a)}}{c^3 \sec(bx + a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(c\*sec(b\*x + a))/(c^3\*sec(b\*x + a)^3), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c \sec(a + bx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))\*\*(5/2),x)

```
[Out] Integral((c*sec(a + b*x))**(-5/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c \sec(bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(5/2), x)
```

$$3.24 \quad \int \frac{1}{(c \sec(a+bx))^{7/2}} dx$$

**Optimal.** Leaf size=100

$$\frac{10\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx), 2\right)\sqrt{c \sec(a+bx)}}{21bc^4} + \frac{10 \sin(a+bx)}{21bc^3\sqrt{c \sec(a+bx)}} + \frac{2 \sin(a+bx)}{7bc(c \sec(a+bx))^{5/2}}$$

[Out] (10\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[c\*Sec[a + b\*x]])/(21\*b\*c^4) + (2\*Sin[a + b\*x])/(7\*b\*c\*(c\*Sec[a + b\*x])^(5/2)) + (10\*Sin[a + b\*x])/(21\*b\*c^3\*Sqrt[c\*Sec[a + b\*x]])

**Rubi [A]** time = 0.059252, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3769, 3771, 2641}

$$\frac{10 \sin(a+bx)}{21bc^3\sqrt{c \sec(a+bx)}} + \frac{10\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx) \middle| 2\right)\sqrt{c \sec(a+bx)}}{21bc^4} + \frac{2 \sin(a+bx)}{7bc(c \sec(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*Sec[a + b\*x])^(-7/2), x]

[Out] (10\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2]\*Sqrt[c\*Sec[a + b\*x]])/(21\*b\*c^4) + (2\*Sin[a + b\*x])/(7\*b\*c\*(c\*Sec[a + b\*x])^(5/2)) + (10\*Sin[a + b\*x])/(21\*b\*c^3\*Sqrt[c\*Sec[a + b\*x]])

### Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]
```

### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]
```

Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{(c \sec(a + bx))^{7/2}} dx &= \frac{2 \sin(a + bx)}{7bc(c \sec(a + bx))^{5/2}} + \frac{5 \int \frac{1}{(c \sec(a + bx))^{3/2}} dx}{7c^2} \\ &= \frac{2 \sin(a + bx)}{7bc(c \sec(a + bx))^{5/2}} + \frac{10 \sin(a + bx)}{21bc^3 \sqrt{c \sec(a + bx)}} + \frac{5 \int \sqrt{c \sec(a + bx)} dx}{21c^4} \\ &= \frac{2 \sin(a + bx)}{7bc(c \sec(a + bx))^{5/2}} + \frac{10 \sin(a + bx)}{21bc^3 \sqrt{c \sec(a + bx)}} + \frac{(5\sqrt{\cos(a + bx)}\sqrt{c \sec(a + bx)}) \int \frac{1}{\sqrt{\cos(a + bx)}} dx}{21c^4} \\ &= \frac{10\sqrt{\cos(a + bx)}F\left(\frac{1}{2}(a + bx) \middle| 2\right) \sqrt{c \sec(a + bx)}}{21bc^4} + \frac{2 \sin(a + bx)}{7bc(c \sec(a + bx))^{5/2}} + \frac{10 \sin(a + bx)}{21bc^3 \sqrt{c \sec(a + bx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0960934, size = 66, normalized size = 0.66

$$\frac{\sqrt{c \sec(a + bx)} \left( 40 \sqrt{\cos(a + bx)} \text{EllipticF}\left(\frac{1}{2}(a + bx), 2\right) + 26 \sin(2(a + bx)) + 3 \sin(4(a + bx)) \right)}{84bc^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*Sec[a + b\*x])^(-7/2), x]

[Out] (Sqrt[c\*Sec[a + b\*x]]\*(40\*Sqrt[Cos[a + b\*x]]\*EllipticF[(a + b\*x)/2, 2] + 26\*Sin[2\*(a + b\*x)] + 3\*Sin[4\*(a + b\*x)]))/(84\*b\*c^4)

**Maple [C]** time = 0.19, size = 153, normalized size = 1.5

$$\frac{2 (\cos(bx + a) + 1)^2 (-1 + \cos(bx + a))}{21 b (\cos(bx + a))^4 (\sin(bx + a))^3} \left( 5 i \text{EllipticF}\left(\frac{i(-1 + \cos(bx + a))}{\sin(bx + a)}, i\right) \sqrt{(\cos(bx + a) + 1)^{-1}} \sqrt{\frac{\cos(bx + a)}{\cos(bx + a) + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*sec(b*x+a))^(7/2),x)`

[Out] 
$$-2/21/b*(\cos(b*x+a)+1)^2*(-1+\cos(b*x+a))*(5*I*EllipticF(I*(-1+\cos(b*x+a))/\sin(b*x+a),I)*(1/(\cos(b*x+a)+1))^(1/2)*(\cos(b*x+a)/(\cos(b*x+a)+1))^(1/2)*\sin(b*x+a)-3*\cos(b*x+a)^4+3*\cos(b*x+a)^3-5*\cos(b*x+a)^2+5*\cos(b*x+a))/(c/\cos(b*x+a))^(7/2)/\cos(b*x+a)^4/\sin(b*x+a)^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c \sec(bx + a))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*sec(b*x+a))^(7/2),x, algorithm="maxima")`

[Out] `integrate((c*sec(b*x + a))^(-7/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c \sec(bx + a)}}{c^4 \sec(bx + a)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*sec(b*x+a))^(7/2),x, algorithm="fricas")`

[Out] `integral(sqrt(c*sec(b*x + a))/(c^4*sec(b*x + a)^4), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*sec(b*x+a))**(7/2),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c \sec(bx + a))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))^(7/2),x, algorithm="giac")

[Out] integrate((c\*sec(b\*x + a))^(-7/2), x)

### 3.25 $\int \sec^{\frac{4}{3}}(a + bx) dx$

**Optimal.** Leaf size=51

$$\frac{3 \sin(a + bx) \sqrt[3]{\sec(a + bx)} \operatorname{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \cos^2(a + bx)\right)}{b \sqrt{\sin^2(a + bx)}}$$

[Out] (3\*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b\*x]^2]\*Sec[a + b\*x]^(1/3)\*Sin[a + b\*x])/(b\*Sqrt[Sin[a + b\*x]^2])

**Rubi [A]** time = 0.023147, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {3772, 2643}

$$\frac{3 \sin(a + bx) \sqrt[3]{\sec(a + bx)} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; \cos^2(a + bx)\right)}{b \sqrt{\sin^2(a + bx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^(4/3), x]

[Out] (3\*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b\*x]^2]\*Sec[a + b\*x]^(1/3)\*Sin[a + b\*x])/(b\*Sqrt[Sin[a + b\*x]^2])

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_.)])^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

#### Rubi steps



$$\int \sec^{\frac{4}{3}}(a + bx) dx = \sqrt[3]{\cos(a + bx)} \sqrt[3]{\sec(a + bx)} \int \frac{1}{\cos^{\frac{4}{3}}(a + bx)} dx$$

$$= \frac{{}_3F_2\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(a + bx)\right) \sqrt[3]{\sec(a + bx)} \sin(a + bx)}{b \sqrt{\sin^2(a + bx)}}$$

**Mathematica [A]** time = 0.0510717, size = 55, normalized size = 1.08

$$\frac{3\sqrt{-\tan^2(a + bx)} \csc(a + bx) \sqrt[3]{\sec(a + bx)} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \sec^2(a + bx)\right)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*x]^(4/3), x]

[Out] (3\*Csc[a + b\*x]\*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[a + b\*x]^2]\*Sec[a + b\*x]^(1/3)\*Sqrt[-Tan[a + b\*x]^2])/(4\*b)

**Maple [F]** time = 0.068, size = 0, normalized size = 0.

$$\int (\sec(bx + a))^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b\*x+a)^(4/3), x)

[Out] int(sec(b\*x+a)^(4/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(bx + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(4/3),x, algorithm="maxima")

[Out] integrate(sec(b\*x + a)^(4/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec(bx + a)^{\frac{4}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(4/3),x, algorithm="fricas")

[Out] integral(sec(b\*x + a)^(4/3), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)\*\*(4/3),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(bx + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(4/3),x, algorithm="giac")

[Out] integrate(sec(b\*x + a)^(4/3), x)

### 3.26 $\int \sec^{\frac{2}{3}}(a + bx) dx$

**Optimal.** Leaf size=51

$$\frac{3 \sin(a + bx) \text{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(a + bx)\right)}{b \sqrt{\sin^2(a + bx)} \sqrt[3]{\sec(a + bx)}}$$

[Out] (-3\*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b\*x]^2]\*Sin[a + b\*x])/(b\*Sec[a + b\*x]^(1/3)\*Sqrt[Sin[a + b\*x]^2])

---

**Rubi [A]** time = 0.0276046, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {3772, 2643}

$$\frac{3 \sin(a + bx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(a + bx)\right)}{b \sqrt{\sin^2(a + bx)} \sqrt[3]{\sec(a + bx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^(2/3), x]

[Out] (-3\*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b\*x]^2]\*Sin[a + b\*x])/(b\*Sec[a + b\*x]^(1/3)\*Sqrt[Sin[a + b\*x]^2])

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

#### Rubi steps

$$\int \sec^{\frac{2}{3}}(a+bx) dx = \cos^{\frac{2}{3}}(a+bx) \sec^{\frac{2}{3}}(a+bx) \int \frac{1}{\cos^{\frac{2}{3}}(a+bx)} dx$$

$$= -\frac{{}_3F_2\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(a+bx)\right) \sin(a+bx)}{b\sqrt[3]{\sec(a+bx)}\sqrt{\sin^2(a+bx)}}$$

**Mathematica [A]** time = 0.0356083, size = 55, normalized size = 1.08

$$\frac{3\sqrt{-\tan^2(a+bx)} \operatorname{csc}(a+bx) \operatorname{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \sec^2(a+bx)\right)}{2b\sqrt[3]{\sec(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*x]^(2/3), x]

[Out] (3\*Csc[a + b\*x]\*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[a + b\*x]^2]\*Sqrt[-Tan[a + b\*x]^2])/(2\*b\*Sec[a + b\*x]^(1/3))

**Maple [F]** time = 0.094, size = 0, normalized size = 0.

$$\int (\sec(bx+a))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b\*x+a)^(2/3), x)

[Out] int(sec(b\*x+a)^(2/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(bx+a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(2/3),x, algorithm="maxima")

[Out] integrate(sec(b\*x + a)^(2/3), x)

---

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec(bx + a)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(2/3),x, algorithm="fricas")

[Out] integral(sec(b\*x + a)^(2/3), x)

---

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \sec^{\frac{2}{3}}(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)\*\*(2/3),x)

[Out] Integral(sec(a + b\*x)\*\*(2/3), x)

---

**Giac** [F] time = 0., size = 0, normalized size = 0.

$$\int \sec(bx + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(2/3),x, algorithm="giac")

[Out] integrate(sec(b\*x + a)^(2/3), x)

### 3.27 $\int \sqrt[3]{\sec(a + bx)} dx$

**Optimal.** Leaf size=53

$$\frac{3 \sin(a + bx) \operatorname{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos^2(a + bx)\right)}{2b \sqrt{\sin^2(a + bx)} \sec^{\frac{2}{3}}(a + bx)}$$

[Out] (-3\*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b\*x]^2]\*Sin[a + b\*x])/(2\*b\*Sec[a + b\*x]^(2/3)\*Sqrt[Sin[a + b\*x]^2])

**Rubi [A]** time = 0.0332359, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {3772, 2643}

$$\frac{3 \sin(a + bx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(a + bx)\right)}{2b \sqrt{\sin^2(a + bx)} \sec^{\frac{2}{3}}(a + bx)}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^(1/3), x]

[Out] (-3\*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b\*x]^2]\*Sin[a + b\*x])/(2\*b\*Sec[a + b\*x]^(2/3)\*Sqrt[Sin[a + b\*x]^2])

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*SIN[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

#### Rubi steps

$$\int \sqrt[3]{\sec(a+bx)} dx = \sqrt[3]{\cos(a+bx)} \sqrt[3]{\sec(a+bx)} \int \frac{1}{\sqrt[3]{\cos(a+bx)}} dx$$

$$= -\frac{{}_3F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(a+bx)\right) \sin(a+bx)}{2b \sec^{\frac{2}{3}}(a+bx) \sqrt{\sin^2(a+bx)}}$$

**Mathematica [A]** time = 0.0336867, size = 53, normalized size = 1.

$$\frac{3\sqrt{-\tan^2(a+bx)} \operatorname{Csc}(a+bx) \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \sec^2(a+bx)\right)}{b \sec^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*x]^(1/3), x]

[Out] (3\*Csc[a + b\*x]\*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[a + b\*x]^2]\*Sqrt[-Tan[a + b\*x]^2])/(b\*Sec[a + b\*x]^(2/3))

**Maple [F]** time = 0.1, size = 0, normalized size = 0.

$$\int \sqrt[3]{\sec(bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b\*x+a)^(1/3), x)

[Out] int(sec(b\*x+a)^(1/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(bx+a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(1/3),x, algorithm="maxima")

[Out] integrate(sec(b\*x + a)^(1/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec(bx + a)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(1/3),x, algorithm="fricas")

[Out] integral(sec(b\*x + a)^(1/3), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{\sec(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)\*\*(1/3),x)

[Out] Integral(sec(a + b\*x)\*\*(1/3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(bx + a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^(1/3),x, algorithm="giac")

[Out] integrate(sec(b\*x + a)^(1/3), x)



$$3.28 \quad \int \frac{1}{\sqrt[3]{\sec(a+bx)}} dx$$

Optimal. Leaf size=53

$$\frac{3 \sin(a+bx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(a+bx)\right)}{4b \sqrt{\sin^2(a+bx)} \sec^{\frac{4}{3}}(a+bx)}$$

[Out] (-3\*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b\*x]^2]\*Sin[a + b\*x])/(4\*b\*Sec[a + b\*x]^(4/3)\*Sqrt[Sin[a + b\*x]^2])

**Rubi [A]** time = 0.0254906, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {3772, 2643}

$$\frac{3 \sin(a+bx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(a+bx)\right)}{4b \sqrt{\sin^2(a+bx)} \sec^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^(-1/3), x]

[Out] (-3\*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b\*x]^2]\*Sin[a + b\*x])/(4\*b\*Sec[a + b\*x]^(4/3)\*Sqrt[Sin[a + b\*x]^2])

### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

### Rubi steps

$$\int \frac{1}{\sqrt[3]{\sec(a+bx)}} dx = \cos^{\frac{2}{3}}(a+bx) \sec^{\frac{2}{3}}(a+bx) \int \sqrt[3]{\cos(a+bx)} dx$$

$$= -\frac{{}_3F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(a+bx)\right) \sin(a+bx)}{4b \sec^{\frac{4}{3}}(a+bx) \sqrt{\sin^2(a+bx)}}$$

**Mathematica [A]** time = 0.0815557, size = 53, normalized size = 1.

$$\frac{3\sqrt{-\tan^2(a+bx)} \operatorname{Csc}(a+bx) \operatorname{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \sec^2(a+bx)\right)}{b \sec^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*x]^(-1/3), x]

[Out] (-3\*Csc[a + b\*x]\*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[a + b\*x]^2]\*Sqrt[-Tan[a + b\*x]^2])/(b\*Sec[a + b\*x]^(4/3))

**Maple [F]** time = 0.093, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\sec(bx+a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(b\*x+a)^(1/3), x)

[Out] int(1/sec(b\*x+a)^(1/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(bx+a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(-1/3), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sec(bx+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(-1/3), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\sec(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(1/3),x)
```

```
[Out] Integral(sec(a + b*x)**(-1/3), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(bx+a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(-1/3), x)
```

$$3.29 \quad \int \frac{1}{\sec^{\frac{2}{3}}(a+bx)} dx$$

**Optimal.** Leaf size=53

$$\frac{3 \sin(a+bx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos^2(a+bx)\right)}{5b \sqrt{\sin^2(a+bx)} \sec^{\frac{5}{3}}(a+bx)}$$

[Out] (-3\*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b\*x]^2]\*Sin[a + b\*x])/(5\*b\*Sec[a + b\*x]^(5/3)\*Sqrt[Sin[a + b\*x]^2])

**Rubi [A]** time = 0.0254632, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {3772, 2643}

$$\frac{3 \sin(a+bx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(a+bx)\right)}{5b \sqrt{\sin^2(a+bx)} \sec^{\frac{5}{3}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^(-2/3), x]

[Out] (-3\*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b\*x]^2]\*Sin[a + b\*x])/(5\*b\*Sec[a + b\*x]^(5/3)\*Sqrt[Sin[a + b\*x]^2])

### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^n], x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^n], x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*SIN[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

### Rubi steps

$$\int \frac{1}{\sec^{\frac{2}{3}}(a+bx)} dx = \sqrt[3]{\cos(a+bx)} \sqrt[3]{\sec(a+bx)} \int \cos^{\frac{2}{3}}(a+bx) dx$$

$$= -\frac{{}_3F_2\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(a+bx)\right) \sin(a+bx)}{5b \sec^{\frac{5}{3}}(a+bx) \sqrt{\sin^2(a+bx)}}$$

**Mathematica [A]** time = 0.0829981, size = 55, normalized size = 1.04

$$\frac{3\sqrt{-\tan^2(a+bx)} \csc(a+bx) \text{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \sec^2(a+bx)\right)}{2b \sec^{\frac{5}{3}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*x]^(-2/3), x]

[Out] (-3\*Csc[a + b\*x]\*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[a + b\*x]^2]\*Sqrt[-Tan[a + b\*x]^2])/(2\*b\*Sec[a + b\*x]^(5/3))

**Maple [F]** time = 0.093, size = 0, normalized size = 0.

$$\int (\sec(bx+a))^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(b\*x+a)^(2/3), x)

[Out] int(1/sec(b\*x+a)^(2/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(bx+a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(b\*x+a)^(2/3),x, algorithm="maxima")

[Out] integrate(sec(b\*x + a)^(-2/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sec^{\frac{2}{3}}(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(b\*x+a)^(2/3),x, algorithm="fricas")

[Out] integral(sec(b\*x + a)^(-2/3), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{2}{3}}(a+bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(b\*x+a)\*\*(2/3),x)

[Out] Integral(sec(a + b\*x)\*\*(-2/3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{2}{3}}(bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(b\*x+a)^(2/3),x, algorithm="giac")

[Out] integrate(sec(b\*x + a)^(-2/3), x)

$$3.30 \quad \int \frac{1}{\sec^{\frac{4}{3}}(a+bx)} dx$$

**Optimal.** Leaf size=53

$$\frac{3 \sin(a+bx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(a+bx)\right)}{7b \sqrt{\sin^2(a+bx)} \sec^{\frac{7}{3}}(a+bx)}$$

[Out] (-3\*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b\*x]^2]\*Sin[a + b\*x])/(7\*b\*Sec[a + b\*x]^(7/3)\*Sqrt[Sin[a + b\*x]^2])

**Rubi [A]** time = 0.0247945, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {3772, 2643}

$$\frac{3 \sin(a+bx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(a+bx)\right)}{7b \sqrt{\sin^2(a+bx)} \sec^{\frac{7}{3}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^(-4/3), x]

[Out] (-3\*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b\*x]^2]\*Sin[a + b\*x])/(7\*b\*Sec[a + b\*x]^(7/3)\*Sqrt[Sin[a + b\*x]^2])

### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^n\_], x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^n\_], x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*SIN[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

### Rubi steps

$$\int \frac{1}{\sec^{\frac{4}{3}}(a+bx)} dx = \cos^{\frac{2}{3}}(a+bx) \sec^{\frac{2}{3}}(a+bx) \int \cos^{\frac{4}{3}}(a+bx) dx$$

$$= -\frac{{}_3F_2\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(a+bx)\right) \sin(a+bx)}{7b \sec^{\frac{7}{3}}(a+bx) \sqrt{\sin^2(a+bx)}}$$

**Mathematica [A]** time = 0.0656554, size = 55, normalized size = 1.04

$$\frac{3\sqrt{-\tan^2(a+bx)} \csc(a+bx) \text{Hypergeometric2F1}\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \sec^2(a+bx)\right)}{4b \sec^{\frac{7}{3}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*x]^(-4/3), x]

[Out] (-3\*Csc[a + b\*x]\*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[a + b\*x]^2]\*Sqrt[-Tan[a + b\*x]^2])/(4\*b\*Sec[a + b\*x]^(7/3))

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int (\sec(bx+a))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(b\*x+a)^(4/3), x)

[Out] int(1/sec(b\*x+a)^(4/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(bx+a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(1/sec(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(-4/3), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sec(bx + a)^{\frac{4}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(-4/3), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{4}{3}}(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(4/3),x)
```

```
[Out] Integral(sec(a + b*x)**(-4/3), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(bx + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(-4/3), x)
```

### 3.31 $\int (c \sec(a + bx))^{4/3} dx$

**Optimal.** Leaf size=54

$$\frac{3c \sin(a + bx) \sqrt[3]{c \sec(a + bx)} \operatorname{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \cos^2(a + bx)\right)}{b \sqrt{\sin^2(a + bx)}}$$

[Out] (3\*c\*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b\*x]^2]\*(c\*Sec[a + b\*x])^(1/3)\*Sin[a + b\*x])/(b\*Sqrt[Sin[a + b\*x]^2])

**Rubi [A]** time = 0.0335746, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3772, 2643}

$$\frac{3c \sin(a + bx) \sqrt[3]{c \sec(a + bx)} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(a + bx)\right)}{b \sqrt{\sin^2(a + bx)}}$$

Antiderivative was successfully verified.

[In] Int[(c\*Sec[a + b\*x])^(4/3), x]

[Out] (3\*c\*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b\*x]^2]\*(c\*Sec[a + b\*x])^(1/3)\*Sin[a + b\*x])/(b\*Sqrt[Sin[a + b\*x]^2])

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*SIN[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

#### Rubi steps

$$\int (c \sec(a + bx))^{4/3} dx = \sqrt[3]{\frac{\cos(a + bx)}{c}} \sqrt[3]{c \sec(a + bx)} \int \frac{1}{\left(\frac{\cos(a+bx)}{c}\right)^{4/3}} dx$$

$$= \frac{3c {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(a + bx)\right) \sqrt[3]{c \sec(a + bx)} \sin(a + bx)}{b\sqrt{\sin^2(a + bx)}}$$

**Mathematica [A]** time = 0.0494979, size = 57, normalized size = 1.06

$$\frac{3\sqrt{-\tan^2(a + bx)} \cot(a + bx) (c \sec(a + bx))^{4/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \sec^2(a + bx)\right)}{4b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c\*Sec[a + b\*x])^(4/3), x]

[Out] (3\*Cot[a + b\*x]\*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[a + b\*x]^2]\*(c\*Sec[a + b\*x])^(4/3)\*Sqrt[-Tan[a + b\*x]^2])/(4\*b)

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int (c \sec(bx + a))^{4/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*sec(b\*x+a))^(4/3), x)

[Out] int((c\*sec(b\*x+a))^(4/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec(bx + a))^{4/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(4/3),x, algorithm="maxima")

[Out] integrate((c\*sec(b\*x + a))^(4/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c \sec(bx + a)\right)^{\frac{1}{3}} c \sec(bx + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(4/3),x, algorithm="fricas")

[Out] integral((c\*sec(b\*x + a))^(1/3)\*c\*sec(b\*x + a), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))\*\*(4/3),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec(bx + a))^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(4/3),x, algorithm="giac")

[Out] integrate((c\*sec(b\*x + a))^(4/3), x)

### 3.32 $\int (c \sec(a + bx))^{2/3} dx$

Optimal. Leaf size=54

$$-\frac{3c \sin(a + bx) \text{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(a + bx)\right)}{b \sqrt{\sin^2(a + bx)} \sqrt[3]{c \sec(a + bx)}}$$

[Out]  $(-3*c*\text{Hypergeometric2F1}[1/6, 1/2, 7/6, \text{Cos}[a + b*x]^2]*\text{Sin}[a + b*x])/(b*(c*\text{Sec}[a + b*x])^{(1/3)}*\text{Sqrt}[\text{Sin}[a + b*x]^2])$

**Rubi [A]** time = 0.0295325, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3772, 2643}

$$-\frac{3c \sin(a + bx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(a + bx)\right)}{b \sqrt{\sin^2(a + bx)} \sqrt[3]{c \sec(a + bx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*\text{Sec}[a + b*x])^{(2/3)}, x]$

[Out]  $(-3*c*\text{Hypergeometric2F1}[1/6, 1/2, 7/6, \text{Cos}[a + b*x]^2]*\text{Sin}[a + b*x])/(b*(c*\text{Sec}[a + b*x])^{(1/3)}*\text{Sqrt}[\text{Sin}[a + b*x]^2])$

#### Rule 3772

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n - 1)}*((\text{Sin}[c + d*x]/b)^{(n - 1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /;$  FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

$\text{Int}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n + 1)}*\text{Hypergeometric2F1}[1/2, (n + 1)/2, (n + 3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n + 1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /;$  FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

#### Rubi steps

$$\int (c \sec(a + bx))^{2/3} dx = \left( \frac{\cos(a + bx)}{c} \right)^{2/3} (c \sec(a + bx))^{2/3} \int \frac{1}{\left( \frac{\cos(a+bx)}{c} \right)^{2/3}} dx$$

$$= -\frac{3 \cos(a + bx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(a + bx)\right) (c \sec(a + bx))^{2/3} \sin(a + bx)}{b \sqrt{\sin^2(a + bx)}}$$

**Mathematica [A]** time = 0.0410417, size = 57, normalized size = 1.06

$$\frac{3 \sqrt{-\tan^2(a + bx)} \cot(a + bx) (c \sec(a + bx))^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \sec^2(a + bx)\right)}{2b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c\*Sec[a + b\*x])^(2/3), x]

[Out] (3\*Cot[a + b\*x]\*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[a + b\*x]^2]\*(c\*Sec[a + b\*x])^(2/3)\*Sqrt[-Tan[a + b\*x]^2])/(2\*b)

**Maple [F]** time = 0.084, size = 0, normalized size = 0.

$$\int (c \sec(bx + a))^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*sec(b\*x+a))^(2/3), x)

[Out] int((c\*sec(b\*x+a))^(2/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec(bx + a))^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(2/3),x, algorithm="maxima")

[Out] integrate((c\*sec(b\*x + a))^(2/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((c \sec (bx + a))^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(2/3),x, algorithm="fricas")

[Out] integral((c\*sec(b\*x + a))^(2/3), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec (a + bx))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))\*\*(2/3),x)

[Out] Integral((c\*sec(a + b\*x))\*\*(2/3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec (bx + a))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(2/3),x, algorithm="giac")

[Out] integrate((c\*sec(b\*x + a))^(2/3), x)

### 3.33 $\int \sqrt[3]{c \sec(a + bx)} dx$

**Optimal.** Leaf size=56

$$\frac{3c \sin(a + bx) \operatorname{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos^2(a + bx)\right)}{2b \sqrt{\sin^2(a + bx)} (c \sec(a + bx))^{2/3}}$$

[Out]  $(-3*c*\operatorname{Hypergeometric2F1}[1/3, 1/2, 4/3, \operatorname{Cos}[a + b*x]^2]*\operatorname{Sin}[a + b*x])/(2*b*(c*\operatorname{Sec}[a + b*x])^{(2/3)}*\operatorname{Sqrt}[\operatorname{Sin}[a + b*x]^2])$

**Rubi [A]** time = 0.0349386, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3772, 2643}

$$\frac{3c \sin(a + bx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(a + bx)\right)}{2b \sqrt{\sin^2(a + bx)} (c \sec(a + bx))^{2/3}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c*\operatorname{Sec}[a + b*x])^{(1/3)}, x]$

[Out]  $(-3*c*\operatorname{Hypergeometric2F1}[1/3, 1/2, 4/3, \operatorname{Cos}[a + b*x]^2]*\operatorname{Sin}[a + b*x])/(2*b*(c*\operatorname{Sec}[a + b*x])^{(2/3)}*\operatorname{Sqrt}[\operatorname{Sin}[a + b*x]^2])$

#### Rule 3772

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(b*\operatorname{Csc}[c + d*x])^{(n - 1)}*((\operatorname{Sin}[c + d*x]/b)^{(n - 1)}*\operatorname{Int}[1/(\operatorname{Sin}[c + d*x]/b)^n, x]), x] /;$  FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

$\operatorname{Int}[(b_.)*\operatorname{sin}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Cos}[c + d*x]*(b*\operatorname{Sin}[c + d*x])^{(n + 1)}*\operatorname{Hypergeometric2F1}[1/2, (n + 1)/2, (n + 3)/2, \operatorname{Sin}[c + d*x]^2])/(b*d*(n + 1)*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]^2]), x] /;$  FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

#### Rubi steps



$$\int \sqrt[3]{c \sec(a+bx)} dx = \sqrt[3]{\frac{\cos(a+bx)}{c}} \sqrt[3]{c \sec(a+bx)} \int \frac{1}{\sqrt[3]{\frac{\cos(a+bx)}{c}}} dx$$

$$= -\frac{3 \cos(a+bx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(a+bx)\right) \sqrt[3]{c \sec(a+bx)} \sin(a+bx)}{2b \sqrt{\sin^2(a+bx)}}$$

**Mathematica [A]** time = 0.0395714, size = 55, normalized size = 0.98

$$\frac{3 \sqrt{-\tan^2(a+bx)} \cot(a+bx) \sqrt[3]{c \sec(a+bx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \sec^2(a+bx)\right)}{b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c\*Sec[a + b\*x])^(1/3), x]

[Out] (3\*Cot[a + b\*x]\*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[a + b\*x]^2]\*(c\*Sec[a + b\*x])^(1/3)\*Sqrt[-Tan[a + b\*x]^2])/b

**Maple [F]** time = 0.105, size = 0, normalized size = 0.

$$\int \sqrt[3]{c \sec(bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*sec(b\*x+a))^(1/3), x)

[Out] int((c\*sec(b\*x+a))^(1/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec(bx+a))^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(1/3),x, algorithm="maxima")

[Out] integrate((c\*sec(b\*x + a))^(1/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c \sec (bx + a)\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(1/3),x, algorithm="fricas")

[Out] integral((c\*sec(b\*x + a))^(1/3), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{c \sec (a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))\*\*(1/3),x)

[Out] Integral((c\*sec(a + b\*x))\*\*(1/3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec (bx + a))^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(1/3),x, algorithm="giac")

[Out] integrate((c\*sec(b\*x + a))^(1/3), x)

$$3.34 \quad \int \frac{1}{\sqrt[3]{c \sec(a+bx)}} dx$$

Optimal. Leaf size=56

$$\frac{3c \sin(a+bx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(a+bx)\right)}{4b \sqrt{\sin^2(a+bx)} (c \sec(a+bx))^{4/3}}$$

[Out] (-3\*c\*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b\*x]^2]\*Sin[a + b\*x])/(4\*b\*(c\*Sec[a + b\*x])^(4/3)\*Sqrt[Sin[a + b\*x]^2])

**Rubi [A]** time = 0.0324897, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3772, 2643}

$$\frac{3c \sin(a+bx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(a+bx)\right)}{4b \sqrt{\sin^2(a+bx)} (c \sec(a+bx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[(c\*Sec[a + b\*x])^(-1/3), x]

[Out] (-3\*c\*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b\*x]^2]\*Sin[a + b\*x])/(4\*b\*(c\*Sec[a + b\*x])^(4/3)\*Sqrt[Sin[a + b\*x]^2])

### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

### Rubi steps

$$\int \frac{1}{\sqrt[3]{c \sec(a+bx)}} dx = \left(\frac{\cos(a+bx)}{c}\right)^{2/3} (c \sec(a+bx))^{2/3} \int \sqrt[3]{\frac{\cos(a+bx)}{c}} dx$$

$$= -\frac{3 \cos^2(a+bx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(a+bx)\right) (c \sec(a+bx))^{2/3} \sin(a+bx)}{4bc \sqrt{\sin^2(a+bx)}}$$

**Mathematica [A]** time = 0.0521933, size = 55, normalized size = 0.98

$$\frac{3\sqrt{-\tan^2(a+bx)} \cot(a+bx) \text{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \sec^2(a+bx)\right)}{b\sqrt[3]{c \sec(a+bx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c\*Sec[a + b\*x])^(-1/3), x]

[Out] (-3\*Cot[a + b\*x]\*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[a + b\*x]^2]\*Sqrt[-Tan[a + b\*x]^2])/(b\*(c\*Sec[a + b\*x])^(1/3))

**Maple [F]** time = 0.094, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{c \sec(bx+a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*sec(b\*x+a))^(1/3), x)

[Out] int(1/(c\*sec(b\*x+a))^(1/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c \sec(bx+a))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))^(1/3),x, algorithm="maxima")

[Out] integrate((c\*sec(b\*x + a))^(1/3), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(c \sec (bx + a))^{\frac{2}{3}}}{c \sec (bx + a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))^(1/3),x, algorithm="fricas")

[Out] integral((c\*sec(b\*x + a))^(2/3)/(c\*sec(b\*x + a)), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{c \sec (a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))\*\*(1/3),x)

[Out] Integral((c\*sec(a + b\*x))\*\*(-1/3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c \sec (bx + a))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))^(1/3),x, algorithm="giac")

[Out] integrate((c\*sec(b\*x + a))^(1/3), x)

$$3.35 \quad \int \frac{1}{(c \sec(a+bx))^{2/3}} dx$$

**Optimal.** Leaf size=56

$$\frac{3c \sin(a+bx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos^2(a+bx)\right)}{5b \sqrt{\sin^2(a+bx)} (c \sec(a+bx))^{5/3}}$$

[Out] (-3\*c\*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b\*x]^2]\*Sin[a + b\*x])/(5\*b\*(c\*Sec[a + b\*x])^(5/3)\*Sqrt[Sin[a + b\*x]^2])

**Rubi [A]** time = 0.0282795, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3772, 2643}

$$\frac{3c \sin(a+bx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(a+bx)\right)}{5b \sqrt{\sin^2(a+bx)} (c \sec(a+bx))^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[(c\*Sec[a + b\*x])^(-2/3), x]

[Out] (-3\*c\*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b\*x]^2]\*Sin[a + b\*x])/(5\*b\*(c\*Sec[a + b\*x])^(5/3)\*Sqrt[Sin[a + b\*x]^2])

### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^ (n\_), x\_Symbol] :> Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^ (n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

### Rubi steps

$$\int \frac{1}{(c \sec(a + bx))^{2/3}} dx = \sqrt[3]{\frac{\cos(a + bx)}{c}} \sqrt[3]{c \sec(a + bx)} \int \left(\frac{\cos(a + bx)}{c}\right)^{2/3} dx$$

$$= -\frac{3 \cos^2(a + bx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(a + bx)\right) \sqrt[3]{c \sec(a + bx)} \sin(a + bx)}{5bc \sqrt{\sin^2(a + bx)}}$$

**Mathematica [A]** time = 0.0526139, size = 57, normalized size = 1.02

$$\frac{3\sqrt{-\tan^2(a + bx)} \cot(a + bx) \text{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \sec^2(a + bx)\right)}{2b(c \sec(a + bx))^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c\*Sec[a + b\*x])^(-2/3), x]

[Out] (-3\*Cot[a + b\*x]\*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[a + b\*x]^2]\*Sqrt[-Tan[a + b\*x]^2])/(2\*b\*(c\*Sec[a + b\*x])^(2/3))

**Maple [F]** time = 0.097, size = 0, normalized size = 0.

$$\int (c \sec(bx + a))^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*sec(b\*x+a))^(2/3), x)

[Out] int(1/(c\*sec(b\*x+a))^(2/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c \sec(bx + a))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(-2/3), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(c \sec(bx + a))^{\frac{1}{3}}}{c \sec(bx + a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*sec(b*x + a))^(1/3)/(c*sec(b*x + a)), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c \sec(a + bx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))**(2/3),x)
```

```
[Out] Integral((c*sec(a + b*x))**(-2/3), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c \sec(bx + a))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(-2/3), x)
```



$$3.36 \quad \int \frac{1}{(c \sec(a+bx))^{4/3}} dx$$

**Optimal.** Leaf size=56

$$\frac{3c \sin(a+bx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(a+bx)\right)}{7b \sqrt{\sin^2(a+bx)} (c \sec(a+bx))^{7/3}}$$

[Out] (-3\*c\*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b\*x]^2]\*Sin[a + b\*x])/(7\*b\*(c\*Sec[a + b\*x])^(7/3)\*Sqrt[Sin[a + b\*x]^2])

**Rubi [A]** time = 0.0286994, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3772, 2643}

$$\frac{3c \sin(a+bx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(a+bx)\right)}{7b \sqrt{\sin^2(a+bx)} (c \sec(a+bx))^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(c\*Sec[a + b\*x])^(-4/3), x]

[Out] (-3\*c\*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b\*x]^2]\*Sin[a + b\*x])/(7\*b\*(c\*Sec[a + b\*x])^(7/3)\*Sqrt[Sin[a + b\*x]^2])

### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

### Rubi steps

$$\int \frac{1}{(c \sec(a + bx))^{4/3}} dx = \left( \frac{\cos(a + bx)}{c} \right)^{2/3} (c \sec(a + bx))^{2/3} \int \left( \frac{\cos(a + bx)}{c} \right)^{4/3} dx$$

$$= -\frac{3 \cos^3(a + bx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(a + bx)\right) (c \sec(a + bx))^{2/3} \sin(a + bx)}{7bc^2 \sqrt{\sin^2(a + bx)}}$$

**Mathematica [A]** time = 0.0688611, size = 57, normalized size = 1.02

$$-\frac{3\sqrt{-\tan^2(a + bx)} \cot(a + bx) \text{Hypergeometric2F1}\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \sec^2(a + bx)\right)}{4b(c \sec(a + bx))^{4/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c\*Sec[a + b\*x])^(-4/3), x]

[Out] (-3\*Cot[a + b\*x]\*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[a + b\*x]^2]\*Sqrt[-Tan[a + b\*x]^2])/(4\*b\*(c\*Sec[a + b\*x])^(4/3))

**Maple [F]** time = 0.054, size = 0, normalized size = 0.

$$\int (c \sec(bx + a))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*sec(b\*x+a))^(4/3), x)

[Out] int(1/(c\*sec(b\*x+a))^(4/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c \sec(bx + a))^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))^(4/3),x, algorithm="maxima")

[Out] integrate((c\*sec(b\*x + a))^(4/3), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(c \sec (bx + a))^{\frac{2}{3}}}{c^2 \sec (bx + a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))^(4/3),x, algorithm="fricas")

[Out] integral((c\*sec(b\*x + a))^(2/3)/(c^2\*sec(b\*x + a)^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c \sec (a + bx))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))\*\*(4/3),x)

[Out] Integral((c\*sec(a + b\*x))\*\*(-4/3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c \sec (bx + a))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*sec(b\*x+a))^(4/3),x, algorithm="giac")

[Out] integrate((c\*sec(b\*x + a))^(4/3), x)

### 3.37 $\int \sec^n(a + bx) dx$

**Optimal.** Leaf size=70

$$\frac{\sin(a + bx) \sec^{n-1}(a + bx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \cos^2(a + bx)\right)}{b(1-n)\sqrt{\sin^2(a + bx)}}$$

[Out] -((Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[a + b\*x]^2]\*Sec[a + b\*x]^(-1 + n)\*Sin[a + b\*x])/(b\*(1 - n)\*Sqrt[Sin[a + b\*x]^2]))

**Rubi [A]** time = 0.0328775, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3772, 2643}

$$\frac{\sin(a + bx) \sec^{n-1}(a + bx) {}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \cos^2(a + bx)\right)}{b(1-n)\sqrt{\sin^2(a + bx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*x]^n,x]

[Out] -((Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[a + b\*x]^2]\*Sec[a + b\*x]^(-1 + n)\*Sin[a + b\*x])/(b\*(1 - n)\*Sqrt[Sin[a + b\*x]^2]))

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^n], x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^n], x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*SIN[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

#### Rubi steps

$$\int \sec^n(a + bx) dx = \cos^n(a + bx) \sec^n(a + bx) \int \cos^{-n}(a + bx) dx$$

$$= -\frac{{}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \cos^2(a + bx)\right) \sec^{-1+n}(a + bx) \sin(a + bx)}{b(1-n)\sqrt{\sin^2(a + bx)}}$$

**Mathematica [A]** time = 0.0495195, size = 61, normalized size = 0.87

$$\frac{\sqrt{-\tan^2(a + bx)} \csc(a + bx) \sec^{n-1}(a + bx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n}{2}, \frac{n+2}{2}, \sec^2(a + bx)\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*x]^n, x]

[Out] (Csc[a + b\*x]\*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[a + b\*x]^2]\*Sec[a + b\*x]^(-1 + n)\*Sqrt[-Tan[a + b\*x]^2])/(b\*n)

**Maple [F]** time = 0.291, size = 0, normalized size = 0.

$$\int (\sec(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b\*x+a)^n, x)

[Out] int(sec(b\*x+a)^n, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(bx + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^n, x, algorithm="maxima")

[Out] integrate(sec(b\*x + a)^n, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sec(bx + a)^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^n,x, algorithm="fricas")

[Out] integral(sec(b\*x + a)^n, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec^n(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)\*\*n,x)

[Out] Integral(sec(a + b\*x)\*\*n, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(bx + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b\*x+a)^n,x, algorithm="giac")

[Out] integrate(sec(b\*x + a)^n, x)

### 3.38 $\int (c \sec(a + bx))^n dx$

**Optimal.** Leaf size=73

$$\frac{c \sin(a + bx)(c \sec(a + bx))^{n-1} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \cos^2(a + bx)\right)}{b(1-n)\sqrt{\sin^2(a + bx)}}$$

[Out] -((c\*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[a + b\*x]^2]\*(c\*Sec[a + b\*x])^(-1 + n)\*Sin[a + b\*x])/(b\*(1 - n)\*Sqrt[Sin[a + b\*x]^2]))

**Rubi [A]** time = 0.0311051, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {3772, 2643}

$$\frac{c \sin(a + bx)(c \sec(a + bx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \cos^2(a + bx)\right)}{b(1-n)\sqrt{\sin^2(a + bx)}}$$

Antiderivative was successfully verified.

[In] Int[(c\*Sec[a + b\*x])^n,x]

[Out] -((c\*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[a + b\*x]^2]\*(c\*Sec[a + b\*x])^(-1 + n)\*Sin[a + b\*x])/(b\*(1 - n)\*Sqrt[Sin[a + b\*x]^2]))

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^n\_, x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^n\_, x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

#### Rubi steps

$$\int (c \sec(a + bx))^n dx = \left(\frac{\cos(a + bx)}{c}\right)^n (c \sec(a + bx))^n \int \left(\frac{\cos(a + bx)}{c}\right)^{-n} dx$$

$$= -\frac{\cos(a + bx) {}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \cos^2(a + bx)\right) (c \sec(a + bx))^n \sin(a + bx)}{b(1-n)\sqrt{\sin^2(a + bx)}}$$

**Mathematica [A]** time = 0.0469835, size = 61, normalized size = 0.84

$$\frac{\sqrt{-\tan^2(a + bx)} \cot(a + bx) (c \sec(a + bx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n}{2}, \frac{n+2}{2}, \sec^2(a + bx)\right)}{bn}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c\*Sec[a + b\*x])^n, x]

[Out] (Cot[a + b\*x]\*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[a + b\*x]^2]\*(c\*Sec[a + b\*x])^n\*sqrt[-Tan[a + b\*x]^2])/(b\*n)

**Maple [F]** time = 0.265, size = 0, normalized size = 0.

$$\int (c \sec(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*sec(b\*x+a))^n, x)

[Out] int((c\*sec(b\*x+a))^n, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((c\*sec(b\*x+a))^n,x, algorithm="maxima")

[Out] integrate((c\*sec(b\*x + a))^n, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((c \sec (bx + a))^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^n,x, algorithm="fricas")

[Out] integral((c\*sec(b\*x + a))^n, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec (a + bx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))\*\*n,x)

[Out] Integral((c\*sec(a + b\*x))\*\*n, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec (bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^n,x, algorithm="giac")

[Out] integrate((c\*sec(b\*x + a))^n, x)

### 3.39 $\int \sec^2(x)^{7/2} dx$

**Optimal.** Leaf size=50

$$\frac{1}{6} \tan(x) \sec^2(x)^{5/2} + \frac{5}{24} \tan(x) \sec^2(x)^{3/2} + \frac{5}{16} \tan(x) \sqrt{\sec^2(x)} + \frac{5}{16} \sinh^{-1}(\tan(x))$$

[Out] (5\*ArcSinh[Tan[x]])/16 + (5\*Sqrt[Sec[x]^2]\*Tan[x])/16 + (5\*(Sec[x]^2)^(3/2)\*Tan[x])/24 + ((Sec[x]^2)^(5/2)\*Tan[x])/6

**Rubi [A]** time = 0.016905, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4122, 195, 215}

$$\frac{1}{6} \tan(x) \sec^2(x)^{5/2} + \frac{5}{24} \tan(x) \sec^2(x)^{3/2} + \frac{5}{16} \tan(x) \sqrt{\sec^2(x)} + \frac{5}{16} \sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2)^(7/2), x]

[Out] (5\*ArcSinh[Tan[x]])/16 + (5\*Sqrt[Sec[x]^2]\*Tan[x])/16 + (5\*(Sec[x]^2)^(3/2)\*Tan[x])/24 + ((Sec[x]^2)^(5/2)\*Tan[x])/6

#### Rule 4122

Int[((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^2)^(p\_), x\_Symbol] := With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[(b\*ff)/f, Subst[Int[(b + b\*ff^2\*x^2)^(p - 1), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned}
\int \sec^2(x)^{7/2} dx &= \text{Subst} \left( \int (1+x^2)^{5/2} dx, x, \tan(x) \right) \\
&= \frac{1}{6} \sec^2(x)^{5/2} \tan(x) + \frac{5}{6} \text{Subst} \left( \int (1+x^2)^{3/2} dx, x, \tan(x) \right) \\
&= \frac{5}{24} \sec^2(x)^{3/2} \tan(x) + \frac{1}{6} \sec^2(x)^{5/2} \tan(x) + \frac{5}{8} \text{Subst} \left( \int \sqrt{1+x^2} dx, x, \tan(x) \right) \\
&= \frac{5}{16} \sqrt{\sec^2(x)} \tan(x) + \frac{5}{24} \sec^2(x)^{3/2} \tan(x) + \frac{1}{6} \sec^2(x)^{5/2} \tan(x) + \frac{5}{16} \text{Subst} \left( \int \frac{1}{\sqrt{1+x^2}} dx, x, \tan(x) \right) \\
&= \frac{5}{16} \sinh^{-1}(\tan(x)) + \frac{5}{16} \sqrt{\sec^2(x)} \tan(x) + \frac{5}{24} \sec^2(x)^{3/2} \tan(x) + \frac{1}{6} \sec^2(x)^{5/2} \tan(x)
\end{aligned}$$

**Mathematica [A]** time = 0.290101, size = 74, normalized size = 1.48

$$\frac{1}{96} \cos(x) \sqrt{\sec^2(x)} \left( \frac{1}{8} (198 \sin(x) + 85 \sin(3x) + 15 \sin(5x)) \sec^6(x) - 30 \log \left( \cos \left( \frac{x}{2} \right) - \sin \left( \frac{x}{2} \right) \right) + 30 \log \left( \sin \left( \frac{x}{2} \right) + \cos \left( \frac{x}{2} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2)^(7/2), x]

[Out] (Cos[x]\*Sqrt[Sec[x]^2]\*(-30\*Log[Cos[x/2] - Sin[x/2]] + 30\*Log[Cos[x/2] + Sin[x/2]] + (Sec[x]^6\*(198\*Sin[x] + 85\*Sin[3\*x] + 15\*Sin[5\*x]))/8))/96

**Maple [A]** time = 0.159, size = 72, normalized size = 1.4

$$-\frac{\cos(x)}{48} \left( 15 \ln \left( -\frac{-1 + \cos(x) + \sin(x)}{\sin(x)} \right) (\cos(x))^6 - 15 \ln \left( -\frac{-1 + \cos(x) - \sin(x)}{\sin(x)} \right) (\cos(x))^6 - 15 (\cos(x))^4 \sin(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sec(x)^2)^(7/2), x)

[Out] -1/48\*(15\*ln(-(-1+cos(x)+sin(x))/sin(x))\*cos(x)^6-15\*ln(-(-1+cos(x)-sin(x))/sin(x))\*cos(x)^6-15\*cos(x)^4\*sin(x)-10\*cos(x)^2\*sin(x)-8\*sin(x))\*cos(x)\*(1/cos(x)^2)^(7/2)

**Maxima [A]** time = 1.74061, size = 57, normalized size = 1.14

$$\frac{1}{6} (\tan(x)^2 + 1)^{\frac{5}{2}} \tan(x) + \frac{5}{24} (\tan(x)^2 + 1)^{\frac{3}{2}} \tan(x) + \frac{5}{16} \sqrt{\tan(x)^2 + 1} \tan(x) + \frac{5}{16} \operatorname{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)^2)^(7/2),x, algorithm="maxima")

[Out] 1/6\*(tan(x)^2 + 1)^(5/2)\*tan(x) + 5/24\*(tan(x)^2 + 1)^(3/2)\*tan(x) + 5/16\*sqrt(tan(x)^2 + 1)\*tan(x) + 5/16\*arcsinh(tan(x))

**Fricas [A]** time = 1.36736, size = 162, normalized size = 3.24

$$\frac{15 \cos(x)^6 \log(\sin(x) + 1) - 15 \cos(x)^6 \log(-\sin(x) + 1) + 2(15 \cos(x)^4 + 10 \cos(x)^2 + 8) \sin(x)}{96 \cos(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)^2)^(7/2),x, algorithm="fricas")

[Out] -1/96\*(15\*cos(x)^6\*log(sin(x) + 1) - 15\*cos(x)^6\*log(-sin(x) + 1) + 2\*(15\*cos(x)^4 + 10\*cos(x)^2 + 8)\*sin(x))/cos(x)^6

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)\*\*2)\*\*(7/2),x)

[Out] Timed out

**Giac [A]** time = 1.28619, size = 80, normalized size = 1.6

$$\frac{5 \log(\sin(x) + 1)}{32 \operatorname{sgn}(\cos(x))} - \frac{5 \log(-\sin(x) + 1)}{32 \operatorname{sgn}(\cos(x))} - \frac{15 \sin(x)^5 - 40 \sin(x)^3 + 33 \sin(x)}{48 (\sin(x)^2 - 1)^3 \operatorname{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 5/32*log(sin(x) + 1)/sgn(cos(x)) - 5/32*log(-sin(x) + 1)/sgn(cos(x)) - 1/48  
*(15*sin(x)^5 - 40*sin(x)^3 + 33*sin(x))/((sin(x)^2 - 1)^3*sgn(cos(x)))
```

### 3.40 $\int \sec^2(x)^{5/2} dx$

**Optimal.** Leaf size=36

$$\frac{1}{4} \tan(x) \sec^2(x)^{3/2} + \frac{3}{8} \tan(x) \sqrt{\sec^2(x)} + \frac{3}{8} \sinh^{-1}(\tan(x))$$

[Out] (3\*ArcSinh[Tan[x]])/8 + (3\*Sqrt[Sec[x]^2]\*Tan[x])/8 + ((Sec[x]^2)^(3/2)\*Tan[x])/4

**Rubi [A]** time = 0.0124014, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4122, 195, 215}

$$\frac{1}{4} \tan(x) \sec^2(x)^{3/2} + \frac{3}{8} \tan(x) \sqrt{\sec^2(x)} + \frac{3}{8} \sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2)^(5/2), x]

[Out] (3\*ArcSinh[Tan[x]])/8 + (3\*Sqrt[Sec[x]^2]\*Tan[x])/8 + ((Sec[x]^2)^(3/2)\*Tan[x])/4

#### Rule 4122

Int[((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)^2])^(p\_), x\_Symbol] := With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[(b\*ff)/f, Subst[Int[(b + b\*ff^2\*x^2)^(p - 1), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned}
\int \sec^2(x)^{5/2} dx &= \text{Subst} \left( \int (1+x^2)^{3/2} dx, x, \tan(x) \right) \\
&= \frac{1}{4} \sec^2(x)^{3/2} \tan(x) + \frac{3}{4} \text{Subst} \left( \int \sqrt{1+x^2} dx, x, \tan(x) \right) \\
&= \frac{3}{8} \sqrt{\sec^2(x)} \tan(x) + \frac{1}{4} \sec^2(x)^{3/2} \tan(x) + \frac{3}{8} \text{Subst} \left( \int \frac{1}{\sqrt{1+x^2}} dx, x, \tan(x) \right) \\
&= \frac{3}{8} \sinh^{-1}(\tan(x)) + \frac{3}{8} \sqrt{\sec^2(x)} \tan(x) + \frac{1}{4} \sec^2(x)^{3/2} \tan(x)
\end{aligned}$$

**Mathematica [A]** time = 0.1402, size = 68, normalized size = 1.89

$$\frac{1}{16} \cos(x) \sqrt{\sec^2(x)} \left( \frac{1}{2} (11 \sin(x) + 3 \sin(3x)) \sec^4(x) - 6 \log \left( \cos \left( \frac{x}{2} \right) - \sin \left( \frac{x}{2} \right) \right) + 6 \log \left( \sin \left( \frac{x}{2} \right) + \cos \left( \frac{x}{2} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2)^(5/2), x]

[Out] (Cos[x]\*Sqrt[Sec[x]^2]\*(-6\*Log[Cos[x/2] - Sin[x/2]] + 6\*Log[Cos[x/2] + Sin[x/2]] + (Sec[x]^4\*(11\*Sin[x] + 3\*Sin[3\*x]))/2))/16

**Maple [B]** time = 0.091, size = 64, normalized size = 1.8

$$\frac{\cos(x)}{8} \left( 3 (\cos(x))^4 \ln \left( -\frac{-1 + \cos(x) - \sin(x)}{\sin(x)} \right) - 3 (\cos(x))^4 \ln \left( -\frac{-1 + \cos(x) + \sin(x)}{\sin(x)} \right) + 3 (\cos(x))^2 \sin(x) + 2 \sin(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sec(x)^2)^(5/2), x)

[Out] 1/8\*(3\*cos(x)^4\*ln(-(-1+cos(x)-sin(x))/sin(x))-3\*cos(x)^4\*ln(-(-1+cos(x)+sin(x))/sin(x))+3\*cos(x)^2\*sin(x)+2\*sin(x))\*cos(x)\*(1/cos(x)^2)^(5/2)

**Maxima [A]** time = 1.64798, size = 41, normalized size = 1.14

$$\frac{1}{4} (\tan(x)^2 + 1)^{\frac{3}{2}} \tan(x) + \frac{3}{8} \sqrt{\tan(x)^2 + 1} \tan(x) + \frac{3}{8} \operatorname{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)^2)^(5/2),x, algorithm="maxima")

[Out] 1/4\*(tan(x)^2 + 1)^(3/2)\*tan(x) + 3/8\*sqrt(tan(x)^2 + 1)\*tan(x) + 3/8\*arcsinh(tan(x))

**Fricas [A]** time = 1.40946, size = 139, normalized size = 3.86

$$\frac{3 \cos(x)^4 \log(\sin(x) + 1) - 3 \cos(x)^4 \log(-\sin(x) + 1) + 2(3 \cos(x)^2 + 2) \sin(x)}{16 \cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)^2)^(5/2),x, algorithm="fricas")

[Out] -1/16\*(3\*cos(x)^4\*log(sin(x) + 1) - 3\*cos(x)^4\*log(-sin(x) + 1) + 2\*(3\*cos(x)^2 + 2)\*sin(x))/cos(x)^4

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)\*\*2)\*\*(5/2),x)

[Out] Timed out

**Giac [B]** time = 1.27814, size = 72, normalized size = 2.

$$\frac{3 \log(\sin(x) + 1)}{16 \operatorname{sgn}(\cos(x))} - \frac{3 \log(-\sin(x) + 1)}{16 \operatorname{sgn}(\cos(x))} - \frac{3 \sin(x)^3 - 5 \sin(x)}{8(\sin(x)^2 - 1)^2 \operatorname{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((sec(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 3/16*log(sin(x) + 1)/sgn(cos(x)) - 3/16*log(-sin(x) + 1)/sgn(cos(x)) - 1/8*  
(3*sin(x)^3 - 5*sin(x))/((sin(x)^2 - 1)^2*sgn(cos(x)))
```

### 3.41 $\int \sec^2(x)^{3/2} dx$

**Optimal.** Leaf size=22

$$\frac{1}{2} \tan(x) \sqrt{\sec^2(x)} + \frac{1}{2} \sinh^{-1}(\tan(x))$$

[Out] ArcSinh[Tan[x]]/2 + (Sqrt[Sec[x]^2]\*Tan[x])/2

**Rubi [A]** time = 0.0085539, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4122, 195, 215}

$$\frac{1}{2} \tan(x) \sqrt{\sec^2(x)} + \frac{1}{2} \sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2)^(3/2), x]

[Out] ArcSinh[Tan[x]]/2 + (Sqrt[Sec[x]^2]\*Tan[x])/2

#### Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

#### Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])
```

#### Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

#### Rubi steps

$$\begin{aligned}
\int \sec^2(x)^{3/2} dx &= \text{Subst} \left( \int \sqrt{1+x^2} dx, x, \tan(x) \right) \\
&= \frac{1}{2} \sqrt{\sec^2(x)} \tan(x) + \frac{1}{2} \text{Subst} \left( \int \frac{1}{\sqrt{1+x^2}} dx, x, \tan(x) \right) \\
&= \frac{1}{2} \sinh^{-1}(\tan(x)) + \frac{1}{2} \sqrt{\sec^2(x)} \tan(x)
\end{aligned}$$

**Mathematica [B]** time = 0.0565225, size = 52, normalized size = 2.36

$$\frac{1}{2} \cos(x) \sqrt{\sec^2(x)} \left( \tan(x) \sec(x) - \log \left( \cos \left( \frac{x}{2} \right) - \sin \left( \frac{x}{2} \right) \right) + \log \left( \sin \left( \frac{x}{2} \right) + \cos \left( \frac{x}{2} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2)^(3/2), x]

[Out] (Cos[x]\*Sqrt[Sec[x]^2]\*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] + Sec[x]\*Tan[x]))/2

**Maple [B]** time = 0.065, size = 53, normalized size = 2.4

$$\frac{\cos(x)}{2} \left( (\cos(x))^2 \ln \left( -\frac{-1 + \cos(x) - \sin(x)}{\sin(x)} \right) - (\cos(x))^2 \ln \left( -\frac{-1 + \cos(x) + \sin(x)}{\sin(x)} \right) + \sin(x) \right) ((\cos(x))^{-2})^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sec(x)^2)^(3/2), x)

[Out] 1/2\*(cos(x)^2\*ln(-(-1+cos(x)-sin(x))/sin(x))-cos(x)^2\*ln(-(-1+cos(x)+sin(x))/sin(x))+sin(x))\*cos(x)\*(1/cos(x)^2)^(3/2)

**Maxima [A]** time = 1.7188, size = 24, normalized size = 1.09

$$\frac{1}{2} \sqrt{\tan(x)^2 + 1} \tan(x) + \frac{1}{2} \operatorname{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)^2)^(3/2),x, algorithm="maxima")

[Out] 1/2\*sqrt(tan(x)^2 + 1)\*tan(x) + 1/2\*arcsinh(tan(x))

**Fricas [B]** time = 1.33071, size = 109, normalized size = 4.95

$$\frac{\cos(x)^2 \log(\sin(x) + 1) - \cos(x)^2 \log(-\sin(x) + 1) + 2 \sin(x)}{4 \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)^2)^(3/2),x, algorithm="fricas")

[Out] -1/4\*(cos(x)^2\*log(sin(x) + 1) - cos(x)^2\*log(-sin(x) + 1) + 2\*sin(x))/cos(x)^2

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (\sec^2(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)\*\*2)\*\*(3/2),x)

[Out] Integral((sec(x)\*\*2)\*\*(3/2), x)

**Giac [B]** time = 1.36668, size = 59, normalized size = 2.68

$$\frac{\log(\sin(x) + 1)}{4 \operatorname{sgn}(\cos(x))} - \frac{\log(-\sin(x) + 1)}{4 \operatorname{sgn}(\cos(x))} - \frac{\sin(x)}{2(\sin(x)^2 - 1)\operatorname{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)^2)^(3/2),x, algorithm="giac")

[Out] 1/4\*log(sin(x) + 1)/sgn(cos(x)) - 1/4\*log(-sin(x) + 1)/sgn(cos(x)) - 1/2\*sin(x)/((sin(x)^2 - 1)\*sgn(cos(x)))

### 3.42 $\int \sqrt{\sec^2(x)} dx$

Optimal. Leaf size=3

$$\sinh^{-1}(\tan(x))$$

[Out] ArcSinh[Tan[x]]

**Rubi [A]** time = 0.005895, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {4122, 215}

$$\sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[x]^2], x]

[Out] ArcSinh[Tan[x]]

#### Rule 4122

Int[((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^2)^(p\_), x\_Symbol] :> With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[(b\*ff)/f, Subst[Int[(b + b\*ff^2\*x^2)^(p - 1), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int \sqrt{\sec^2(x)} dx &= \text{Subst} \left( \int \frac{1}{\sqrt{1+x^2}} dx, x, \tan(x) \right) \\ &= \sinh^{-1}(\tan(x)) \end{aligned}$$

**Mathematica [B]** time = 0.0093418, size = 44, normalized size = 14.67

$$\cos(x)\sqrt{\sec^2(x)}\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[x]^2], x]

[Out] Cos[x]\*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])\*Sqrt[Sec[x]^2]

**Maple [B]** time = 0.071, size = 21, normalized size = 7.

$$-2 \cos(x) \operatorname{Artanh}\left(\frac{-1 + \cos(x)}{\sin(x)}\right) \sqrt{(\cos(x))^{-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sec(x)^2)^(1/2), x)

[Out] -2\*cos(x)\*arctanh((-1+cos(x))/sin(x))\*(1/cos(x)^2)^(1/2)

**Maxima [A]** time = 1.73177, size = 4, normalized size = 1.33

$$\operatorname{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)^2)^(1/2), x, algorithm="maxima")

[Out] arcsinh(tan(x))

**Fricas [B]** time = 1.42133, size = 61, normalized size = 20.33

$$-\frac{1}{2} \log(\sin(x) + 1) + \frac{1}{2} \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(sin(x) + 1) + 1/2*log(-sin(x) + 1)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sec^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sec(x)**2), x)
```

**Giac [B]** time = 1.33872, size = 47, normalized size = 15.67

$$\frac{\log\left(\left|\frac{1}{\sin(x)} + \sin(x) + 2\right|\right)}{4 \operatorname{sgn}(\cos(x))} - \frac{\log\left(\left|\frac{1}{\sin(x)} + \sin(x) - 2\right|\right)}{4 \operatorname{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(1/sin(x) + sin(x) + 2))/sgn(cos(x)) - 1/4*log(abs(1/sin(x) + sin(x) - 2))/sgn(cos(x))
```

$$3.43 \quad \int \frac{1}{\sqrt{\sec^2(x)}} dx$$

Optimal. Leaf size=11

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

[Out] Tan[x]/Sqrt[Sec[x]^2]

**Rubi [A]** time = 0.0073548, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {4122, 191}

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[Sec[x]^2],x]

[Out] Tan[x]/Sqrt[Sec[x]^2]

#### Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

#### Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

#### Rubi steps



$$\int \frac{1}{\sqrt{\sec^2(x)}} dx = \text{Subst} \left( \int \frac{1}{(1+x^2)^{3/2}} dx, x, \tan(x) \right)$$

$$= \frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

**Mathematica [A]** time = 0.0058971, size = 11, normalized size = 1.

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[Sec[x]^2], x]

[Out] Tan[x]/Sqrt[Sec[x]^2]

**Maple [A]** time = 0.074, size = 14, normalized size = 1.3

$$\frac{\sin(x)}{\cos(x)} \frac{1}{\sqrt{(\cos(x))^{-2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sec(x)^2)^(1/2), x)

[Out] sin(x)/(1/cos(x)^2)^(1/2)/cos(x)

**Maxima [A]** time = 1.18009, size = 15, normalized size = 1.36

$$\frac{\tan(x)}{\sqrt{\tan(x)^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] tan(x)/sqrt(tan(x)^2 + 1)
```

---

**Fricas [A]** time = 1.2399, size = 12, normalized size = 1.09

$$-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sin(x)
```

---

**Sympy [A]** time = 0.482606, size = 10, normalized size = 0.91

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2)**(1/2),x)
```

```
[Out] tan(x)/sqrt(sec(x)**2)
```

---

**Giac [A]** time = 1.27162, size = 8, normalized size = 0.73

$$\operatorname{sgn}(\cos(x))\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sgn(cos(x))*sin(x)
```

$$3.44 \quad \int \frac{1}{\sec^2(x)^{3/2}} dx$$

**Optimal.** Leaf size=29

$$\frac{2 \tan(x)}{3\sqrt{\sec^2(x)}} + \frac{\tan(x)}{3 \sec^2(x)^{3/2}}$$

[Out] Tan[x]/(3\*(Sec[x]^2)^(3/2)) + (2\*Tan[x])/(3\*Sqrt[Sec[x]^2])

**Rubi [A]** time = 0.0105769, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4122, 192, 191}

$$\frac{2 \tan(x)}{3\sqrt{\sec^2(x)}} + \frac{\tan(x)}{3 \sec^2(x)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2)^(-3/2), x]

[Out] Tan[x]/(3\*(Sec[x]^2)^(3/2)) + (2\*Tan[x])/(3\*Sqrt[Sec[x]^2])

#### Rule 4122

Int[((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^2)^(p\_), x\_Symbol] := With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[(b\*ff)/f, Subst[Int[(b + b\*ff^2\*x^2)^(p - 1), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]

#### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sec^2(x)^{3/2}} dx &= \text{Subst} \left( \int \frac{1}{(1+x^2)^{5/2}} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{3 \sec^2(x)^{3/2}} + \frac{2}{3} \text{Subst} \left( \int \frac{1}{(1+x^2)^{3/2}} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{3 \sec^2(x)^{3/2}} + \frac{2 \tan(x)}{3 \sqrt{\sec^2(x)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0149289, size = 23, normalized size = 0.79

$$\frac{(9 \sin(x) + \sin(3x)) \sec(x)}{12 \sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2)^(-3/2), x]

[Out] (Sec[x]\*(9\*Sin[x] + Sin[3\*x]))/(12\*Sqrt[Sec[x]^2])

**Maple [A]** time = 0.059, size = 21, normalized size = 0.7

$$\frac{\sin(x) ((\cos(x))^2 + 2)}{3 (\cos(x))^3} ((\cos(x))^{-2})^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sec(x)^2)^(3/2), x)

[Out] 1/3\*sin(x)\*(cos(x)^2+2)/cos(x)^3/(1/cos(x)^2)^(3/2)

**Maxima [A]** time = 1.08934, size = 34, normalized size = 1.17

$$\frac{2 \tan(x)}{3 \sqrt{\tan(x)^2 + 1}} + \frac{\tan(x)}{3 (\tan(x)^2 + 1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sec(x)^2)^(3/2),x, algorithm="maxima")`

[Out]  $2/3*\tan(x)/\sqrt{\tan(x)^2 + 1} + 1/3*\tan(x)/(\tan(x)^2 + 1)^{(3/2)}$

**Fricas [A]** time = 1.31181, size = 38, normalized size = 1.31

$$-\frac{1}{3}(\cos(x)^2 + 2)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sec(x)^2)^(3/2),x, algorithm="fricas")`

[Out]  $-1/3*(\cos(x)^2 + 2)*\sin(x)$

**Sympy [A]** time = 1.5059, size = 27, normalized size = 0.93

$$\frac{2 \tan^3(x)}{3 (\sec^2(x))^{\frac{3}{2}}} + \frac{\tan(x)}{(\sec^2(x))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sec(x)**2)**(3/2),x)`

[Out]  $2*\tan(x)**3/(3*(\sec(x)**2)**(3/2)) + \tan(x)/(\sec(x)**2)**(3/2)$

**Giac [A]** time = 1.35019, size = 22, normalized size = 0.76

$$-\frac{1}{3}\operatorname{sgn}(\cos(x))\sin(x)^3 + \operatorname{sgn}(\cos(x))\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sec(x)^2)^(3/2),x, algorithm="giac")`

[Out]  $-1/3*\operatorname{sgn}(\cos(x))*\sin(x)^3 + \operatorname{sgn}(\cos(x))*\sin(x)$

$$3.45 \quad \int \frac{1}{\sec^2(x)^{5/2}} dx$$

**Optimal.** Leaf size=43

$$\frac{8 \tan(x)}{15\sqrt{\sec^2(x)}} + \frac{4 \tan(x)}{15 \sec^2(x)^{3/2}} + \frac{\tan(x)}{5 \sec^2(x)^{5/2}}$$

[Out] Tan[x]/(5\*(Sec[x]^2)^(5/2)) + (4\*Tan[x])/(15\*(Sec[x]^2)^(3/2)) + (8\*Tan[x])/(15\*Sqrt[Sec[x]^2])

**Rubi [A]** time = 0.0149745, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4122, 192, 191}

$$\frac{8 \tan(x)}{15\sqrt{\sec^2(x)}} + \frac{4 \tan(x)}{15 \sec^2(x)^{3/2}} + \frac{\tan(x)}{5 \sec^2(x)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2)^(-5/2), x]

[Out] Tan[x]/(5\*(Sec[x]^2)^(5/2)) + (4\*Tan[x])/(15\*(Sec[x]^2)^(3/2)) + (8\*Tan[x])/(15\*Sqrt[Sec[x]^2])

#### Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)^2]^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

#### Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]
```

#### Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sec^2(x)^{5/2}} dx &= \text{Subst} \left( \int \frac{1}{(1+x^2)^{7/2}} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{5 \sec^2(x)^{5/2}} + \frac{4}{5} \text{Subst} \left( \int \frac{1}{(1+x^2)^{5/2}} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{5 \sec^2(x)^{5/2}} + \frac{4 \tan(x)}{15 \sec^2(x)^{3/2}} + \frac{8}{15} \text{Subst} \left( \int \frac{1}{(1+x^2)^{3/2}} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{5 \sec^2(x)^{5/2}} + \frac{4 \tan(x)}{15 \sec^2(x)^{3/2}} + \frac{8 \tan(x)}{15 \sqrt{\sec^2(x)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0249409, size = 31, normalized size = 0.72

$$\frac{(150 \sin(x) + 25 \sin(3x) + 3 \sin(5x)) \sec(x)}{240 \sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2)^(-5/2), x]

[Out] (Sec[x]\*(150\*Sin[x] + 25\*Sin[3\*x] + 3\*Sin[5\*x]))/(240\*Sqrt[Sec[x]^2])

**Maple [A]** time = 0.072, size = 29, normalized size = 0.7

$$\frac{\sin(x) \left( 3 (\cos(x))^4 + 4 (\cos(x))^2 + 8 \right)}{15 (\cos(x))^5} \left( (\cos(x))^{-2} \right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sec(x)^2)^(5/2), x)

[Out] 1/15\*sin(x)\*(3\*cos(x)^4+4\*cos(x)^2+8)/cos(x)^5/(1/cos(x)^2)^(5/2)

**Maxima [A]** time = 1.16518, size = 50, normalized size = 1.16

$$\frac{8 \tan(x)}{15 \sqrt{\tan(x)^2 + 1}} + \frac{4 \tan(x)}{15 (\tan(x)^2 + 1)^{\frac{3}{2}}} + \frac{\tan(x)}{5 (\tan(x)^2 + 1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2)^(5/2),x, algorithm="maxima")

[Out] 8/15\*tan(x)/sqrt(tan(x)^2 + 1) + 4/15\*tan(x)/(tan(x)^2 + 1)^(3/2) + 1/5\*tan(x)/(tan(x)^2 + 1)^(5/2)

**Fricas [A]** time = 1.44419, size = 59, normalized size = 1.37

$$-\frac{1}{15} (3 \cos(x)^4 + 4 \cos(x)^2 + 8) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2)^(5/2),x, algorithm="fricas")

[Out] -1/15\*(3\*cos(x)^4 + 4\*cos(x)^2 + 8)\*sin(x)

**Sympy [A]** time = 17.704, size = 44, normalized size = 1.02

$$\frac{8 \tan^5(x)}{15 (\sec^2(x))^{\frac{5}{2}}} + \frac{4 \tan^3(x)}{3 (\sec^2(x))^{\frac{5}{2}}} + \frac{\tan(x)}{(\sec^2(x))^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)\*\*2)\*\*(5/2),x)

[Out] 8\*tan(x)\*\*5/(15\*(sec(x)\*\*2)\*\*(5/2)) + 4\*tan(x)\*\*3/(3\*(sec(x)\*\*2)\*\*(5/2)) + tan(x)/(sec(x)\*\*2)\*\*(5/2)



**Giac [A]** time = 1.25735, size = 34, normalized size = 0.79

$$\frac{1}{5} \operatorname{sgn}(\cos(x)) \sin(x)^5 - \frac{2}{3} \operatorname{sgn}(\cos(x)) \sin(x)^3 + \operatorname{sgn}(\cos(x)) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2)^(5/2),x, algorithm="giac")

[Out] 1/5\*sgn(cos(x))\*sin(x)^5 - 2/3\*sgn(cos(x))\*sin(x)^3 + sgn(cos(x))\*sin(x)

$$3.46 \quad \int \frac{1}{\sec^2(x)^{7/2}} dx$$

**Optimal.** Leaf size=57

$$\frac{16 \tan(x)}{35\sqrt{\sec^2(x)}} + \frac{8 \tan(x)}{35 \sec^2(x)^{3/2}} + \frac{6 \tan(x)}{35 \sec^2(x)^{5/2}} + \frac{\tan(x)}{7 \sec^2(x)^{7/2}}$$

[Out] Tan[x]/(7\*(Sec[x]^2)^(7/2)) + (6\*Tan[x])/(35\*(Sec[x]^2)^(5/2)) + (8\*Tan[x])/(35\*(Sec[x]^2)^(3/2)) + (16\*Tan[x])/(35\*Sqrt[Sec[x]^2])

**Rubi [A]** time = 0.0188078, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4122, 192, 191}

$$\frac{16 \tan(x)}{35\sqrt{\sec^2(x)}} + \frac{8 \tan(x)}{35 \sec^2(x)^{3/2}} + \frac{6 \tan(x)}{35 \sec^2(x)^{5/2}} + \frac{\tan(x)}{7 \sec^2(x)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2)^(-7/2), x]

[Out] Tan[x]/(7\*(Sec[x]^2)^(7/2)) + (6\*Tan[x])/(35\*(Sec[x]^2)^(5/2)) + (8\*Tan[x])/(35\*(Sec[x]^2)^(3/2)) + (16\*Tan[x])/(35\*Sqrt[Sec[x]^2])

#### Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)^2]^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

#### Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]
```

#### Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sec^2(x)^{7/2}} dx &= \text{Subst} \left( \int \frac{1}{(1+x^2)^{9/2}} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{7 \sec^2(x)^{7/2}} + \frac{6}{7} \text{Subst} \left( \int \frac{1}{(1+x^2)^{7/2}} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{7 \sec^2(x)^{7/2}} + \frac{6 \tan(x)}{35 \sec^2(x)^{5/2}} + \frac{24}{35} \text{Subst} \left( \int \frac{1}{(1+x^2)^{5/2}} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{7 \sec^2(x)^{7/2}} + \frac{6 \tan(x)}{35 \sec^2(x)^{5/2}} + \frac{8 \tan(x)}{35 \sec^2(x)^{3/2}} + \frac{16}{35} \text{Subst} \left( \int \frac{1}{(1+x^2)^{3/2}} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{7 \sec^2(x)^{7/2}} + \frac{6 \tan(x)}{35 \sec^2(x)^{5/2}} + \frac{8 \tan(x)}{35 \sec^2(x)^{3/2}} + \frac{16 \tan(x)}{35 \sqrt{\sec^2(x)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0376147, size = 37, normalized size = 0.65

$$\frac{(1225 \sin(x) + 245 \sin(3x) + 49 \sin(5x) + 5 \sin(7x)) \sec(x)}{2240 \sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2)^(-7/2), x]

[Out] (Sec[x]\*(1225\*Sin[x] + 245\*Sin[3\*x] + 49\*Sin[5\*x] + 5\*Sin[7\*x]))/(2240\*sqrt[Sec[x]^2])

**Maple [A]** time = 0.088, size = 35, normalized size = 0.6

$$\frac{\sin(x) \left( 5 (\cos(x))^6 + 6 (\cos(x))^4 + 8 (\cos(x))^2 + 16 \right)}{35 (\cos(x))^7} \left( (\cos(x))^{-2} \right)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sec(x)^2)^(7/2), x)

[Out]  $1/35*\sin(x)*(5*\cos(x)^6+6*\cos(x)^4+8*\cos(x)^2+16)/\cos(x)^7/(1/\cos(x)^2)^{(7/2)}$

**Maxima [A]** time = 1.06939, size = 66, normalized size = 1.16

$$\frac{16 \tan(x)}{35 \sqrt{\tan(x)^2 + 1}} + \frac{8 \tan(x)}{35 (\tan(x)^2 + 1)^{3/2}} + \frac{6 \tan(x)}{35 (\tan(x)^2 + 1)^{5/2}} + \frac{\tan(x)}{7 (\tan(x)^2 + 1)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sec(x)^2)^(7/2),x, algorithm="maxima")`

[Out]  $16/35*\tan(x)/\sqrt{\tan(x)^2 + 1} + 8/35*\tan(x)/(\tan(x)^2 + 1)^{(3/2)} + 6/35*\tan(x)/(\tan(x)^2 + 1)^{(5/2)} + 1/7*\tan(x)/(\tan(x)^2 + 1)^{(7/2)}$

**Fricas [A]** time = 1.4212, size = 78, normalized size = 1.37

$$-\frac{1}{35} (5 \cos(x)^6 + 6 \cos(x)^4 + 8 \cos(x)^2 + 16) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sec(x)^2)^(7/2),x, algorithm="fricas")`

[Out]  $-1/35*(5*\cos(x)^6 + 6*\cos(x)^4 + 8*\cos(x)^2 + 16)*\sin(x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sec(x)**2)**(7/2),x)`

[Out] Timed out

---

**Giac [A]** time = 1.4046, size = 46, normalized size = 0.81

$$-\frac{1}{7} \operatorname{sgn}(\cos(x)) \sin(x)^7 + \frac{3}{5} \operatorname{sgn}(\cos(x)) \sin(x)^5 - \operatorname{sgn}(\cos(x)) \sin(x)^3 + \operatorname{sgn}(\cos(x)) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2)^(7/2),x, algorithm="giac")

[Out] -1/7\*sgn(cos(x))\*sin(x)^7 + 3/5\*sgn(cos(x))\*sin(x)^5 - sgn(cos(x))\*sin(x)^3  
+ sgn(cos(x))\*sin(x)

### 3.47 $\int \left(a \sec^2(x)\right)^{7/2} dx$

**Optimal.** Leaf size=84

$$\frac{5}{16}a^3 \tan(x)\sqrt{a \sec^2(x)} + \frac{5}{24}a^2 \tan(x) \left(a \sec^2(x)\right)^{3/2} + \frac{5}{16}a^{7/2} \tanh^{-1}\left(\frac{\sqrt{a} \tan(x)}{\sqrt{a \sec^2(x)}}\right) + \frac{1}{6}a \tan(x) \left(a \sec^2(x)\right)^{5/2}$$

[Out] (5\*a^(7/2)\*ArcTanh[(Sqrt[a]\*Tan[x])/Sqrt[a\*Sec[x]^2]])/16 + (5\*a^3\*Sqrt[a\*Sec[x]^2]\*Tan[x])/16 + (5\*a^2\*(a\*Sec[x]^2)^(3/2)\*Tan[x])/24 + (a\*(a\*Sec[x]^2)^(5/2)\*Tan[x])/6

**Rubi [A]** time = 0.0413025, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4122, 195, 217, 206}

$$\frac{5}{16}a^3 \tan(x)\sqrt{a \sec^2(x)} + \frac{5}{24}a^2 \tan(x) \left(a \sec^2(x)\right)^{3/2} + \frac{5}{16}a^{7/2} \tanh^{-1}\left(\frac{\sqrt{a} \tan(x)}{\sqrt{a \sec^2(x)}}\right) + \frac{1}{6}a \tan(x) \left(a \sec^2(x)\right)^{5/2}$$

Antiderivative was successfully verified.

[In] Int[(a\*Sec[x]^2)^(7/2), x]

[Out] (5\*a^(7/2)\*ArcTanh[(Sqrt[a]\*Tan[x])/Sqrt[a\*Sec[x]^2]])/16 + (5\*a^3\*Sqrt[a\*Sec[x]^2]\*Tan[x])/16 + (5\*a^2\*(a\*Sec[x]^2)^(3/2)\*Tan[x])/24 + (a\*(a\*Sec[x]^2)^(5/2)\*Tan[x])/6

#### Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)^2]^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

#### Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])
```

#### Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int (a \sec^2(x))^{7/2} dx &= a \operatorname{Subst}\left(\int (a + ax^2)^{5/2} dx, x, \tan(x)\right) \\
&= \frac{1}{6}a (a \sec^2(x))^{5/2} \tan(x) + \frac{1}{6}(5a^2) \operatorname{Subst}\left(\int (a + ax^2)^{3/2} dx, x, \tan(x)\right) \\
&= \frac{5}{24}a^2 (a \sec^2(x))^{3/2} \tan(x) + \frac{1}{6}a (a \sec^2(x))^{5/2} \tan(x) + \frac{1}{8}(5a^3) \operatorname{Subst}\left(\int \sqrt{a + ax^2} dx, x, \tan(x)\right) \\
&= \frac{5}{16}a^3 \sqrt{a \sec^2(x)} \tan(x) + \frac{5}{24}a^2 (a \sec^2(x))^{3/2} \tan(x) + \frac{1}{6}a (a \sec^2(x))^{5/2} \tan(x) + \frac{1}{16}(5a^4) \operatorname{Subst}\left(\int \sqrt{a + ax^2} dx, x, \tan(x)\right) \\
&= \frac{5}{16}a^3 \sqrt{a \sec^2(x)} \tan(x) + \frac{5}{24}a^2 (a \sec^2(x))^{3/2} \tan(x) + \frac{1}{6}a (a \sec^2(x))^{5/2} \tan(x) + \frac{1}{16}(5a^4) \operatorname{Subst}\left(\int \sqrt{a + ax^2} dx, x, \tan(x)\right) \\
&= \frac{5}{16}a^{7/2} \tanh^{-1}\left(\frac{\sqrt{a} \tan(x)}{\sqrt{a \sec^2(x)}}\right) + \frac{5}{16}a^3 \sqrt{a \sec^2(x)} \tan(x) + \frac{5}{24}a^2 (a \sec^2(x))^{3/2} \tan(x) + \frac{1}{6}a (a \sec^2(x))^{5/2} \tan(x) + \frac{1}{16}(5a^4) \operatorname{Subst}\left(\int \sqrt{a + ax^2} dx, x, \tan(x)\right)
\end{aligned}$$

**Mathematica [A]** time = 0.124033, size = 78, normalized size = 0.93

$$\frac{1}{96} \cos^7(x) (a \sec^2(x))^{7/2} \left( \frac{1}{8} (198 \sin(x) + 85 \sin(3x) + 15 \sin(5x)) \sec^6(x) - 30 \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + 30 \log\left(\sin\left(\frac{x}{2}\right)\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^2)^(7/2), x]
```

```
[Out] (Cos[x]^7*(a*Sec[x]^2)^(7/2)*(-30*Log[Cos[x/2] - Sin[x/2]] + 30*Log[Cos[x/2]
+ Sin[x/2]] + (Sec[x]^6*(198*Sin[x] + 85*Sin[3*x] + 15*Sin[5*x]))/8))/96
```

**Maple [A]** time = 0.145, size = 74, normalized size = 0.9

$$\frac{\cos(x)}{48} \left( 15 \ln\left(-\frac{-1 + \cos(x) - \sin(x)}{\sin(x)}\right) (\cos(x))^6 - 15 \ln\left(-\frac{-1 + \cos(x) + \sin(x)}{\sin(x)}\right) (\cos(x))^6 + 15 (\cos(x))^4 \sin(x) + \dots \right)$$





```

*x)^2 + 36*a^3*cos(2*x)^2 + a^3*sin(12*x)^2 + 36*a^3*sin(10*x)^2 + 225*a^3*
sin(8*x)^2 + 400*a^3*sin(6*x)^2 + 225*a^3*sin(4*x)^2 + 180*a^3*sin(4*x)*sin
(2*x) + 36*a^3*sin(2*x)^2 + 12*a^3*cos(2*x) + a^3 + 2*(6*a^3*cos(10*x) + 15
*a^3*cos(8*x) + 20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*c
os(12*x) + 12*(15*a^3*cos(8*x) + 20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*
cos(2*x) + a^3)*cos(10*x) + 30*(20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*c
os(2*x) + a^3)*cos(8*x) + 40*(15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*cos(6
*x) + 30*(6*a^3*cos(2*x) + a^3)*cos(4*x) + 2*(6*a^3*sin(10*x) + 15*a^3*sin(
8*x) + 20*a^3*sin(6*x) + 15*a^3*sin(4*x) + 6*a^3*sin(2*x))*sin(12*x) + 12*(
15*a^3*sin(8*x) + 20*a^3*sin(6*x) + 15*a^3*sin(4*x) + 6*a^3*sin(2*x))*sin(1
0*x) + 30*(20*a^3*sin(6*x) + 15*a^3*sin(4*x) + 6*a^3*sin(2*x))*sin(8*x) + 1
20*(5*a^3*sin(4*x) + 2*a^3*sin(2*x))*sin(6*x))*log(cos(x)^2 + sin(x)^2 - 2*
sin(x) + 1) - 4*(15*a^3*cos(11*x) + 85*a^3*cos(9*x) + 198*a^3*cos(7*x) - 19
8*a^3*cos(5*x) - 85*a^3*cos(3*x) - 15*a^3*cos(x))*sin(12*x) + 60*(6*a^3*cos
(10*x) + 15*a^3*cos(8*x) + 20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*
x) + a^3)*sin(11*x) - 24*(85*a^3*cos(9*x) + 198*a^3*cos(7*x) - 198*a^3*cos(
5*x) - 85*a^3*cos(3*x) - 15*a^3*cos(x))*sin(10*x) + 340*(15*a^3*cos(8*x) +
20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*sin(9*x) - 60*(19
8*a^3*cos(7*x) - 198*a^3*cos(5*x) - 85*a^3*cos(3*x) - 15*a^3*cos(x))*sin(8*
x) + 792*(20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*sin(7*x
) + 80*(198*a^3*cos(5*x) + 85*a^3*cos(3*x) + 15*a^3*cos(x))*sin(6*x) - 792*
(15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*sin(5*x) + 300*(17*a^3*cos(3*x) +
3*a^3*cos(x))*sin(4*x) - 340*(6*a^3*cos(2*x) + a^3)*sin(3*x))*sqrt(a)/(2*(6
*cos(10*x) + 15*cos(8*x) + 20*cos(6*x) + 15*cos(4*x) + 6*cos(2*x) + 1)*cos(
12*x) + cos(12*x)^2 + 12*(15*cos(8*x) + 20*cos(6*x) + 15*cos(4*x) + 6*cos(2
*x) + 1)*cos(10*x) + 36*cos(10*x)^2 + 30*(20*cos(6*x) + 15*cos(4*x) + 6*cos
(2*x) + 1)*cos(8*x) + 225*cos(8*x)^2 + 40*(15*cos(4*x) + 6*cos(2*x) + 1)*co
s(6*x) + 400*cos(6*x)^2 + 30*(6*cos(2*x) + 1)*cos(4*x) + 225*cos(4*x)^2 + 3
6*cos(2*x)^2 + 2*(6*sin(10*x) + 15*sin(8*x) + 20*sin(6*x) + 15*sin(4*x) + 6
*sin(2*x))*sin(12*x) + sin(12*x)^2 + 12*(15*sin(8*x) + 20*sin(6*x) + 15*sin
(4*x) + 6*sin(2*x))*sin(10*x) + 36*sin(10*x)^2 + 30*(20*sin(6*x) + 15*sin(4
*x) + 6*sin(2*x))*sin(8*x) + 225*sin(8*x)^2 + 120*(5*sin(4*x) + 2*sin(2*x))
*sin(6*x) + 400*sin(6*x)^2 + 225*sin(4*x)^2 + 180*sin(4*x)*sin(2*x) + 36*si
n(2*x)^2 + 12*cos(2*x) + 1)

```

---

**Fricas [A]** time = 1.47792, size = 186, normalized size = 2.21

$$\frac{\left(15 a^3 \cos (x)^6 \log \left(-\frac{\sin (x)-1}{\sin (x)+1}\right)-2\left(15 a^3 \cos (x)^4+10 a^3 \cos (x)^2+8 a^3\right) \sin (x)\right) \sqrt{\frac{a}{\cos (x)^2}}}{96 \cos (x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^2)^(7/2),x, algorithm="fricas")

[Out]  $-1/96*(15*a^3*\cos(x)^6*\log(-(\sin(x) - 1)/(\sin(x) + 1))) - 2*(15*a^3*\cos(x)^4 + 10*a^3*\cos(x)^2 + 8*a^3*\sin(x))*\sqrt{a/\cos(x)^2}/\cos(x)^5$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)\*\*2)\*\*(7/2),x)

[Out] Timed out

**Giac [A]** time = 1.30087, size = 107, normalized size = 1.27

$$\frac{1}{96} \left( 15 a^3 \log(\sin(x) + 1) \operatorname{sgn}(\cos(x)) - 15 a^3 \log(-\sin(x) + 1) \operatorname{sgn}(\cos(x)) - \frac{2(15 a^3 \operatorname{sgn}(\cos(x)) \sin(x)^5 - 40 a^3 \operatorname{sgn}(\cos(x)) \sin(x)^3 + 33 a^3 \operatorname{sgn}(\cos(x)) \sin(x))}{(\sin(x)^2 - 1)^3} \right) \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^2)^(7/2),x, algorithm="giac")

[Out]  $1/96*(15*a^3*\log(\sin(x) + 1)*\operatorname{sgn}(\cos(x)) - 15*a^3*\log(-\sin(x) + 1)*\operatorname{sgn}(\cos(x)) - 2*(15*a^3*\operatorname{sgn}(\cos(x))*\sin(x)^5 - 40*a^3*\operatorname{sgn}(\cos(x))*\sin(x)^3 + 33*a^3*\operatorname{sgn}(\cos(x))*\sin(x))/(\sin(x)^2 - 1)^3*\sqrt{a}$

### 3.48 $\int (a \sec^2(x))^{5/2} dx$

**Optimal.** Leaf size=65

$$\frac{3}{8}a^2 \tan(x)\sqrt{a \sec^2(x)} + \frac{3}{8}a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a} \tan(x)}{\sqrt{a \sec^2(x)}}\right) + \frac{1}{4}a \tan(x) (a \sec^2(x))^{3/2}$$

[Out] (3\*a^(5/2)\*ArcTanh[(Sqrt[a]\*Tan[x])/Sqrt[a\*Sec[x]^2]])/8 + (3\*a^2\*Sqrt[a\*Sec[x]^2]\*Tan[x])/8 + (a\*(a\*Sec[x]^2)^(3/2)\*Tan[x])/4

**Rubi [A]** time = 0.0313453, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4122, 195, 217, 206}

$$\frac{3}{8}a^2 \tan(x)\sqrt{a \sec^2(x)} + \frac{3}{8}a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a} \tan(x)}{\sqrt{a \sec^2(x)}}\right) + \frac{1}{4}a \tan(x) (a \sec^2(x))^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(a\*Sec[x]^2)^(5/2),x]

[Out] (3\*a^(5/2)\*ArcTanh[(Sqrt[a]\*Tan[x])/Sqrt[a\*Sec[x]^2]])/8 + (3\*a^2\*Sqrt[a\*Sec[x]^2]\*Tan[x])/8 + (a\*(a\*Sec[x]^2)^(3/2)\*Tan[x])/4

#### Rule 4122

Int[((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^2)^(p\_), x\_Symbol] :> With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[(b\*ff)/f, Subst[Int[(b + b\*ff^2\*x^2)^(p - 1), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 217

`Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

### Rule 206

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

### Rubi steps

$$\begin{aligned}
 \int (a \sec^2(x))^{5/2} dx &= a \operatorname{Subst} \left( \int (a + ax^2)^{3/2} dx, x, \tan(x) \right) \\
 &= \frac{1}{4} a (a \sec^2(x))^{3/2} \tan(x) + \frac{1}{4} (3a^2) \operatorname{Subst} \left( \int \sqrt{a + ax^2} dx, x, \tan(x) \right) \\
 &= \frac{3}{8} a^2 \sqrt{a \sec^2(x)} \tan(x) + \frac{1}{4} a (a \sec^2(x))^{3/2} \tan(x) + \frac{1}{8} (3a^3) \operatorname{Subst} \left( \int \frac{1}{\sqrt{a + ax^2}} dx, x, \tan(x) \right) \\
 &= \frac{3}{8} a^2 \sqrt{a \sec^2(x)} \tan(x) + \frac{1}{4} a (a \sec^2(x))^{3/2} \tan(x) + \frac{1}{8} (3a^3) \operatorname{Subst} \left( \int \frac{1}{1 - ax^2} dx, x, \frac{\tan(x)}{\sqrt{a \sec^2(x)}} \right) \\
 &= \frac{3}{8} a^{5/2} \tanh^{-1} \left( \frac{\sqrt{a} \tan(x)}{\sqrt{a \sec^2(x)}} \right) + \frac{3}{8} a^2 \sqrt{a \sec^2(x)} \tan(x) + \frac{1}{4} a (a \sec^2(x))^{3/2} \tan(x)
 \end{aligned}$$

**Mathematica [A]** time = 0.120652, size = 72, normalized size = 1.11

$$\frac{1}{16} \cos^5(x) (a \sec^2(x))^{5/2} \left( \frac{1}{2} (11 \sin(x) + 3 \sin(3x)) \sec^4(x) - 6 \log \left( \cos \left( \frac{x}{2} \right) - \sin \left( \frac{x}{2} \right) \right) + 6 \log \left( \sin \left( \frac{x}{2} \right) + \cos \left( \frac{x}{2} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a\*Sec[x]^2)^(5/2), x]

[Out] (Cos[x]^5\*(a\*Sec[x]^2)^(5/2)\*(-6\*Log[Cos[x/2] - Sin[x/2]] + 6\*Log[Cos[x/2] + Sin[x/2]] + (Sec[x]^4\*(11\*Sin[x] + 3\*Sin[3\*x]))/2))/16

**Maple [A]** time = 0.087, size = 66, normalized size = 1.

$$\frac{\cos(x)}{8} \left( 3 (\cos(x))^4 \ln \left( -\frac{-1 + \cos(x) - \sin(x)}{\sin(x)} \right) - 3 (\cos(x))^4 \ln \left( -\frac{-1 + \cos(x) + \sin(x)}{\sin(x)} \right) + 3 (\cos(x))^2 \sin(x) + 2 \sin(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a*\sec(x)^2)^{(5/2}),x)$

[Out]  $\frac{1}{8}*(3*\cos(x)^4*\ln(-(-1+\cos(x)-\sin(x))/\sin(x))-3*\cos(x)^4*\ln(-(-1+\cos(x)+\sin(x))/\sin(x))+3*\cos(x)^2*\sin(x)+2*\sin(x))*\cos(x)*(a/\cos(x)^2)^{(5/2)}$

**Maxima [B]** time = 2.67276, size = 1500, normalized size = 23.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a*\sec(x)^2)^{(5/2}),x, \text{algorithm}="maxima")$

[Out]  $\frac{1}{16}*(176*a^2*\cos(3*x)*\sin(2*x) + 48*a^2*\cos(x)*\sin(2*x) - 48*a^2*\cos(2*x)*\sin(x) - 12*a^2*\sin(x) + 4*(3*a^2*\sin(7*x) + 11*a^2*\sin(5*x) - 11*a^2*\sin(3*x) - 3*a^2*\sin(x))*\cos(8*x) - 24*(2*a^2*\sin(6*x) + 3*a^2*\sin(4*x) + 2*a^2*\sin(2*x))*\cos(7*x) + 16*(11*a^2*\sin(5*x) - 11*a^2*\sin(3*x) - 3*a^2*\sin(x))*\cos(6*x) - 88*(3*a^2*\sin(4*x) + 2*a^2*\sin(2*x))*\cos(5*x) - 24*(11*a^2*\sin(3*x) + 3*a^2*\sin(x))*\cos(4*x) + 3*(a^2*\cos(8*x)^2 + 16*a^2*\cos(6*x)^2 + 36*a^2*\cos(4*x)^2 + 16*a^2*\cos(2*x)^2 + a^2*\sin(8*x)^2 + 16*a^2*\sin(6*x)^2 + 36*a^2*\sin(4*x)^2 + 48*a^2*\sin(4*x)*\sin(2*x) + 16*a^2*\sin(2*x)^2 + 8*a^2*\cos(2*x) + a^2 + 2*(4*a^2*\cos(6*x) + 6*a^2*\cos(4*x) + 4*a^2*\cos(2*x) + a^2)*\cos(8*x) + 8*(6*a^2*\cos(4*x) + 4*a^2*\cos(2*x) + a^2)*\cos(6*x) + 12*(4*a^2*\cos(2*x) + a^2)*\cos(4*x) + 4*(2*a^2*\sin(6*x) + 3*a^2*\sin(4*x) + 2*a^2*\sin(2*x))*\sin(8*x) + 16*(3*a^2*\sin(4*x) + 2*a^2*\sin(2*x))*\sin(6*x))*\log(\cos(x)^2 + \sin(x)^2 + 2*\sin(x) + 1) - 3*(a^2*\cos(8*x)^2 + 16*a^2*\cos(6*x)^2 + 36*a^2*\cos(4*x)^2 + 16*a^2*\cos(2*x)^2 + a^2*\sin(8*x)^2 + 16*a^2*\sin(6*x)^2 + 36*a^2*\sin(4*x)^2 + 48*a^2*\sin(4*x)*\sin(2*x) + 16*a^2*\sin(2*x)^2 + 8*a^2*\cos(2*x) + a^2 + 2*(4*a^2*\cos(6*x) + 6*a^2*\cos(4*x) + 4*a^2*\cos(2*x) + a^2)*\cos(8*x) + 8*(6*a^2*\cos(4*x) + 4*a^2*\cos(2*x) + a^2)*\cos(6*x) + 12*(4*a^2*\cos(2*x) + a^2)*\cos(4*x) + 4*(2*a^2*\sin(6*x) + 3*a^2*\sin(4*x) + 2*a^2*\sin(2*x))*\sin(8*x) + 16*(3*a^2*\sin(4*x) + 2*a^2*\sin(2*x))*\sin(6*x))*\log(\cos(x)^2 + \sin(x)^2 - 2*\sin(x) + 1) - 4*(3*a^2*\cos(7*x) + 11*a^2*\cos(5*x) - 11*a^2*\cos(3*x) - 3*a^2*\cos(x))*\sin(8*x) + 12*(4*a^2*\cos(6*x) + 6*a^2*\cos(4*x) + 4*a^2*\cos(2*x) + a^2)*\sin(7*x) - 16*(11*a^2*\cos(5*x) - 11*a^2*\cos(3*x) - 3*a^2*\cos(x))*\sin(6*x) + 44*(6*a^2*\cos(4*x) + 4*a^2*\cos(2*x) + a^2)*\sin(5*x) + 24*(11*a^2*\cos(3*x) + 3*a^2*\cos(x))*\sin(4*x) - 44*(4*a^2*\cos(2*x) + a^2)*\sin(3*x))*\sqrt{a}/(2*(4*\cos(6*x) + 6*\cos(4*x) + 4*\cos(2*x) + 1)*\cos(8*x) + \cos(8*x)^2 + 8*(6*\cos(4*x) + 4*\cos(2*x) + 1)*\cos(6*x) + 16*\cos(6*x)^2 + 12*(4*\cos(2*x) + 1)*\cos(4*x) + 36*\cos(4*x)^2 + 16*\cos(2*x)^2 + 4*(2*\sin(6*x) + 3*\sin(4*x)$

) + 2\*sin(2\*x))\*sin(8\*x) + sin(8\*x)^2 + 16\*(3\*sin(4\*x) + 2\*sin(2\*x))\*sin(6\*x) + 16\*sin(6\*x)^2 + 36\*sin(4\*x)^2 + 48\*sin(4\*x)\*sin(2\*x) + 16\*sin(2\*x)^2 + 8\*cos(2\*x) + 1)

**Fricas [A]** time = 1.56971, size = 159, normalized size = 2.45

$$-\frac{\left(3a^2 \cos(x)^4 \log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right) - 2\left(3a^2 \cos(x)^2 + 2a^2\right) \sin(x)\right) \sqrt{\frac{a}{\cos(x)^2}}}{16 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^2)^(5/2),x, algorithm="fricas")

[Out] -1/16\*(3\*a^2\*cos(x)^4\*log(-(sin(x) - 1)/(sin(x) + 1)) - 2\*(3\*a^2\*cos(x)^2 + 2\*a^2)\*sin(x))\*sqrt(a/cos(x)^2)/cos(x)^3

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)\*\*2)\*\*(5/2),x)

[Out] Timed out

**Giac [A]** time = 1.40925, size = 90, normalized size = 1.38

$$\frac{1}{16} \left( 3a^2 \log(\sin(x) + 1) \operatorname{sgn}(\cos(x)) - 3a^2 \log(-\sin(x) + 1) \operatorname{sgn}(\cos(x)) - \frac{2\left(3a^2 \operatorname{sgn}(\cos(x)) \sin(x)^3 - 5a^2 \operatorname{sgn}(\cos(x))\right)}{(\sin(x)^2 - 1)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^2)^(5/2),x, algorithm="giac")

```
[Out] 1/16*(3*a^2*log(sin(x) + 1)*sgn(cos(x)) - 3*a^2*log(-sin(x) + 1)*sgn(cos(x)) - 2*(3*a^2*sgn(cos(x))*sin(x)^3 - 5*a^2*sgn(cos(x))*sin(x))/(sin(x)^2 - 1)^2)*sqrt(a)
```

### 3.49 $\int \left(a \sec^2(x)\right)^{3/2} dx$

**Optimal.** Leaf size=46

$$\frac{1}{2}a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \tan(x)}{\sqrt{a \sec^2(x)}}\right) + \frac{1}{2}a \tan(x) \sqrt{a \sec^2(x)}$$

[Out]  $(a^{(3/2)} * \text{ArcTanh}[(\text{Sqrt}[a] * \text{Tan}[x]) / \text{Sqrt}[a * \text{Sec}[x]^2]]) / 2 + (a * \text{Sqrt}[a * \text{Sec}[x]^2] * \text{Tan}[x]) / 2$

**Rubi [A]** time = 0.0223066, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4122, 195, 217, 206}

$$\frac{1}{2}a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \tan(x)}{\sqrt{a \sec^2(x)}}\right) + \frac{1}{2}a \tan(x) \sqrt{a \sec^2(x)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a * \text{Sec}[x]^2)^{(3/2)}, x]$

[Out]  $(a^{(3/2)} * \text{ArcTanh}[(\text{Sqrt}[a] * \text{Tan}[x]) / \text{Sqrt}[a * \text{Sec}[x]^2]]) / 2 + (a * \text{Sqrt}[a * \text{Sec}[x]^2] * \text{Tan}[x]) / 2$

#### Rule 4122

$\text{Int}[(b * \sec[e + f * x] + (f * x)^2)^{p}, x\_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Tan}[e + f * x], x]\}, \text{Dist}[(b * ff) / f, \text{Subst}[\text{Int}[(b + b * ff^2 * x^2)^{p - 1}, x], x, \text{Tan}[e + f * x] / ff], x] /; \text{FreeQ}[\{b, e, f, p\}, x] \&\& \text{!IntegerQ}[p]$

#### Rule 195

$\text{Int}[(a + (b * x)^n)^p, x\_Symbol] \rightarrow \text{Simp}[(x * (a + b * x^n)^p) / (n * p + 1), x] + \text{Dist}[(a * n * p) / (n * p + 1), \text{Int}[(a + b * x^n)^{p - 1}, x], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[p, 0] \&\& (\text{IntegerQ}[2 * p] \|\| (\text{EqQ}[n, 2] \&\& \text{IntegerQ}[4 * p]) \|\| (\text{EqQ}[n, 2] \&\& \text{IntegerQ}[3 * p]) \|\| \text{LtQ}[\text{Denominator}[p + 1/n], \text{Denominator}[p]])$

#### Rule 217



```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int (a \sec^2(x))^{3/2} dx &= a \operatorname{Subst} \left( \int \sqrt{a + ax^2} dx, x, \tan(x) \right) \\ &= \frac{1}{2} a \sqrt{a \sec^2(x)} \tan(x) + \frac{1}{2} a^2 \operatorname{Subst} \left( \int \frac{1}{\sqrt{a + ax^2}} dx, x, \tan(x) \right) \\ &= \frac{1}{2} a \sqrt{a \sec^2(x)} \tan(x) + \frac{1}{2} a^2 \operatorname{Subst} \left( \int \frac{1}{1 - ax^2} dx, x, \frac{\tan(x)}{\sqrt{a \sec^2(x)}} \right) \\ &= \frac{1}{2} a^{3/2} \tanh^{-1} \left( \frac{\sqrt{a} \tan(x)}{\sqrt{a \sec^2(x)}} \right) + \frac{1}{2} a \sqrt{a \sec^2(x)} \tan(x) \end{aligned}$$

**Mathematica [A]** time = 0.050598, size = 55, normalized size = 1.2

$$\frac{1}{2} a \cos(x) \sqrt{a \sec^2(x)} \left( \tan(x) \sec(x) - \log \left( \cos \left( \frac{x}{2} \right) - \sin \left( \frac{x}{2} \right) \right) + \log \left( \sin \left( \frac{x}{2} \right) + \cos \left( \frac{x}{2} \right) \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^2)^(3/2), x]
```

```
[Out] (a*Cos[x]*Sqrt[a*Sec[x]^2]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[
x/2]] + Sec[x]*Tan[x]))/2
```

**Maple [A]** time = 0.061, size = 55, normalized size = 1.2

$$\frac{\cos(x)}{2} \left( (\cos(x))^2 \ln \left( -\frac{-1 + \cos(x) - \sin(x)}{\sin(x)} \right) - (\cos(x))^2 \ln \left( -\frac{-1 + \cos(x) + \sin(x)}{\sin(x)} \right) + \sin(x) \right) \left( \frac{a}{(\cos(x))^2} \right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*sec(x)^2)^(3/2),x)

[Out] 1/2\*(cos(x)^2\*ln(-(-1+cos(x)-sin(x))/sin(x))-cos(x)^2\*ln(-(-1+cos(x)+sin(x))/sin(x))+sin(x))\*cos(x)\*(a/cos(x)^2)^(3/2)

**Maxima [B]** time = 1.94571, size = 437, normalized size = 9.5

$$\frac{(8a \cos(3x) \sin(2x) - 8a \cos(x) \sin(2x) + 8a \cos(2x) \sin(x) - 4(a \sin(3x) - a \sin(x)) \cos(4x) - (a \cos(4x)^2 + 4a \cos(2x)^2 + \sin(4x)^2 + 4a \sin(4x) \sin(2x) + 4a \sin(2x)^2 + 2(2a \cos(2x) + a) \cos(4x) + 4a \cos(2x) + a) \log(\cos(x)^2 + \sin(x)^2 + 2 \sin(x) + 1) + (a \cos(4x)^2 + 4a \cos(2x)^2 + a \sin(4x)^2 + 4a \sin(4x) \sin(2x) + 4a \sin(2x)^2 + 2(2a \cos(2x) + a) \cos(4x) + 4a \cos(2x) + a) \log(\cos(x)^2 + \sin(x)^2 - 2 \sin(x) + 1) + 4(a \cos(3x) - a \cos(x)) \sin(4x) - 4(2a \cos(2x) + a) \sin(3x) + 4a \sin(x)) \sqrt{a} / (2(2 \cos(2x) + 1) \cos(4x) + \cos(4x)^2 + 4 \cos(2x)^2 + \sin(4x)^2 + 4 \sin(4x) \sin(2x) + 4 \sin(2x)^2 + 4 \cos(2x) + 1)}{4 \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^2)^(3/2),x, algorithm="maxima")

[Out] -1/4\*(8\*a\*cos(3\*x)\*sin(2\*x) - 8\*a\*cos(x)\*sin(2\*x) + 8\*a\*cos(2\*x)\*sin(x) - 4\*(a\*sin(3\*x) - a\*sin(x))\*cos(4\*x) - (a\*cos(4\*x)^2 + 4\*a\*cos(2\*x)^2 + a\*sin(4\*x)^2 + 4\*a\*sin(4\*x)\*sin(2\*x) + 4\*a\*sin(2\*x)^2 + 2\*(2\*a\*cos(2\*x) + a)\*cos(4\*x) + 4\*a\*cos(2\*x) + a)\*log(cos(x)^2 + sin(x)^2 + 2\*sin(x) + 1) + (a\*cos(4\*x)^2 + 4\*a\*cos(2\*x)^2 + a\*sin(4\*x)^2 + 4\*a\*sin(4\*x)\*sin(2\*x) + 4\*a\*sin(2\*x)^2 + 2\*(2\*a\*cos(2\*x) + a)\*cos(4\*x) + 4\*a\*cos(2\*x) + a)\*log(cos(x)^2 + sin(x)^2 - 2\*sin(x) + 1) + 4\*(a\*cos(3\*x) - a\*cos(x))\*sin(4\*x) - 4\*(2\*a\*cos(2\*x) + a)\*sin(3\*x) + 4\*a\*sin(x))\*sqrt(a)/(2\*(2\*cos(2\*x) + 1)\*cos(4\*x) + cos(4\*x)^2 + 4\*cos(2\*x)^2 + sin(4\*x)^2 + 4\*sin(4\*x)\*sin(2\*x) + 4\*sin(2\*x)^2 + 4\*cos(2\*x) + 1)

**Fricas [A]** time = 1.47143, size = 119, normalized size = 2.59

$$\frac{\left(a \cos(x)^2 \log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right) - 2a \sin(x)\right) \sqrt{\frac{a}{\cos(x)^2}}}{4 \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^2)^(3/2),x, algorithm="fricas")

[Out] -1/4\*(a\*cos(x)^2\*log(-(sin(x) - 1)/(sin(x) + 1)) - 2\*a\*sin(x))\*sqrt(a/cos(x)^2)/cos(x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a \sec^2(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)\*\*2)\*\*(3/2),x)

[Out] Integral((a\*sec(x)\*\*2)\*\*(3/2), x)

**Giac [A]** time = 1.30607, size = 57, normalized size = 1.24

$$\frac{1}{4} \left( \log(\sin(x) + 1) \operatorname{sgn}(\cos(x)) - \log(-\sin(x) + 1) \operatorname{sgn}(\cos(x)) - \frac{2 \operatorname{sgn}(\cos(x)) \sin(x)}{\sin(x)^2 - 1} \right) a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^2)^(3/2),x, algorithm="giac")

[Out] 1/4\*(log(sin(x) + 1)\*sgn(cos(x)) - log(-sin(x) + 1)\*sgn(cos(x)) - 2\*sgn(cos(x))\*sin(x)/(sin(x)^2 - 1))\*a^(3/2)

### 3.50 $\int \sqrt{a \sec^2(x)} dx$

**Optimal.** Leaf size=25

$$\sqrt{a} \tanh^{-1} \left( \frac{\sqrt{a} \tan(x)}{\sqrt{a \sec^2(x)}} \right)$$

[Out] Sqrt[a]\*ArcTanh[(Sqrt[a]\*Tan[x])/Sqrt[a\*Sec[x]^2]]

**Rubi [A]** time = 0.0148376, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {4122, 217, 206}

$$\sqrt{a} \tanh^{-1} \left( \frac{\sqrt{a} \tan(x)}{\sqrt{a \sec^2(x)}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a\*Sec[x]^2], x]

[Out] Sqrt[a]\*ArcTanh[(Sqrt[a]\*Tan[x])/Sqrt[a\*Sec[x]^2]]

#### Rule 4122

Int[((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)^2])^(p\_), x\_Symbol] :> With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[(b\*ff)/f, Subst[Int[(b + b\*ff^2\*x^2)^(p - 1), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\begin{aligned}\int \sqrt{a \sec^2(x)} dx &= a \operatorname{Subst} \left( \int \frac{1}{\sqrt{a + ax^2}} dx, x, \tan(x) \right) \\ &= a \operatorname{Subst} \left( \int \frac{1}{1 - ax^2} dx, x, \frac{\tan(x)}{\sqrt{a \sec^2(x)}} \right) \\ &= \sqrt{a} \tanh^{-1} \left( \frac{\sqrt{a} \tan(x)}{\sqrt{a \sec^2(x)}} \right)\end{aligned}$$

**Mathematica [A]** time = 0.0078522, size = 46, normalized size = 1.84

$$\cos(x) \sqrt{a \sec^2(x)} \left( \log \left( \sin \left( \frac{x}{2} \right) + \cos \left( \frac{x}{2} \right) \right) - \log \left( \cos \left( \frac{x}{2} \right) - \sin \left( \frac{x}{2} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a\*Sec[x]^2], x]

[Out] Cos[x]\*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])\*Sqrt[a\*Sec[x]^2]

**Maple [A]** time = 0.056, size = 23, normalized size = 0.9

$$-2 \cos(x) \operatorname{Artanh} \left( \frac{-1 + \cos(x)}{\sin(x)} \right) \sqrt{\frac{a}{(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*sec(x)^2)^(1/2), x)

[Out] -2\*cos(x)\*arctanh((-1+cos(x))/sin(x))\*(a/cos(x)^2)^(1/2)

**Maxima [A]** time = 1.93595, size = 51, normalized size = 2.04

$$\frac{1}{2} \sqrt{a} \left( \log \left( \cos^2(x) + \sin^2(x) + 2 \sin(x) + 1 \right) - \log \left( \cos^2(x) + \sin^2(x) - 2 \sin(x) + 1 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^2)^(1/2),x, algorithm="maxima")

[Out] 1/2\*sqrt(a)\*(log(cos(x)^2 + sin(x)^2 + 2\*sin(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2\*sin(x) + 1))

**Fricas [A]** time = 1.46416, size = 171, normalized size = 6.84

$$\left[ -\frac{1}{2} \sqrt{\frac{a}{\cos(x)^2}} \cos(x) \log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right), -\sqrt{-a} \arctan\left(\frac{\sqrt{-a} \sqrt{\frac{a}{\cos(x)^2}} \cos(x) \sin(x)}{a}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^2)^(1/2),x, algorithm="fricas")

[Out] [-1/2\*sqrt(a/cos(x)^2)\*cos(x)\*log(-(sin(x) - 1)/(sin(x) + 1)), -sqrt(-a)\*arctan(sqrt(-a)\*sqrt(a/cos(x)^2)\*cos(x)\*sin(x)/a)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sec^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)\*\*2)\*\*(1/2),x)

[Out] Integral(sqrt(a\*sec(x)\*\*2), x)

**Giac [A]** time = 1.34264, size = 42, normalized size = 1.68

$$\frac{1}{4} \sqrt{a} \left( \log\left(\left|\frac{1}{\sin(x)} + \sin(x) + 2\right|\right) - \log\left(\left|\frac{1}{\sin(x)} + \sin(x) - 2\right|\right) \right) \operatorname{sgn}(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^2)^(1/2),x, algorithm="giac")

```
[Out] 1/4*sqrt(a)*(log(abs(1/sin(x) + sin(x) + 2)) - log(abs(1/sin(x) + sin(x) - 2))) *sgn(cos(x))
```

$$3.51 \quad \int \frac{1}{\sqrt{a \sec^2(x)}} dx$$

**Optimal.** Leaf size=13

$$\frac{\tan(x)}{\sqrt{a \sec^2(x)}}$$

[Out] Tan[x]/Sqrt[a\*Sec[x]^2]

**Rubi [A]** time = 0.0286669, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4122, 191}

$$\frac{\tan(x)}{\sqrt{a \sec^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a\*Sec[x]^2],x]

[Out] Tan[x]/Sqrt[a\*Sec[x]^2]

#### Rule 4122

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

#### Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

#### Rubi steps



$$\int \frac{1}{\sqrt{a \sec^2(x)}} dx = a \operatorname{Subst} \left( \int \frac{1}{(a + ax^2)^{3/2}} dx, x, \tan(x) \right)$$

$$= \frac{\tan(x)}{\sqrt{a \sec^2(x)}}$$

**Mathematica [A]** time = 0.0053793, size = 13, normalized size = 1.

$$\frac{\tan(x)}{\sqrt{a \sec^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a\*Sec[x]^2], x]

[Out] Tan[x]/Sqrt[a\*Sec[x]^2]

**Maple [A]** time = 0.069, size = 16, normalized size = 1.2

$$\frac{\sin(x)}{\cos(x)} \frac{1}{\sqrt{\frac{a}{(\cos(x))^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a\*sec(x)^2)^(1/2), x)

[Out] sin(x)/(a/cos(x)^2)^(1/2)/cos(x)

**Maxima [A]** time = 1.88085, size = 8, normalized size = 0.62

$$\frac{\sin(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)^2)^(1/2), x, algorithm="maxima")

[Out]  $\sin(x)/\sqrt{a}$

---

**Fricas [A]** time = 1.44531, size = 46, normalized size = 3.54

$$\frac{\sqrt{\frac{a}{\cos(x)^2}} \cos(x) \sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*sec(x)^2)^(1/2),x, algorithm="fricas")`

[Out] `sqrt(a/cos(x)^2)*cos(x)*sin(x)/a`

---

**Sympy [A]** time = 0.53778, size = 15, normalized size = 1.15

$$\frac{\tan(x)}{\sqrt{a}\sqrt{\sec^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*sec(x)**2)**(1/2),x)`

[Out] `tan(x)/(sqrt(a)*sqrt(sec(x)**2))`

---

**Giac [A]** time = 1.22898, size = 15, normalized size = 1.15

$$\frac{\sin(x)}{\sqrt{a}\operatorname{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*sec(x)^2)^(1/2),x, algorithm="giac")`

[Out] `sin(x)/(sqrt(a)*sgn(cos(x)))`

$$3.52 \quad \int \frac{1}{(a \sec^2(x))^{3/2}} dx$$

**Optimal.** Leaf size=36

$$\frac{2 \tan(x)}{3a\sqrt{a \sec^2(x)}} + \frac{\tan(x)}{3(a \sec^2(x))^{3/2}}$$

[Out] Tan[x]/(3\*(a\*Sec[x]^2)^(3/2)) + (2\*Tan[x])/(3\*a\*Sqrt[a\*Sec[x]^2])

**Rubi [A]** time = 0.0182093, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {4122, 192, 191}

$$\frac{2 \tan(x)}{3a\sqrt{a \sec^2(x)}} + \frac{\tan(x)}{3(a \sec^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a\*Sec[x]^2)^(-3/2), x]

[Out] Tan[x]/(3\*(a\*Sec[x]^2)^(3/2)) + (2\*Tan[x])/(3\*a\*Sqrt[a\*Sec[x]^2])

#### Rule 4122

Int[((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^2)^(p\_), x\_Symbol] :> With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[(b\*ff)/f, Subst[Int[(b + b\*ff^2\*x^2)^(p - 1), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]

#### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a \sec^2(x))^{3/2}} dx &= a \operatorname{Subst} \left( \int \frac{1}{(a + ax^2)^{5/2}} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{3 (a \sec^2(x))^{3/2}} + \frac{2}{3} \operatorname{Subst} \left( \int \frac{1}{(a + ax^2)^{3/2}} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{3 (a \sec^2(x))^{3/2}} + \frac{2 \tan(x)}{3a \sqrt{a \sec^2(x)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0178713, size = 27, normalized size = 0.75

$$\frac{(9 \sin(x) + \sin(3x)) \sec^3(x)}{12 (a \sec^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*Sec[x]^2)^(-3/2), x]

[Out] (Sec[x]^3\*(9\*Sin[x] + Sin[3\*x]))/(12\*(a\*Sec[x]^2)^(3/2))

**Maple [A]** time = 0.057, size = 23, normalized size = 0.6

$$\frac{\sin(x) \left( (\cos(x))^2 + 2 \right)}{3 (\cos(x))^3} \left( \frac{a}{(\cos(x))^2} \right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a\*sec(x)^2)^(3/2), x)

[Out] 1/3\*sin(x)\*(cos(x)^2+2)/cos(x)^3/(a/cos(x)^2)^(3/2)

**Maxima [A]** time = 1.94539, size = 19, normalized size = 0.53

$$\frac{\sin(3x) + 9 \sin(x)}{12 a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)^2)^(3/2),x, algorithm="maxima")

[Out] 1/12\*(sin(3\*x) + 9\*sin(x))/a^(3/2)

**Fricas [A]** time = 1.4321, size = 74, normalized size = 2.06

$$\frac{(\cos(x)^3 + 2 \cos(x)) \sqrt{\frac{a}{\cos(x)^2}} \sin(x)}{3 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)^2)^(3/2),x, algorithm="fricas")

[Out] 1/3\*(cos(x)^3 + 2\*cos(x))\*sqrt(a/cos(x)^2)\*sin(x)/a^2

**Sympy [A]** time = 1.48685, size = 37, normalized size = 1.03

$$\frac{2 \tan^3(x)}{3 a^{\frac{3}{2}} (\sec^2(x))^{\frac{3}{2}}} + \frac{\tan(x)}{a^{\frac{3}{2}} (\sec^2(x))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)\*\*2)\*\*(3/2),x)

[Out] 2\*tan(x)\*\*3/(3\*a\*\*(3/2)\*(sec(x)\*\*2)\*\*(3/2)) + tan(x)/(a\*\*(3/2)\*(sec(x)\*\*2)\*\*(3/2))

**Giac [B]** time = 1.37879, size = 78, normalized size = 2.17

$$\frac{2 \left( 3 \left( \frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right) \right)^2 \operatorname{sgn}\left(-\tan\left(\frac{1}{2}x\right)^2 + 1\right) - 4 \operatorname{sgn}\left(-\tan\left(\frac{1}{2}x\right)^2 + 1\right) \right)}{3 a^{\frac{3}{2}} \left( \frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right) \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(1/tan(1/2*x) + tan(1/2*x))^2*sgn(-tan(1/2*x)^2 + 1) - 4*sgn(-tan(1/2*x)^2 + 1))/(a^(3/2)*(1/tan(1/2*x) + tan(1/2*x))^3)
```

$$3.53 \quad \int \frac{1}{(a \sec^2(x))^{5/2}} dx$$

**Optimal.** Leaf size=55

$$\frac{8 \tan(x)}{15a^2 \sqrt{a \sec^2(x)}} + \frac{4 \tan(x)}{15a (a \sec^2(x))^{3/2}} + \frac{\tan(x)}{5 (a \sec^2(x))^{5/2}}$$

[Out] Tan[x]/(5\*(a\*Sec[x]^2)^(5/2)) + (4\*Tan[x])/((15\*a\*(a\*Sec[x]^2)^(3/2))) + (8\*Tan[x])/((15\*a^2\*Sqrt[a\*Sec[x]^2]))

**Rubi [A]** time = 0.0266371, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {4122, 192, 191}

$$\frac{8 \tan(x)}{15a^2 \sqrt{a \sec^2(x)}} + \frac{4 \tan(x)}{15a (a \sec^2(x))^{3/2}} + \frac{\tan(x)}{5 (a \sec^2(x))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a\*Sec[x]^2)^(-5/2), x]

[Out] Tan[x]/(5\*(a\*Sec[x]^2)^(5/2)) + (4\*Tan[x])/((15\*a\*(a\*Sec[x]^2)^(3/2))) + (8\*Tan[x])/((15\*a^2\*Sqrt[a\*Sec[x]^2]))

#### Rule 4122

Int[((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^2)^(p\_), x\_Symbol] :> With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[(b\*ff)/f, Subst[Int[(b + b\*ff^2\*x^2)^(p - 1), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]

#### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

#### Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{(a \sec^2(x))^{5/2}} dx &= a \operatorname{Subst} \left( \int \frac{1}{(a + ax^2)^{7/2}} dx, x, \tan(x) \right) \\ &= \frac{\tan(x)}{5 (a \sec^2(x))^{5/2}} + \frac{4}{5} \operatorname{Subst} \left( \int \frac{1}{(a + ax^2)^{5/2}} dx, x, \tan(x) \right) \\ &= \frac{\tan(x)}{5 (a \sec^2(x))^{5/2}} + \frac{4 \tan(x)}{15a (a \sec^2(x))^{3/2}} + \frac{8 \operatorname{Subst} \left( \int \frac{1}{(a+ax^2)^{3/2}} dx, x, \tan(x) \right)}{15a} \\ &= \frac{\tan(x)}{5 (a \sec^2(x))^{5/2}} + \frac{4 \tan(x)}{15a (a \sec^2(x))^{3/2}} + \frac{8 \tan(x)}{15a^2 \sqrt{a \sec^2(x)}} \end{aligned}$$

**Mathematica [A]** time = 0.0278019, size = 36, normalized size = 0.65

$$\frac{(150 \sin(x) + 25 \sin(3x) + 3 \sin(5x)) \cos(x) \sqrt{a \sec^2(x)}}{240a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^2)^(-5/2), x]
```

```
[Out] (Cos[x]*Sqrt[a*Sec[x]^2]*(150*Sin[x] + 25*Sin[3*x] + 3*Sin[5*x]))/(240*a^3)
```

**Maple [A]** time = 0.064, size = 31, normalized size = 0.6

$$\frac{\sin(x) \left( 3 (\cos(x))^4 + 4 (\cos(x))^2 + 8 \right)}{15 (\cos(x))^5} \left( \frac{a}{(\cos(x))^2} \right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^2)^(5/2), x)
```



[Out]  $1/15*\sin(x)*(3*\cos(x)^4+4*\cos(x)^2+8)/\cos(x)^5/(a/\cos(x)^2)^{(5/2)}$

---

**Maxima [A]** time = 1.83415, size = 30, normalized size = 0.55

$$\frac{3 \sin(5x) + 25 \sin(3x) + 150 \sin(x)}{240 a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*sec(x)^2)^(5/2),x, algorithm="maxima")`

[Out]  $1/240*(3*\sin(5*x) + 25*\sin(3*x) + 150*\sin(x))/a^{(5/2)}$

---

**Fricas [A]** time = 1.44471, size = 96, normalized size = 1.75

$$\frac{(3 \cos(x)^5 + 4 \cos(x)^3 + 8 \cos(x)) \sqrt{\frac{a}{\cos(x)^2}} \sin(x)}{15 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*sec(x)^2)^(5/2),x, algorithm="fricas")`

[Out]  $1/15*(3*\cos(x)^5 + 4*\cos(x)^3 + 8*\cos(x))*\sqrt{a/\cos(x)^2}*\sin(x)/a^3$

---

**Sympy [A]** time = 17.4853, size = 60, normalized size = 1.09

$$\frac{8 \tan^5(x)}{15 a^{\frac{5}{2}} (\sec^2(x))^{\frac{5}{2}}} + \frac{4 \tan^3(x)}{3 a^{\frac{5}{2}} (\sec^2(x))^{\frac{5}{2}}} + \frac{\tan(x)}{a^{\frac{5}{2}} (\sec^2(x))^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*sec(x)**2)**(5/2),x)`

[Out]  $8*\tan(x)**5/(15*a**(5/2)*(sec(x)**2)**(5/2)) + 4*\tan(x)**3/(3*a**(5/2)*(sec(x)**2)**(5/2)) + \tan(x)/(a**(5/2)*(sec(x)**2)**(5/2))$

---

**Giac [A]** time = 1.44072, size = 113, normalized size = 2.05

$$\frac{2 \left( 15 \left( \frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right) \right)^4 \operatorname{sgn}\left(-\tan\left(\frac{1}{2}x\right)^2 + 1\right) - 40 \left( \frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right) \right)^2 \operatorname{sgn}\left(-\tan\left(\frac{1}{2}x\right)^2 + 1\right) + 48 \operatorname{sgn}\left(-\tan\left(\frac{1}{2}x\right)^2 + 1\right) \right)}{15 a^{\frac{5}{2}} \left( \frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right) \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)^2)^(5/2),x, algorithm="giac")

[Out] 2/15\*(15\*(1/tan(1/2\*x) + tan(1/2\*x))^4\*sgn(-tan(1/2\*x)^2 + 1) - 40\*(1/tan(1/2\*x) + tan(1/2\*x))^2\*sgn(-tan(1/2\*x)^2 + 1) + 48\*sgn(-tan(1/2\*x)^2 + 1))/(a^(5/2)\*(1/tan(1/2\*x) + tan(1/2\*x))^5)

$$3.54 \quad \int \frac{1}{(a \sec^2(x))^{7/2}} dx$$

**Optimal.** Leaf size=74

$$\frac{16 \tan(x)}{35a^3 \sqrt{a \sec^2(x)}} + \frac{8 \tan(x)}{35a^2 (a \sec^2(x))^{3/2}} + \frac{6 \tan(x)}{35a (a \sec^2(x))^{5/2}} + \frac{\tan(x)}{7 (a \sec^2(x))^{7/2}}$$

[Out] Tan[x]/(7\*(a\*Sec[x]^2)^(7/2)) + (6\*Tan[x])/(35\*a\*(a\*Sec[x]^2)^(5/2)) + (8\*Tan[x])/(35\*a^2\*(a\*Sec[x]^2)^(3/2)) + (16\*Tan[x])/(35\*a^3\*Sqrt[a\*Sec[x]^2])

**Rubi [A]** time = 0.0349597, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {4122, 192, 191}

$$\frac{16 \tan(x)}{35a^3 \sqrt{a \sec^2(x)}} + \frac{8 \tan(x)}{35a^2 (a \sec^2(x))^{3/2}} + \frac{6 \tan(x)}{35a (a \sec^2(x))^{5/2}} + \frac{\tan(x)}{7 (a \sec^2(x))^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(a\*Sec[x]^2)^(-7/2), x]

[Out] Tan[x]/(7\*(a\*Sec[x]^2)^(7/2)) + (6\*Tan[x])/(35\*a\*(a\*Sec[x]^2)^(5/2)) + (8\*Tan[x])/(35\*a^2\*(a\*Sec[x]^2)^(3/2)) + (16\*Tan[x])/(35\*a^3\*Sqrt[a\*Sec[x]^2])

#### Rule 4122

Int[((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^2)^(p\_), x\_Symbol] :> With[{ff = FreeFactors[Tan[e + f\*x], x]}, Dist[(b\*ff)/f, Subst[Int[(b + b\*ff^2\*x^2)^(p - 1), x], x, Tan[e + f\*x]/ff], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]

#### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{(a \sec^2(x))^{7/2}} dx &= a \operatorname{Subst} \left( \int \frac{1}{(a + ax^2)^{9/2}} dx, x, \tan(x) \right) \\
 &= \frac{\tan(x)}{7(a \sec^2(x))^{7/2}} + \frac{6}{7} \operatorname{Subst} \left( \int \frac{1}{(a + ax^2)^{7/2}} dx, x, \tan(x) \right) \\
 &= \frac{\tan(x)}{7(a \sec^2(x))^{7/2}} + \frac{6 \tan(x)}{35a(a \sec^2(x))^{5/2}} + \frac{24 \operatorname{Subst} \left( \int \frac{1}{(a+ax^2)^{5/2}} dx, x, \tan(x) \right)}{35a} \\
 &= \frac{\tan(x)}{7(a \sec^2(x))^{7/2}} + \frac{6 \tan(x)}{35a(a \sec^2(x))^{5/2}} + \frac{8 \tan(x)}{35a^2(a \sec^2(x))^{3/2}} + \frac{16 \operatorname{Subst} \left( \int \frac{1}{(a+ax^2)^{3/2}} dx, x, \tan(x) \right)}{35a^2} \\
 &= \frac{\tan(x)}{7(a \sec^2(x))^{7/2}} + \frac{6 \tan(x)}{35a(a \sec^2(x))^{5/2}} + \frac{8 \tan(x)}{35a^2(a \sec^2(x))^{3/2}} + \frac{16 \tan(x)}{35a^3 \sqrt{a \sec^2(x)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0347235, size = 42, normalized size = 0.57

$$\frac{(1225 \sin(x) + 245 \sin(3x) + 49 \sin(5x) + 5 \sin(7x)) \cos(x) \sqrt{a \sec^2(x)}}{2240a^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*Sec[x]^2)^(-7/2), x]

[Out] (Cos[x]\*Sqrt[a\*Sec[x]^2]\*(1225\*Sin[x] + 245\*Sin[3\*x] + 49\*Sin[5\*x] + 5\*Sin[7\*x]))/(2240\*a^4)

**Maple [A]** time = 0.081, size = 37, normalized size = 0.5

$$\frac{\sin(x) (5 (\cos(x))^6 + 6 (\cos(x))^4 + 8 (\cos(x))^2 + 16)}{35 (\cos(x))^7} \left( \frac{a}{(\cos(x))^2} \right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a*sec(x)^2)^(7/2),x)`

[Out]  $1/35*\sin(x)*(5*\cos(x)^6+6*\cos(x)^4+8*\cos(x)^2+16)/\cos(x)^7/(a/\cos(x)^2)^(7/2)$

**Maxima [A]** time = 1.7842, size = 38, normalized size = 0.51

$$\frac{5 \sin(7x) + 49 \sin(5x) + 245 \sin(3x) + 1225 \sin(x)}{2240 a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*sec(x)^2)^(7/2),x, algorithm="maxima")`

[Out]  $1/2240*(5*\sin(7*x) + 49*\sin(5*x) + 245*\sin(3*x) + 1225*\sin(x))/a^(7/2)$

**Fricas [A]** time = 1.45123, size = 115, normalized size = 1.55

$$\frac{(5 \cos(x)^7 + 6 \cos(x)^5 + 8 \cos(x)^3 + 16 \cos(x)) \sqrt{\frac{a}{\cos(x)^2}} \sin(x)}{35 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*sec(x)^2)^(7/2),x, algorithm="fricas")`

[Out]  $1/35*(5*\cos(x)^7 + 6*\cos(x)^5 + 8*\cos(x)^3 + 16*\cos(x))*\sqrt{a/\cos(x)^2}*\sin(x)/a^4$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)\*\*2)\*\*(7/2),x)

[Out] Timed out

**Giac [A]** time = 1.6085, size = 149, normalized size = 2.01

$$\frac{2 \left( 35 \left( \frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right) \right)^6 \operatorname{sgn}\left(-\tan\left(\frac{1}{2}x\right)^2 + 1\right) - 140 \left( \frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right) \right)^4 \operatorname{sgn}\left(-\tan\left(\frac{1}{2}x\right)^2 + 1\right) + 336 \left( \frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right) \right)^2 \operatorname{sgn}\left(-\tan\left(\frac{1}{2}x\right)^2 + 1\right) - 320 \operatorname{sgn}\left(-\tan\left(\frac{1}{2}x\right)^2 + 1\right) \right)}{35 a^{\frac{7}{2}} \left( \frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right) \right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)^2)^(7/2),x, algorithm="giac")

[Out] 2/35\*(35\*(1/tan(1/2\*x) + tan(1/2\*x))^6\*sgn(-tan(1/2\*x)^2 + 1) - 140\*(1/tan(1/2\*x) + tan(1/2\*x))^4\*sgn(-tan(1/2\*x)^2 + 1) + 336\*(1/tan(1/2\*x) + tan(1/2\*x))^2\*sgn(-tan(1/2\*x)^2 + 1) - 320\*sgn(-tan(1/2\*x)^2 + 1))/(a^(7/2)\*(1/tan(1/2\*x) + tan(1/2\*x))^7)

### 3.55 $\int (a \sec^3(x))^{5/2} dx$

**Optimal.** Leaf size=117

$$\frac{2}{13}a^2 \tan(x) \sec^4(x) \sqrt{a \sec^3(x)} + \frac{22}{117}a^2 \tan(x) \sec^2(x) \sqrt{a \sec^3(x)} + \frac{154}{585}a^2 \tan(x) \sqrt{a \sec^3(x)} - \frac{154}{195}a^2 \cos^2(x) E\left(\frac{x}{2} \middle| 2\right) \sqrt{a \sec^3(x)}$$

```
[Out] (-154*a^2*Cos[x]^(3/2)*EllipticE[x/2, 2]*Sqrt[a*Sec[x]^3])/195 + (154*a^2*Cos[x]*Sqrt[a*Sec[x]^3]*Sin[x])/195 + (154*a^2*Sqrt[a*Sec[x]^3]*Tan[x])/585 + (22*a^2*Sec[x]^2*Sqrt[a*Sec[x]^3]*Tan[x])/117 + (2*a^2*Sec[x]^4*Sqrt[a*Sec[x]^3]*Tan[x])/13
```

**Rubi [A]** time = 0.0522832, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4123, 3768, 3771, 2639}

$$\frac{2}{13}a^2 \tan(x) \sec^4(x) \sqrt{a \sec^3(x)} + \frac{22}{117}a^2 \tan(x) \sec^2(x) \sqrt{a \sec^3(x)} + \frac{154}{585}a^2 \tan(x) \sqrt{a \sec^3(x)} - \frac{154}{195}a^2 \cos^2(x) E\left(\frac{x}{2} \middle| 2\right) \sqrt{a \sec^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^3)^(5/2), x]
```

```
[Out] (-154*a^2*Cos[x]^(3/2)*EllipticE[x/2, 2]*Sqrt[a*Sec[x]^3])/195 + (154*a^2*Cos[x]*Sqrt[a*Sec[x]^3]*Sin[x])/195 + (154*a^2*Sqrt[a*Sec[x]^3]*Tan[x])/585 + (22*a^2*Sec[x]^2*Sqrt[a*Sec[x]^3]*Tan[x])/117 + (2*a^2*Sec[x]^4*Sqrt[a*Sec[x]^3]*Tan[x])/13
```

#### Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := Dist[(b^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPart[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p]
```

#### Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)])*(b_.)^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^n, x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int (a \sec^3(x))^{5/2} dx &= \frac{(a^2 \sqrt{a \sec^3(x)}) \int \sec^{\frac{15}{2}}(x) dx}{\sec^{\frac{3}{2}}(x)} \\
 &= \frac{2}{13} a^2 \sec^4(x) \sqrt{a \sec^3(x)} \tan(x) + \frac{(11 a^2 \sqrt{a \sec^3(x)}) \int \sec^{\frac{11}{2}}(x) dx}{13 \sec^{\frac{3}{2}}(x)} \\
 &= \frac{22}{117} a^2 \sec^2(x) \sqrt{a \sec^3(x)} \tan(x) + \frac{2}{13} a^2 \sec^4(x) \sqrt{a \sec^3(x)} \tan(x) + \frac{(77 a^2 \sqrt{a \sec^3(x)}) \int \sec^{\frac{7}{2}}(x) dx}{117 \sec^{\frac{3}{2}}(x)} \\
 &= \frac{154}{585} a^2 \sqrt{a \sec^3(x)} \tan(x) + \frac{22}{117} a^2 \sec^2(x) \sqrt{a \sec^3(x)} \tan(x) + \frac{2}{13} a^2 \sec^4(x) \sqrt{a \sec^3(x)} \tan(x) + \frac{(77 a^2 \sqrt{a \sec^3(x)}) \int \sec^{\frac{3}{2}}(x) dx}{117 \sec^{\frac{3}{2}}(x)} \\
 &= \frac{154}{195} a^2 \cos(x) \sqrt{a \sec^3(x)} \sin(x) + \frac{154}{585} a^2 \sqrt{a \sec^3(x)} \tan(x) + \frac{22}{117} a^2 \sec^2(x) \sqrt{a \sec^3(x)} \tan(x) + \frac{2}{13} a^2 \sec^4(x) \sqrt{a \sec^3(x)} \tan(x) \\
 &= \frac{154}{195} a^2 \cos(x) \sqrt{a \sec^3(x)} \sin(x) + \frac{154}{585} a^2 \sqrt{a \sec^3(x)} \tan(x) + \frac{22}{117} a^2 \sec^2(x) \sqrt{a \sec^3(x)} \tan(x) + \frac{2}{13} a^2 \sec^4(x) \sqrt{a \sec^3(x)} \tan(x) \\
 &= -\frac{154}{195} a^2 \cos^{\frac{3}{2}}(x) E\left(\frac{x}{2} \middle| 2\right) \sqrt{a \sec^3(x)} + \frac{154}{195} a^2 \cos(x) \sqrt{a \sec^3(x)} \sin(x) + \frac{154}{585} a^2 \sqrt{a \sec^3(x)} \tan(x) + \frac{22}{117} a^2 \sec^2(x) \sqrt{a \sec^3(x)} \tan(x) + \frac{2}{13} a^2 \sec^4(x) \sqrt{a \sec^3(x)} \tan(x)
 \end{aligned}$$

**Mathematica [A]** time = 0.0933119, size = 59, normalized size = 0.5

$$-\frac{2}{585} a \sec(x) (a \sec^3(x))^{3/2} \left( -45 \tan(x) - 231 \sin(x) \cos^5(x) - 77 \sin(x) \cos^3(x) + 231 \cos^{\frac{11}{2}}(x) E\left(\frac{x}{2} \middle| 2\right) - 55 \sin(x) \cos(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a\*Sec[x]^3)^(5/2), x]



[Out]  $(-2*a*\text{Sec}[x]*(a*\text{Sec}[x]^3)^{(3/2)}*(231*\text{Cos}[x]^{(11/2)}*\text{EllipticE}[x/2, 2] - 55*\text{Cos}[x]*\text{Sin}[x] - 77*\text{Cos}[x]^3*\text{Sin}[x] - 231*\text{Cos}[x]^5*\text{Sin}[x] - 45*\text{Tan}[x]))/585$

**Maple [C]** time = 0.368, size = 223, normalized size = 1.9

$$\frac{2(\cos(x)+1)^2(-1+\cos(x))^2\cos(x)}{585(\sin(x))^5} \left( 231i(\cos(x))^7\sin(x)\sqrt{\frac{\cos(x)}{\cos(x)+1}}\sqrt{(\cos(x)+1)^{-1}}\text{EllipticF}\left(\frac{i(-1+\cos(x))}{\sin(x)}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a*\text{sec}(x)^3)^{(5/2)}, x)$

[Out]  $-2/585*(\cos(x)+1)^2*(-1+\cos(x))^2*(231*I*\cos(x)^7*\sin(x)*(\cos(x)/(\cos(x)+1))^{(1/2)}*(1/(\cos(x)+1))^{(1/2)}*\text{EllipticF}(I*(-1+\cos(x))/\sin(x), I)-231*I*\cos(x)^7*\sin(x)*(\cos(x)/(\cos(x)+1))^{(1/2)}*(1/(\cos(x)+1))^{(1/2)}*\text{EllipticE}(I*(-1+\cos(x))/\sin(x), I)+231*I*\cos(x)^6*\sin(x)*(\cos(x)/(\cos(x)+1))^{(1/2)}*(1/(\cos(x)+1))^{(1/2)}*\text{EllipticF}(I*(-1+\cos(x))/\sin(x), I)-231*I*\cos(x)^6*\sin(x)*(\cos(x)/(\cos(x)+1))^{(1/2)}*(1/(\cos(x)+1))^{(1/2)}*\text{EllipticE}(I*(-1+\cos(x))/\sin(x), I)+231*\cos(x)^7-154*\cos(x)^6-22*\cos(x)^4-10*\cos(x)^2-45)*\cos(x)*(a/\cos(x)^3)^{(5/2)}/\sin(x)^5$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (a \sec(x)^3)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a*\text{sec}(x)^3)^{(5/2)}, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}((a*\text{sec}(x)^3)^{(5/2)}, x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a \sec(x)^3} a^2 \sec(x)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(x)^3)*a^2*sec(x)^6, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**3)**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (a \sec(x)^3)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(x)^3)^(5/2), x)
```

### 3.56 $\int (a \sec^3(x))^{3/2} dx$

**Optimal.** Leaf size=65

$$\frac{10}{21}a \cos^2(x) \text{EllipticF}\left(\frac{x}{2}, 2\right) \sqrt{a \sec^3(x)} + \frac{10}{21}a \sin(x) \sqrt{a \sec^3(x)} + \frac{2}{7}a \tan(x) \sec(x) \sqrt{a \sec^3(x)}$$

[Out] (10\*a\*cos[x]^(3/2)\*EllipticF[x/2, 2]\*Sqrt[a\*Sec[x]^3])/21 + (10\*a\*Sqrt[a\*Sec[x]^3]\*Sin[x])/21 + (2\*a\*Sec[x]\*Sqrt[a\*Sec[x]^3]\*Tan[x])/7

**Rubi [A]** time = 0.0353327, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4123, 3768, 3771, 2641}

$$\frac{10}{21}a \sin(x) \sqrt{a \sec^3(x)} + \frac{2}{7}a \tan(x) \sec(x) \sqrt{a \sec^3(x)} + \frac{10}{21}a \cos^2(x) F\left(\frac{x}{2} \middle| 2\right) \sqrt{a \sec^3(x)}$$

Antiderivative was successfully verified.

[In] Int[(a\*Sec[x]^3)^(3/2), x]

[Out] (10\*a\*cos[x]^(3/2)\*EllipticF[x/2, 2]\*Sqrt[a\*Sec[x]^3])/21 + (10\*a\*Sqrt[a\*Sec[x]^3]\*Sin[x])/21 + (2\*a\*Sec[x]\*Sqrt[a\*Sec[x]^3]\*Tan[x])/7

#### Rule 4123

Int[((b\_.)\*((c\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_))^(p\_), x\_Symbol] := Dist[(b^IntPart[p]\*(b\*(c\*Sec[e + f\*x])^n)^FracPart[p])/(c\*Sec[e + f\*x])^(n\*FracPart[p]), Int[(c\*Sec[e + f\*x])^(n\*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p]

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*cos[c + d\*x]\*(b\*csc[c + d\*x])^(n-1))/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*csc[c + d\*x])^n\*sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int (a \sec^3(x))^{3/2} dx &= \frac{(a\sqrt{a \sec^3(x)}) \int \sec^{\frac{9}{2}}(x) dx}{\sec^{\frac{3}{2}}(x)} \\
 &= \frac{2}{7} a \sec(x) \sqrt{a \sec^3(x)} \tan(x) + \frac{(5a\sqrt{a \sec^3(x)}) \int \sec^{\frac{5}{2}}(x) dx}{7 \sec^{\frac{3}{2}}(x)} \\
 &= \frac{10}{21} a \sqrt{a \sec^3(x)} \sin(x) + \frac{2}{7} a \sec(x) \sqrt{a \sec^3(x)} \tan(x) + \frac{(5a\sqrt{a \sec^3(x)}) \int \sqrt{\sec(x)} dx}{21 \sec^{\frac{3}{2}}(x)} \\
 &= \frac{10}{21} a \sqrt{a \sec^3(x)} \sin(x) + \frac{2}{7} a \sec(x) \sqrt{a \sec^3(x)} \tan(x) + \frac{1}{21} \left( 5a \cos^{\frac{3}{2}}(x) \sqrt{a \sec^3(x)} \right) \int \frac{1}{\sqrt{\cos(x)}} dx \\
 &= \frac{10}{21} a \cos^{\frac{3}{2}}(x) F\left(\frac{x}{2} \middle| 2\right) \sqrt{a \sec^3(x)} + \frac{10}{21} a \sqrt{a \sec^3(x)} \sin(x) + \frac{2}{7} a \sec(x) \sqrt{a \sec^3(x)} \tan(x)
 \end{aligned}$$

**Mathematica [A]** time = 0.0346709, size = 43, normalized size = 0.66

$$\frac{2}{21} a \sec(x) \sqrt{a \sec^3(x)} \left( 5 \cos^{\frac{5}{2}}(x) \text{EllipticF}\left(\frac{x}{2}, 2\right) + 3 \tan(x) + 5 \sin(x) \cos(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a\*Sec[x]^3)^(3/2), x]

```
[Out] (2*a*Sec[x]*Sqrt[a*Sec[x]^3]*(5*Cos[x]^(5/2)*EllipticF[x/2, 2] + 5*Cos[x]*Sin[x] + 3*Tan[x]))/21
```

**Maple [C]** time = 0.182, size = 87, normalized size = 1.3

$$-\frac{2(\cos(x)+1)^2(-1+\cos(x))\cos(x)}{21(\sin(x))^3} \left( 5i(\cos(x))^3 \sin(x) \sqrt{(\cos(x)+1)^{-1}} \sqrt{\frac{\cos(x)}{\cos(x)+1}} \text{EllipticF}\left(\frac{i(-1+\cos(x))}{\sin(x)}, i\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*sec(x)^3)^(3/2),x)`

[Out]  $-2/21*(\cos(x)+1)^2*(-1+\cos(x))*(5*I*\cos(x)^3*\sin(x)*(1/(\cos(x)+1))^{1/2}*(\cos(x)/(\cos(x)+1))^{1/2}*\text{EllipticF}(I*(-1+\cos(x))/\sin(x),I)-5*\cos(x)^3+5*\cos(x)^2-3*\cos(x)+3)*\cos(x)*(a/\cos(x)^3)^{3/2}/\sin(x)^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (a \sec(x)^3)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*sec(x)^3)^(3/2),x, algorithm="maxima")`

[Out] `integrate((a*sec(x)^3)^(3/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a \sec(x)^3} a \sec(x)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*sec(x)^3)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(a*sec(x)^3)*a*sec(x)^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a \sec^3(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sec(x)**3)**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (a \sec(x)^3)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(x)^3)^(3/2), x)
```

### 3.57 $\int \sqrt{a \sec^3(x)} dx$

**Optimal.** Leaf size=42

$$2 \sin(x) \cos(x) \sqrt{a \sec^3(x)} - 2 \cos^{\frac{3}{2}}(x) E\left(\frac{x}{2} \middle| 2\right) \sqrt{a \sec^3(x)}$$

[Out]  $-2 \cos[x]^{(3/2)} \text{EllipticE}[x/2, 2] \text{Sqrt}[a \text{Sec}[x]^3] + 2 \cos[x] \text{Sqrt}[a \text{Sec}[x]^3] \sin[x]$

**Rubi [A]** time = 0.0256077, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4123, 3768, 3771, 2639}

$$2 \sin(x) \cos(x) \sqrt{a \sec^3(x)} - 2 \cos^{\frac{3}{2}}(x) E\left(\frac{x}{2} \middle| 2\right) \sqrt{a \sec^3(x)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[a \text{Sec}[x]^3], x]$

[Out]  $-2 \cos[x]^{(3/2)} \text{EllipticE}[x/2, 2] \text{Sqrt}[a \text{Sec}[x]^3] + 2 \cos[x] \text{Sqrt}[a \text{Sec}[x]^3] \sin[x]$

#### Rule 4123

$\text{Int}[(c \cdot \sec(e + f \cdot x))^n]^{(p)}, x\_Symbol] \rightarrow \text{Dist}[(b \cdot \text{IntPart}[p] \cdot (c \cdot \sec[e + f \cdot x])^n)^{\text{FracPart}[p]} / (c \cdot \sec[e + f \cdot x])^{(n \cdot \text{FracPart}[p])}], \text{Int}[(c \cdot \sec[e + f \cdot x])^{(n \cdot p)}, x], x] /;$  FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p]

#### Rule 3768

$\text{Int}[(\csc(c + d \cdot x))^n]^{(b)}, x\_Symbol] \rightarrow -\text{Simp}[(b \cdot \cos[c + d \cdot x]) \cdot (b \cdot \csc[c + d \cdot x])^{(n-1)} / (d \cdot (n-1)), x] + \text{Dist}[(b^2 \cdot (n-2)) / (n-1), \text{Int}[(b \cdot \csc[c + d \cdot x])^{(n-2)}, x], x] /;$  FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

$\text{Int}[(\csc(c + d \cdot x))^n]^{(b)}, x\_Symbol] \rightarrow \text{Dist}[(b \cdot \csc[c + d \cdot x])^n \cdot \sin[c + d \cdot x]^n, \text{Int}[1/\sin[c + d \cdot x]^n, x], x] /;$  FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a \sec^3(x)} dx &= \frac{\sqrt{a \sec^3(x)} \int \sec^{\frac{3}{2}}(x) dx}{\sec^{\frac{3}{2}}(x)} \\
&= 2 \cos(x) \sqrt{a \sec^3(x)} \sin(x) - \frac{\sqrt{a \sec^3(x)} \int \frac{1}{\sqrt{\sec(x)}} dx}{\sec^{\frac{3}{2}}(x)} \\
&= 2 \cos(x) \sqrt{a \sec^3(x)} \sin(x) - \left( \cos^{\frac{3}{2}}(x) \sqrt{a \sec^3(x)} \right) \int \sqrt{\cos(x)} dx \\
&= -2 \cos^{\frac{3}{2}}(x) E\left(\frac{x}{2} \middle| 2\right) \sqrt{a \sec^3(x)} + 2 \cos(x) \sqrt{a \sec^3(x)} \sin(x)
\end{aligned}$$

**Mathematica [A]** time = 0.0171513, size = 32, normalized size = 0.76

$$2 \cos(x) \sqrt{a \sec^3(x)} \left( \sin(x) - \sqrt{\cos(x)} E\left(\frac{x}{2} \middle| 2\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a\*Sec[x]^3], x]

[Out] 2\*Cos[x]\*Sqrt[a\*Sec[x]^3]\*(-(Sqrt[Cos[x]]\*EllipticE[x/2, 2]) + Sin[x])

**Maple [C]** time = 0.205, size = 191, normalized size = 4.6

$$2 \frac{(\cos(x) + 1)^2 (-1 + \cos(x))^2 \cos(x)}{(\sin(x))^5} \left( i \cos(x) \sin(x) \sqrt{(\cos(x) + 1)^{-1}} \sqrt{\frac{\cos(x)}{\cos(x) + 1}} \text{EllipticE}\left(\frac{i(-1 + \cos(x))}{\sin(x)}, i\right) - i \sqrt{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*sec(x)^3)^(1/2), x)



```
[Out] 2*(cos(x)+1)^2*(-1+cos(x))^2*(I*cos(x)*sin(x)*(1/(cos(x)+1))^(1/2)*(cos(x)/
(cos(x)+1))^(1/2)*EllipticE(I*(-1+cos(x))/sin(x),I)-I*(1/(cos(x)+1))^(1/2)*
(cos(x)/(cos(x)+1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)*cos(x)*sin(x)+I
*sin(x)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticE(I*(-1+cos(
x))/sin(x),I)-I*EllipticF(I*(-1+cos(x))/sin(x),I)*(1/(cos(x)+1))^(1/2)*(cos
(x)/(cos(x)+1))^(1/2)*sin(x)-cos(x)+1)*cos(x)*(a/cos(x)^3)^(1/2)/sin(x)^5
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sec(x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(x)^3), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a \sec(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(x)^3), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sec^3(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**3)**(1/2),x)
```

[Out] `Integral(sqrt(a*sec(x)**3), x)`

---

**Giac** [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sec(x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*sec(x)^3)^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(a*sec(x)^3), x)`

$$3.58 \quad \int \frac{1}{\sqrt{a \sec^3(x)}} dx$$

Optimal. Leaf size=44

$$\frac{2\text{EllipticF}\left(\frac{x}{2}, 2\right)}{3 \cos^{\frac{3}{2}}(x) \sqrt{a \sec^3(x)}} + \frac{2 \tan(x)}{3 \sqrt{a \sec^3(x)}}$$

[Out] (2\*EllipticF[x/2, 2])/(3\*Cos[x]^(3/2)\*Sqrt[a\*Sec[x]^3]) + (2\*Tan[x])/(3\*Sqrt[a\*Sec[x]^3])

**Rubi [A]** time = 0.0270201, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4123, 3769, 3771, 2641}

$$\frac{2 \tan(x)}{3 \sqrt{a \sec^3(x)}} + \frac{2F\left(\frac{x}{2} \middle| 2\right)}{3 \cos^{\frac{3}{2}}(x) \sqrt{a \sec^3(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a\*Sec[x]^3], x]

[Out] (2\*EllipticF[x/2, 2])/(3\*Cos[x]^(3/2)\*Sqrt[a\*Sec[x]^3]) + (2\*Tan[x])/(3\*Sqrt[a\*Sec[x]^3])

#### Rule 4123

Int[((b\_.)\*((c\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_))^(p\_), x\_Symbol] :> Dist[(b ^IntPart[p]\*(b\*(c\*Sec[e + f\*x])^n)^FracPart[p])/(c\*Sec[e + f\*x])^(n\*FracPart[p]), Int[(c\*Sec[e + f\*x])^(n\*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] & !IntegerQ[p]

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n], x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt{a \sec^3(x)}} dx &= \frac{\sec^{\frac{3}{2}}(x) \int \frac{1}{\sec^{\frac{3}{2}}(x)} dx}{\sqrt{a \sec^3(x)}} \\
 &= \frac{2 \tan(x)}{3\sqrt{a \sec^3(x)}} + \frac{\sec^{\frac{3}{2}}(x) \int \sqrt{\sec(x)} dx}{3\sqrt{a \sec^3(x)}} \\
 &= \frac{2 \tan(x)}{3\sqrt{a \sec^3(x)}} + \frac{\int \frac{1}{\sqrt{\cos(x)}} dx}{3 \cos^{\frac{3}{2}}(x) \sqrt{a \sec^3(x)}} \\
 &= \frac{2F\left(\frac{x}{2} \middle| 2\right)}{3 \cos^{\frac{3}{2}}(x) \sqrt{a \sec^3(x)}} + \frac{2 \tan(x)}{3\sqrt{a \sec^3(x)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0398245, size = 31, normalized size = 0.7

$$\frac{2 \left( \frac{\text{EllipticF}\left(\frac{x}{2}, 2\right)}{\cos^{\frac{3}{2}}(x)} + \tan(x) \right)}{3\sqrt{a \sec^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sec[x]^3], x]
```

```
[Out] (2*(EllipticF[x/2, 2]/Cos[x]^(3/2) + Tan[x]))/(3*Sqrt[a*Sec[x]^3])
```

**Maple [C]** time = 0.14, size = 76, normalized size = 1.7

$$\frac{(-2 + 2 \cos(x)) (\cos(x) + 1)^2}{3 (\cos(x))^2 (\sin(x))^3} \left( -i \operatorname{EllipticF} \left( \frac{i(-1 + \cos(x))}{\sin(x)}, i \right) \sqrt{(\cos(x) + 1)^{-1}} \sqrt{\frac{\cos(x)}{\cos(x) + 1}} \sin(x) + (\cos(x))^2 - \cos(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a*sec(x)^3)^(1/2),x)`

[Out] `2/3*(-1+cos(x))*(-I*EllipticF(I*(-1+cos(x))/sin(x),I)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*sin(x)+cos(x)^2-cos(x))*(cos(x)+1)^2/cos(x)^2/sin(x)^3/(a/cos(x)^3)^(1/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a \sec(x)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*sec(x)^3)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt(a*sec(x)^3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{\sqrt{a \sec(x)^3}}{a \sec(x)^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*sec(x)^3)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(a*sec(x)^3)/(a*sec(x)^3), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a \sec^3(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)\*\*3)\*\*(1/2), x)

[Out] Integral(1/sqrt(a\*sec(x)\*\*3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a \sec(x)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)^3)^(1/2), x, algorithm="giac")

[Out] integrate(1/sqrt(a\*sec(x)^3), x)

$$3.59 \quad \int \frac{1}{(a \sec^3(x))^{3/2}} dx$$

**Optimal.** Leaf size=73

$$\frac{14 \sin(x)}{45a\sqrt{a \sec^3(x)}} + \frac{2 \sin(x) \cos^2(x)}{9a\sqrt{a \sec^3(x)}} + \frac{14E\left(\frac{x}{2} \middle| 2\right)}{15a \cos^{\frac{3}{2}}(x)\sqrt{a \sec^3(x)}}$$

[Out] (14\*EllipticE[x/2, 2])/(15\*a\*Cos[x]^(3/2)\*Sqrt[a\*Sec[x]^3]) + (14\*Sin[x])/(45\*a\*Sqrt[a\*Sec[x]^3]) + (2\*Cos[x]^2\*Sin[x])/(9\*a\*Sqrt[a\*Sec[x]^3])

**Rubi [A]** time = 0.0367949, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4123, 3769, 3771, 2639}

$$\frac{14 \sin(x)}{45a\sqrt{a \sec^3(x)}} + \frac{2 \sin(x) \cos^2(x)}{9a\sqrt{a \sec^3(x)}} + \frac{14E\left(\frac{x}{2} \middle| 2\right)}{15a \cos^{\frac{3}{2}}(x)\sqrt{a \sec^3(x)}}$$

Antiderivative was successfully verified.

[In] Int[(a\*Sec[x]^3)^(-3/2), x]

[Out] (14\*EllipticE[x/2, 2])/(15\*a\*Cos[x]^(3/2)\*Sqrt[a\*Sec[x]^3]) + (14\*Sin[x])/(45\*a\*Sqrt[a\*Sec[x]^3]) + (2\*Cos[x]^2\*Sin[x])/(9\*a\*Sqrt[a\*Sec[x]^3])

#### Rule 4123

Int[((b\_.)\*((c\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_))^(p\_), x\_Symbol] := Dist[(b ^IntPart[p]\*(b\*(c\*Sec[e + f\*x])^n)^FracPart[p])/(c\*Sec[e + f\*x]^(n\*FracPart[p])), Int[(c\*Sec[e + f\*x])^(n\*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] & & !IntegerQ[p]

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n], x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(a \sec^3(x))^{3/2}} dx &= \frac{\sec^{\frac{3}{2}}(x) \int \frac{1}{\sec^2(x)} dx}{a \sqrt{a} \sec^3(x)} \\
&= \frac{2 \cos^2(x) \sin(x)}{9a \sqrt{a} \sec^3(x)} + \frac{\left(7 \sec^{\frac{3}{2}}(x)\right) \int \frac{1}{\sec^2(x)} dx}{9a \sqrt{a} \sec^3(x)} \\
&= \frac{14 \sin(x)}{45a \sqrt{a} \sec^3(x)} + \frac{2 \cos^2(x) \sin(x)}{9a \sqrt{a} \sec^3(x)} + \frac{\left(7 \sec^{\frac{3}{2}}(x)\right) \int \frac{1}{\sqrt{\sec(x)}} dx}{15a \sqrt{a} \sec^3(x)} \\
&= \frac{14 \sin(x)}{45a \sqrt{a} \sec^3(x)} + \frac{2 \cos^2(x) \sin(x)}{9a \sqrt{a} \sec^3(x)} + \frac{7 \int \sqrt{\cos(x)} dx}{15a \cos^{\frac{3}{2}}(x) \sqrt{a} \sec^3(x)} \\
&= \frac{14E\left(\frac{x}{2} \middle| 2\right)}{15a \cos^{\frac{3}{2}}(x) \sqrt{a} \sec^3(x)} + \frac{14 \sin(x)}{45a \sqrt{a} \sec^3(x)} + \frac{2 \cos^2(x) \sin(x)}{9a \sqrt{a} \sec^3(x)}
\end{aligned}$$

**Mathematica [A]** time = 0.0940153, size = 43, normalized size = 0.59

$$\frac{33 \sin(x) + 5 \sin(3x) + \frac{84E\left(\frac{x}{2} \middle| 2\right)}{\cos^{\frac{3}{2}}(x)}}{90a \sqrt{a} \sec^3(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^3)^(-3/2), x]
```

```
[Out] ((84*EllipticE[x/2, 2])/Cos[x]^(3/2) + 33*Sin[x] + 5*Sin[3*x])/(90*a*Sqrt[a
*Sec[x]^3])
```



---

**Maple [C]** time = 0.156, size = 198, normalized size = 2.7

$$-\frac{2}{45 (\cos(x))^5 \sin(x)} \left( 5 (\cos(x))^6 - 21 i \sqrt{(\cos(x)+1)^{-1}} \sqrt{\frac{\cos(x)}{\cos(x)+1}} \operatorname{EllipticF}\left(\frac{i(-1+\cos(x))}{\sin(x)}, i\right) \cos(x) \sin(x) + 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a\*sec(x)^3)^(3/2),x)

[Out] -2/45\*(5\*cos(x)^6-21\*I\*(1/(cos(x)+1))^(1/2)\*(cos(x)/(cos(x)+1))^(1/2)\*EllipticF(I\*(-1+cos(x))/sin(x),I)\*cos(x)\*sin(x)+21\*I\*cos(x)\*sin(x)\*(1/(cos(x)+1))^(1/2)\*(cos(x)/(cos(x)+1))^(1/2)\*EllipticE(I\*(-1+cos(x))/sin(x),I)-21\*I\*EllipticF(I\*(-1+cos(x))/sin(x),I)\*(1/(cos(x)+1))^(1/2)\*(cos(x)/(cos(x)+1))^(1/2)\*sin(x)+21\*I\*sin(x)\*(1/(cos(x)+1))^(1/2)\*(cos(x)/(cos(x)+1))^(1/2)\*EllipticE(I\*(-1+cos(x))/sin(x),I)+2\*cos(x)^4+14\*cos(x)^2-21\*cos(x))/cos(x)^5/sin(x)/(a/cos(x)^3)^(3/2)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a \sec(x)^3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)^3)^(3/2),x, algorithm="maxima")

[Out] integrate((a\*sec(x)^3)^(-3/2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{a \sec(x)^3}}{a^2 \sec(x)^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(x)^3)/(a^2*sec(x)^6), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a \sec^3(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sec(x)**3)**(-3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a \sec(x)^3)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(x)^3)^(-3/2), x)
```

$$3.60 \quad \int \frac{1}{(a \sec^3(x))^{5/2}} dx$$

**Optimal.** Leaf size=117

$$\frac{26 \operatorname{EllipticF}\left(\frac{x}{2}, 2\right)}{77a^2 \cos^{\frac{3}{2}}(x) \sqrt{a \sec^3(x)}} + \frac{26 \tan(x)}{77a^2 \sqrt{a \sec^3(x)}} + \frac{2 \sin(x) \cos^5(x)}{15a^2 \sqrt{a \sec^3(x)}} + \frac{26 \sin(x) \cos^3(x)}{165a^2 \sqrt{a \sec^3(x)}} + \frac{78 \sin(x) \cos(x)}{385a^2 \sqrt{a \sec^3(x)}}$$

[Out] (26\*EllipticF[x/2, 2])/(77\*a^2\*Cos[x]^(3/2)\*Sqrt[a\*Sec[x]^3]) + (78\*Cos[x]\*Sin[x])/(385\*a^2\*Sqrt[a\*Sec[x]^3]) + (26\*Cos[x]^3\*Sin[x])/(165\*a^2\*Sqrt[a\*Sec[x]^3]) + (2\*Cos[x]^5\*Sin[x])/(15\*a^2\*Sqrt[a\*Sec[x]^3]) + (26\*Tan[x])/(77\*a^2\*Sqrt[a\*Sec[x]^3])

**Rubi [A]** time = 0.0561162, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {4123, 3769, 3771, 2641}

$$\frac{26 \tan(x)}{77a^2 \sqrt{a \sec^3(x)}} + \frac{2 \sin(x) \cos^5(x)}{15a^2 \sqrt{a \sec^3(x)}} + \frac{26 \sin(x) \cos^3(x)}{165a^2 \sqrt{a \sec^3(x)}} + \frac{26F\left(\frac{x}{2} \middle| 2\right)}{77a^2 \cos^{\frac{3}{2}}(x) \sqrt{a \sec^3(x)}} + \frac{78 \sin(x) \cos(x)}{385a^2 \sqrt{a \sec^3(x)}}$$

Antiderivative was successfully verified.

[In] Int[(a\*Sec[x]^3)^(-5/2), x]

[Out] (26\*EllipticF[x/2, 2])/(77\*a^2\*Cos[x]^(3/2)\*Sqrt[a\*Sec[x]^3]) + (78\*Cos[x]\*Sin[x])/(385\*a^2\*Sqrt[a\*Sec[x]^3]) + (26\*Cos[x]^3\*Sin[x])/(165\*a^2\*Sqrt[a\*Sec[x]^3]) + (2\*Cos[x]^5\*Sin[x])/(15\*a^2\*Sqrt[a\*Sec[x]^3]) + (26\*Tan[x])/(77\*a^2\*Sqrt[a\*Sec[x]^3])

#### Rule 4123

Int[((b\_.)\*((c\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_))^(p\_), x\_Symbol] := Dist[(b^IntPart[p]\*(b\*(c\*Sec[e + f\*x])^n)^FracPart[p])/(c\*Sec[e + f\*x])^(n\*FracPart[p]), Int[(c\*Sec[e + f\*x])^(n\*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p]

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c +

$d*x])^{(n + 2), x], x] /; \text{FreeQ}\{b, c, d\}, x] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$   
 $]$

### Rule 3771

$\text{Int}[(\text{csc}[c_.] + (d_.)*(x_)]*(b_.)^{(n_)}, x\_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^{n*} \text{Sin}[c + d*x]^{n}, \text{Int}[1/\text{Sin}[c + d*x]^{n}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \ \&\& \ \text{EqQ}[n^2, 1/4]$

### Rule 2641

$\text{Int}[1/\text{Sqrt}[\text{sin}[c_.] + (d_.)*(x_)]], x\_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

### Rubi steps

$$\begin{aligned} \int \frac{1}{(a \sec^3(x))^{5/2}} dx &= \frac{\sec^3(x) \int \frac{1}{\sec^2(x)} dx}{a^2 \sqrt{a} \sec^3(x)} \\ &= \frac{2 \cos^5(x) \sin(x)}{15a^2 \sqrt{a} \sec^3(x)} + \frac{\left(13 \sec^3(x)\right) \int \frac{1}{\sec^2(x)} dx}{15a^2 \sqrt{a} \sec^3(x)} \\ &= \frac{26 \cos^3(x) \sin(x)}{165a^2 \sqrt{a} \sec^3(x)} + \frac{2 \cos^5(x) \sin(x)}{15a^2 \sqrt{a} \sec^3(x)} + \frac{\left(39 \sec^3(x)\right) \int \frac{1}{\sec^2(x)} dx}{55a^2 \sqrt{a} \sec^3(x)} \\ &= \frac{78 \cos(x) \sin(x)}{385a^2 \sqrt{a} \sec^3(x)} + \frac{26 \cos^3(x) \sin(x)}{165a^2 \sqrt{a} \sec^3(x)} + \frac{2 \cos^5(x) \sin(x)}{15a^2 \sqrt{a} \sec^3(x)} + \frac{\left(39 \sec^3(x)\right) \int \frac{1}{\sec^2(x)} dx}{77a^2 \sqrt{a} \sec^3(x)} \\ &= \frac{78 \cos(x) \sin(x)}{385a^2 \sqrt{a} \sec^3(x)} + \frac{26 \cos^3(x) \sin(x)}{165a^2 \sqrt{a} \sec^3(x)} + \frac{2 \cos^5(x) \sin(x)}{15a^2 \sqrt{a} \sec^3(x)} + \frac{26 \tan(x)}{77a^2 \sqrt{a} \sec^3(x)} + \frac{\left(13 \sec^3(x)\right) \int \sqrt{\sec(x)} dx}{77a^2 \sqrt{a} \sec^3(x)} \\ &= \frac{78 \cos(x) \sin(x)}{385a^2 \sqrt{a} \sec^3(x)} + \frac{26 \cos^3(x) \sin(x)}{165a^2 \sqrt{a} \sec^3(x)} + \frac{2 \cos^5(x) \sin(x)}{15a^2 \sqrt{a} \sec^3(x)} + \frac{26 \tan(x)}{77a^2 \sqrt{a} \sec^3(x)} + \frac{13 \int \frac{1}{\sqrt{\cos(x)}} dx}{77a^2 \cos^3(x) \sqrt{a} \sec^3(x)} \\ &= \frac{26F\left(\frac{x}{2} \middle| 2\right)}{77a^2 \cos^3(x) \sqrt{a} \sec^3(x)} + \frac{78 \cos(x) \sin(x)}{385a^2 \sqrt{a} \sec^3(x)} + \frac{26 \cos^3(x) \sin(x)}{165a^2 \sqrt{a} \sec^3(x)} + \frac{2 \cos^5(x) \sin(x)}{15a^2 \sqrt{a} \sec^3(x)} + \frac{26 \tan(x)}{77a^2 \sqrt{a} \sec^3(x)} \end{aligned}$$

**Mathematica [A]** time = 0.096753, size = 59, normalized size = 0.5

$$\frac{\cos(x)\sqrt{a\sec^3(x)}\left(24960\sqrt{\cos(x)}\text{EllipticF}\left(\frac{x}{2}, 2\right) + 19122\sin(2x) + 4406\sin(4x) + 826\sin(6x) + 77\sin(8x)\right)}{73920a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*Sec[x]^3)^(-5/2), x]

[Out] (Cos[x]\*Sqrt[a\*Sec[x]^3]\*(24960\*Sqrt[Cos[x]]\*EllipticF[x/2, 2] + 19122\*Sin[2\*x] + 4406\*Sin[4\*x] + 826\*Sin[6\*x] + 77\*Sin[8\*x]))/(73920\*a^3)

**Maple [C]** time = 0.206, size = 114, normalized size = 1.

$$\frac{(-2 + 2 \cos(x)) (\cos(x) + 1)^2}{1155 (\cos(x))^8 (\sin(x))^3} \left( 77 (\cos(x))^8 - 77 (\cos(x))^7 + 91 (\cos(x))^6 - 91 (\cos(x))^5 - 195 i \text{EllipticF}\left(\frac{i(-1 + \cos(x))}{\sin(x)}, I\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a\*sec(x)^3)^(5/2), x)

[Out] 2/1155\*(-1+cos(x))\*(77\*cos(x)^8-77\*cos(x)^7+91\*cos(x)^6-91\*cos(x)^5-195\*I\*EllipticF(I\*(-1+cos(x))/sin(x), I)\*(1/(cos(x)+1))^(1/2)\*(cos(x)/(cos(x)+1))^(1/2)\*sin(x)+117\*cos(x)^4-117\*cos(x)^3+195\*cos(x)^2-195\*cos(x))\*(cos(x)+1)^2/cos(x)^8/sin(x)^3/(a/cos(x)^3)^(5/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a \sec(x)^3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)^3)^(5/2), x, algorithm="maxima")

[Out] integrate((a\*sec(x)^3)^(-5/2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\sqrt{a \sec(x)^3}}{a^3 \sec(x)^9}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)^3)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(a\*sec(x)^3)/(a^3\*sec(x)^9), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a \sec^3(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)\*\*3)\*\*(5/2),x)

[Out] Integral((a\*sec(x)\*\*3)\*\*(-5/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a \sec(x)^3)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)^3)^(5/2),x, algorithm="giac")

[Out] integrate((a\*sec(x)^3)^(-5/2), x)

### 3.61 $\int \left(a \sec^4(x)\right)^{7/2} dx$

**Optimal.** Leaf size=163

$$a^3 \sin(x) \cos(x) \sqrt{a \sec^4(x)} + \frac{1}{13} a^3 \sin^2(x) \tan^{11}(x) \sqrt{a \sec^4(x)} + \frac{6}{11} a^3 \sin^2(x) \tan^9(x) \sqrt{a \sec^4(x)} + \frac{5}{3} a^3 \sin^2(x) \tan^7(x) \sqrt{a \sec^4(x)}$$

```
[Out] a^3*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x] + 2*a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]
+ 3*a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^3 + (20*a^3*Sqrt[a*Sec[x]^4]*Sin[
x]^2*Tan[x]^5)/7 + (5*a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^7)/3 + (6*a^3*Sq
rt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^9)/11 + (a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x
]^11)/13
```

**Rubi [A]** time = 0.0390895, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4123, 3767}

$$a^3 \sin(x) \cos(x) \sqrt{a \sec^4(x)} + \frac{1}{13} a^3 \sin^2(x) \tan^{11}(x) \sqrt{a \sec^4(x)} + \frac{6}{11} a^3 \sin^2(x) \tan^9(x) \sqrt{a \sec^4(x)} + \frac{5}{3} a^3 \sin^2(x) \tan^7(x) \sqrt{a \sec^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^4)^(7/2), x]
```

```
[Out] a^3*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x] + 2*a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]
+ 3*a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^3 + (20*a^3*Sqrt[a*Sec[x]^4]*Sin[
x]^2*Tan[x]^5)/7 + (5*a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^7)/3 + (6*a^3*Sq
rt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^9)/11 + (a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x
]^11)/13
```

#### Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

#### Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

d}, x] && IGtQ[n/2, 0]

### Rubi steps

$$\begin{aligned} \int (a \sec^4(x))^{7/2} dx &= \left( a^3 \cos^2(x) \sqrt{a \sec^4(x)} \right) \int \sec^{14}(x) dx \\ &= - \left( \left( a^3 \cos^2(x) \sqrt{a \sec^4(x)} \right) \text{Subst} \left( \int (1 + 6x^2 + 15x^4 + 20x^6 + 15x^8 + 6x^{10} + x^{12}) dx, x, -\tan(x) \right) \right) \\ &= a^3 \cos(x) \sqrt{a \sec^4(x)} \sin(x) + 2a^3 \sqrt{a \sec^4(x)} \sin^2(x) \tan(x) + 3a^3 \sqrt{a \sec^4(x)} \sin^2(x) \tan^3(x) + \frac{20}{7} a^3 \end{aligned}$$

**Mathematica [A]** time = 0.168441, size = 54, normalized size = 0.33

$$\frac{\sin(x) \cos(x) (2380 \cos(2x) + 1093 \cos(4x) + 378 \cos(6x) + 92 \cos(8x) + 14 \cos(10x) + \cos(12x) + 2048) (a \sec^4(x))^{7/2}}{6006}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*Sec[x]^4)^(7/2), x]

[Out] (Cos[x]\*(2048 + 2380\*Cos[2\*x] + 1093\*Cos[4\*x] + 378\*Cos[6\*x] + 92\*Cos[8\*x] + 14\*Cos[10\*x] + Cos[12\*x])\*(a\*Sec[x]^4)^(7/2)\*Sin[x])/6006

**Maple [A]** time = 0.233, size = 53, normalized size = 0.3

$$\frac{(1024 (\cos(x))^{12} + 512 (\cos(x))^{10} + 384 (\cos(x))^8 + 320 (\cos(x))^6 + 280 (\cos(x))^4 + 252 (\cos(x))^2 + 231) \sin(x) \cos(x)}{3003}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*sec(x)^4)^(7/2), x)

[Out] 1/3003\*(1024\*cos(x)^12+512\*cos(x)^10+384\*cos(x)^8+320\*cos(x)^6+280\*cos(x)^4+252\*cos(x)^2+231)\*(a/cos(x)^4)^(7/2)\*sin(x)\*cos(x)



**Maxima [A]** time = 1.64345, size = 82, normalized size = 0.5

$$\frac{1}{13} a^{\frac{7}{2}} \tan(x)^{13} + \frac{6}{11} a^{\frac{7}{2}} \tan(x)^{11} + \frac{5}{3} a^{\frac{7}{2}} \tan(x)^9 + \frac{20}{7} a^{\frac{7}{2}} \tan(x)^7 + 3 a^{\frac{7}{2}} \tan(x)^5 + 2 a^{\frac{7}{2}} \tan(x)^3 + a^{\frac{7}{2}} \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^4)^(7/2),x, algorithm="maxima")

[Out] 1/13\*a^(7/2)\*tan(x)^13 + 6/11\*a^(7/2)\*tan(x)^11 + 5/3\*a^(7/2)\*tan(x)^9 + 20/7\*a^(7/2)\*tan(x)^7 + 3\*a^(7/2)\*tan(x)^5 + 2\*a^(7/2)\*tan(x)^3 + a^(7/2)\*tan(x)

**Fricas [A]** time = 1.58708, size = 228, normalized size = 1.4

$$\frac{(1024 a^3 \cos(x)^{12} + 512 a^3 \cos(x)^{10} + 384 a^3 \cos(x)^8 + 320 a^3 \cos(x)^6 + 280 a^3 \cos(x)^4 + 252 a^3 \cos(x)^2 + 231 a^3) \sqrt{\frac{a}{\cos(x)}}}{3003 \cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^4)^(7/2),x, algorithm="fricas")

[Out] 1/3003\*(1024\*a^3\*cos(x)^12 + 512\*a^3\*cos(x)^10 + 384\*a^3\*cos(x)^8 + 320\*a^3\*cos(x)^6 + 280\*a^3\*cos(x)^4 + 252\*a^3\*cos(x)^2 + 231\*a^3)\*sqrt(a/cos(x)^4)\*sin(x)/cos(x)^11

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)\*\*4)\*\*(7/2),x)

[Out] Timed out

**Giac [A]** time = 1.32692, size = 90, normalized size = 0.55

$$\frac{1}{3003} \left( 231 a^3 \tan(x)^{13} + 1638 a^3 \tan(x)^{11} + 5005 a^3 \tan(x)^9 + 8580 a^3 \tan(x)^7 + 9009 a^3 \tan(x)^5 + 6006 a^3 \tan(x)^3 + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(7/2),x, algorithm="giac")
```

```
[Out] 1/3003*(231*a^3*tan(x)^13 + 1638*a^3*tan(x)^11 + 5005*a^3*tan(x)^9 + 8580*a^3*tan(x)^7 + 9009*a^3*tan(x)^5 + 6006*a^3*tan(x)^3 + 3003*a^3*tan(x))*sqrt(a)
```

### 3.62 $\int \left(a \sec^4(x)\right)^{5/2} dx$

**Optimal.** Leaf size=117

$$a^2 \sin(x) \cos(x) \sqrt{a \sec^4(x)} + \frac{1}{9} a^2 \sin^2(x) \tan^7(x) \sqrt{a \sec^4(x)} + \frac{4}{7} a^2 \sin^2(x) \tan^5(x) \sqrt{a \sec^4(x)} + \frac{6}{5} a^2 \sin^2(x) \tan^3(x) \sqrt{a \sec^4(x)}$$

```
[Out] a^2*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x] + (4*a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x])
/3 + (6*a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^3)/5 + (4*a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^5)/7 + (a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^7)/9
```

**Rubi [A]** time = 0.0303353, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4123, 3767}

$$a^2 \sin(x) \cos(x) \sqrt{a \sec^4(x)} + \frac{1}{9} a^2 \sin^2(x) \tan^7(x) \sqrt{a \sec^4(x)} + \frac{4}{7} a^2 \sin^2(x) \tan^5(x) \sqrt{a \sec^4(x)} + \frac{6}{5} a^2 \sin^2(x) \tan^3(x) \sqrt{a \sec^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^4)^(5/2), x]
```

```
[Out] a^2*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x] + (4*a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x])
/3 + (6*a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^3)/5 + (4*a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^5)/7 + (a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^7)/9
```

#### Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

#### Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

#### Rubi steps

$$\begin{aligned} \int (a \sec^4(x))^{5/2} dx &= \left( a^2 \cos^2(x) \sqrt{a \sec^4(x)} \right) \int \sec^{10}(x) dx \\ &= - \left( \left( a^2 \cos^2(x) \sqrt{a \sec^4(x)} \right) \text{Subst} \left( \int (1 + 4x^2 + 6x^4 + 4x^6 + x^8) dx, x, -\tan(x) \right) \right) \\ &= a^2 \cos(x) \sqrt{a \sec^4(x)} \sin(x) + \frac{4}{3} a^2 \sqrt{a \sec^4(x)} \sin^2(x) \tan(x) + \frac{6}{5} a^2 \sqrt{a \sec^4(x)} \sin^2(x) \tan^3(x) + \frac{4}{7} a^2 \end{aligned}$$

**Mathematica [A]** time = 0.0911082, size = 42, normalized size = 0.36

$$\frac{1}{315} \sin(x) \cos(x) (130 \cos(2x) + 46 \cos(4x) + 10 \cos(6x) + \cos(8x) + 128) (a \sec^4(x))^{5/2}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*Sec[x]^4)^(5/2), x]

[Out] (Cos[x]\*(128 + 130\*Cos[2\*x] + 46\*Cos[4\*x] + 10\*Cos[6\*x] + Cos[8\*x])\*(a\*Sec[x]^4)^(5/2)\*Sin[x])/315

**Maple [A]** time = 0.113, size = 41, normalized size = 0.4

$$\frac{(128 (\cos(x))^8 + 64 (\cos(x))^6 + 48 (\cos(x))^4 + 40 (\cos(x))^2 + 35) \cos(x) \sin(x)}{315} \left( \frac{a}{(\cos(x))^4} \right)^{5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*sec(x)^4)^(5/2), x)

[Out] 1/315\*(128\*cos(x)^8+64\*cos(x)^6+48\*cos(x)^4+40\*cos(x)^2+35)\*cos(x)\*sin(x)\*(a/cos(x)^4)^(5/2)

**Maxima [A]** time = 1.79159, size = 58, normalized size = 0.5

$$\frac{1}{9} a^{\frac{5}{2}} \tan(x)^9 + \frac{4}{7} a^{\frac{5}{2}} \tan(x)^7 + \frac{6}{5} a^{\frac{5}{2}} \tan(x)^5 + \frac{4}{3} a^{\frac{5}{2}} \tan(x)^3 + a^{\frac{5}{2}} \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^4)^(5/2),x, algorithm="maxima")

[Out]  $1/9*a^{(5/2)}*\tan(x)^9 + 4/7*a^{(5/2)}*\tan(x)^7 + 6/5*a^{(5/2)}*\tan(x)^5 + 4/3*a^{(5/2)}*\tan(x)^3 + a^{(5/2)}*\tan(x)$

**Fricas [A]** time = 1.4071, size = 165, normalized size = 1.41

$$\frac{(128 a^2 \cos(x)^8 + 64 a^2 \cos(x)^6 + 48 a^2 \cos(x)^4 + 40 a^2 \cos(x)^2 + 35 a^2) \sqrt{\frac{a}{\cos(x)^4}} \sin(x)}{315 \cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^4)^(5/2),x, algorithm="fricas")

[Out]  $1/315*(128*a^2*\cos(x)^8 + 64*a^2*\cos(x)^6 + 48*a^2*\cos(x)^4 + 40*a^2*\cos(x)^2 + 35*a^2)*\sqrt{a/\cos(x)^4}*\sin(x)/\cos(x)^7$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)\*\*4)\*\*(5/2),x)

[Out] Timed out

**Giac [A]** time = 1.24393, size = 66, normalized size = 0.56

$$\frac{1}{315} (35 a^2 \tan(x)^9 + 180 a^2 \tan(x)^7 + 378 a^2 \tan(x)^5 + 420 a^2 \tan(x)^3 + 315 a^2 \tan(x)) \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^4)^(5/2),x, algorithm="giac")

[Out]  $\frac{1}{315}(35a^2\tan(x)^9 + 180a^2\tan(x)^7 + 378a^2\tan(x)^5 + 420a^2\tan(x)^3 + 315a^2\tan(x))\sqrt{a}$

### 3.63 $\int \left(a \sec^4(x)\right)^{3/2} dx$

**Optimal.** Leaf size=61

$$a \sin(x) \cos(x) \sqrt{a \sec^4(x)} + \frac{1}{5} a \sin^2(x) \tan^3(x) \sqrt{a \sec^4(x)} + \frac{2}{3} a \sin^2(x) \tan(x) \sqrt{a \sec^4(x)}$$

[Out] a\*cos[x]\*Sqrt[a\*Sec[x]^4]\*Sin[x] + (2\*a\*Sqrt[a\*Sec[x]^4]\*Sin[x]^2\*Tan[x])/3 + (a\*Sqrt[a\*Sec[x]^4]\*Sin[x]^2\*Tan[x]^3)/5

**Rubi [A]** time = 0.0220653, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4123, 3767}

$$a \sin(x) \cos(x) \sqrt{a \sec^4(x)} + \frac{1}{5} a \sin^2(x) \tan^3(x) \sqrt{a \sec^4(x)} + \frac{2}{3} a \sin^2(x) \tan(x) \sqrt{a \sec^4(x)}$$

Antiderivative was successfully verified.

[In] Int[(a\*Sec[x]^4)^(3/2), x]

[Out] a\*cos[x]\*Sqrt[a\*Sec[x]^4]\*Sin[x] + (2\*a\*Sqrt[a\*Sec[x]^4]\*Sin[x]^2\*Tan[x])/3 + (a\*Sqrt[a\*Sec[x]^4]\*Sin[x]^2\*Tan[x]^3)/5

#### Rule 4123

Int[((b\_.)\*((c\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_))^(p\_), x\_Symbol] :> Dist[(b ^IntPart[p]\*(b\*(c\*Sec[e + f\*x])^n)^FracPart[p])/(c\*Sec[e + f\*x])^(n\*FracPart[p]), Int[(c\*Sec[e + f\*x])^(n\*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] & & !IntegerQ[p]

#### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)^(n\_)], x\_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

#### Rubi steps

$$\begin{aligned}
\int (a \sec^4(x))^{3/2} dx &= \left( a \cos^2(x) \sqrt{a \sec^4(x)} \right) \int \sec^6(x) dx \\
&= - \left( \left( a \cos^2(x) \sqrt{a \sec^4(x)} \right) \text{Subst} \left( \int (1 + 2x^2 + x^4) dx, x, -\tan(x) \right) \right) \\
&= a \cos(x) \sqrt{a \sec^4(x)} \sin(x) + \frac{2}{3} a \sqrt{a \sec^4(x)} \sin^2(x) \tan(x) + \frac{1}{5} a \sqrt{a \sec^4(x)} \sin^2(x) \tan^3(x)
\end{aligned}$$

**Mathematica [A]** time = 0.0571263, size = 30, normalized size = 0.49

$$\frac{1}{15} \sin(x) \cos(x) (6 \cos(2x) + \cos(4x) + 8) (a \sec^4(x))^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*Sec[x]^4)^(3/2), x]

[Out] (Cos[x]\*(8 + 6\*Cos[2\*x] + Cos[4\*x])\*(a\*Sec[x]^4)^(3/2)\*Sin[x])/15

**Maple [A]** time = 0.067, size = 29, normalized size = 0.5

$$\frac{(8 (\cos(x))^4 + 4 (\cos(x))^2 + 3) \cos(x) \sin(x)}{15} \left( \frac{a}{(\cos(x))^4} \right)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*sec(x)^4)^(3/2), x)

[Out] 1/15\*(8\*cos(x)^4+4\*cos(x)^2+3)\*cos(x)\*sin(x)\*(a/cos(x)^4)^(3/2)

**Maxima [A]** time = 1.69223, size = 34, normalized size = 0.56

$$\frac{1}{5} a^{3/2} \tan(x)^5 + \frac{2}{3} a^{3/2} \tan(x)^3 + a^{3/2} \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^4)^(3/2), x, algorithm="maxima")



[Out]  $1/5*a^{(3/2)}*\tan(x)^5 + 2/3*a^{(3/2)}*\tan(x)^3 + a^{(3/2)}*\tan(x)$

---

**Fricas [A]** time = 1.4485, size = 101, normalized size = 1.66

$$\frac{(8 a \cos(x)^4 + 4 a \cos(x)^2 + 3 a) \sqrt{\frac{a}{\cos(x)^4}} \sin(x)}{15 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*sec(x)^4)^(3/2),x, algorithm="fricas")`

[Out]  $1/15*(8*a*\cos(x)^4 + 4*a*\cos(x)^2 + 3*a)*\sqrt{a/\cos(x)^4}*\sin(x)/\cos(x)^3$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a \sec^4(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*sec(x)**4)**(3/2),x)`

[Out] `Integral((a*sec(x)**4)**(3/2), x)`

---

**Giac [A]** time = 1.30526, size = 30, normalized size = 0.49

$$\frac{1}{15} (3 \tan(x)^5 + 10 \tan(x)^3 + 15 \tan(x)) a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*sec(x)^4)^(3/2),x, algorithm="giac")`

[Out]  $1/15*(3*\tan(x)^5 + 10*\tan(x)^3 + 15*\tan(x))*a^{(3/2)}$

### 3.64 $\int \sqrt{a \sec^4(x)} dx$

**Optimal.** Leaf size=15

$$\sin(x) \cos(x) \sqrt{a \sec^4(x)}$$

[Out] Cos[x]\*Sqrt[a\*Sec[x]^4]\*Sin[x]

**Rubi [A]** time = 0.0158969, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {4123, 3767, 8}

$$\sin(x) \cos(x) \sqrt{a \sec^4(x)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a\*Sec[x]^4], x]

[Out] Cos[x]\*Sqrt[a\*Sec[x]^4]\*Sin[x]

#### Rule 4123

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := Dist[(b
^IntPart[p]*(b*(c*Sec[e + f*x])^n)^FracPart[p])/(c*Sec[e + f*x])^(n*FracPar
t[p]), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

#### Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

#### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

#### Rubi steps

$$\begin{aligned}
 \int \sqrt{a \sec^4(x)} dx &= \left( \cos^2(x) \sqrt{a \sec^4(x)} \right) \int \sec^2(x) dx \\
 &= - \left( \left( \cos^2(x) \sqrt{a \sec^4(x)} \right) \text{Subst} \left( \int 1 dx, x, -\tan(x) \right) \right) \\
 &= \cos(x) \sqrt{a \sec^4(x)} \sin(x)
 \end{aligned}$$

**Mathematica [A]** time = 0.0054477, size = 15, normalized size = 1.

$$\sin(x) \cos(x) \sqrt{a \sec^4(x)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a\*Sec[x]^4],x]

[Out] Cos[x]\*Sqrt[a\*Sec[x]^4]\*Sin[x]

**Maple [A]** time = 0.063, size = 14, normalized size = 0.9

$$\cos(x) \sin(x) \sqrt{\frac{a}{(\cos(x))^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*sec(x)^4)^(1/2),x)

[Out] cos(x)\*sin(x)\*(a/cos(x)^4)^(1/2)

**Maxima [A]** time = 1.73949, size = 8, normalized size = 0.53

$$\sqrt{a} \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^4)^(1/2),x, algorithm="maxima")

[Out] sqrt(a)\*tan(x)

---

**Fricas [A]** time = 1.36385, size = 43, normalized size = 2.87

$$\sqrt{\frac{a}{\cos(x)^4}} \cos(x) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^4)^(1/2),x, algorithm="fricas")

[Out] sqrt(a/cos(x)^4)\*cos(x)\*sin(x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sec^4(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)\*\*4)\*\*(1/2),x)

[Out] Integral(sqrt(a\*sec(x)\*\*4), x)

---

**Giac [A]** time = 1.2576, size = 8, normalized size = 0.53

$$\sqrt{a} \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*sec(x)^4)^(1/2),x, algorithm="giac")

[Out] sqrt(a)\*tan(x)

$$3.65 \quad \int \frac{1}{\sqrt{a \sec^4(x)}} dx$$

Optimal. Leaf size=36

$$\frac{x \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{\tan(x)}{2\sqrt{a \sec^4(x)}}$$

[Out] (x\*Sec[x]^2)/(2\*Sqrt[a\*Sec[x]^4]) + Tan[x]/(2\*Sqrt[a\*Sec[x]^4])

**Rubi [A]** time = 0.0146596, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {4123, 2635, 8}

$$\frac{x \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{\tan(x)}{2\sqrt{a \sec^4(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a\*Sec[x]^4],x]

[Out] (x\*Sec[x]^2)/(2\*Sqrt[a\*Sec[x]^4]) + Tan[x]/(2\*Sqrt[a\*Sec[x]^4])

#### Rule 4123

Int[((b\_.)\*((c\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_))^(p\_), x\_Symbol] :> Dist[(b ^IntPart[p]\*(b\*(c\*Sec[e + f\*x])^n)^FracPart[p])/(c\*Sec[e + f\*x])^(n\*FracPart[p]), Int[(c\*Sec[e + f\*x])^(n\*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] & & !IntegerQ[p]

#### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x] \*(b\*Sin[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a \sec^4(x)}} dx &= \frac{\sec^2(x) \int \cos^2(x) dx}{\sqrt{a \sec^4(x)}} \\ &= \frac{\tan(x)}{2\sqrt{a \sec^4(x)}} + \frac{\sec^2(x) \int 1 dx}{2\sqrt{a \sec^4(x)}} \\ &= \frac{x \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{\tan(x)}{2\sqrt{a \sec^4(x)}} \end{aligned}$$

**Mathematica [A]** time = 0.0234295, size = 23, normalized size = 0.64

$$\frac{\tan(x) + x \sec^2(x)}{2\sqrt{a \sec^4(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a\*Sec[x]^4],x]

[Out] (x\*Sec[x]^2 + Tan[x])/(2\*Sqrt[a\*Sec[x]^4])

**Maple [A]** time = 0.074, size = 22, normalized size = 0.6

$$\frac{\cos(x) \sin(x) + x}{2 (\cos(x))^2} \frac{1}{\sqrt{\frac{a}{(\cos(x))^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a\*sec(x)^4)^(1/2),x)

[Out] 1/2\*(cos(x)\*sin(x)+x)/cos(x)^2/(a/cos(x)^4)^(1/2)

**Maxima [A]** time = 1.66388, size = 34, normalized size = 0.94

$$\frac{x}{2\sqrt{a}} + \frac{\tan(x)}{2(\sqrt{a} \tan(x)^2 + \sqrt{a})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*sec(x)^4)^(1/2),x, algorithm="maxima")`

[Out] `1/2*x/sqrt(a) + 1/2*tan(x)/(sqrt(a)*tan(x)^2 + sqrt(a))`

**Fricas [A]** time = 1.4198, size = 74, normalized size = 2.06

$$\frac{(\cos(x)^3 \sin(x) + x \cos(x)^2) \sqrt{\frac{a}{\cos(x)^4}}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*sec(x)^4)^(1/2),x, algorithm="fricas")`

[Out] `1/2*(cos(x)^3*sin(x) + x*cos(x)^2)*sqrt(a/cos(x)^4)/a`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a \sec^4(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*sec(x)**4)**(1/2),x)`

[Out] `Integral(1/sqrt(a*sec(x)**4), x)`

**Giac [A]** time = 1.31144, size = 53, normalized size = 1.47

$$-\frac{1}{2} \sqrt{a} \left( \frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor - x}{a} - \frac{\tan(x)}{(\tan(x)^2 + 1)a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*((pi*floor(x/pi + 1/2) - x)/a - tan(x)/((tan(x)^2 + 1)*a))
```



$$3.66 \quad \int \frac{1}{(a \sec^4(x))^{3/2}} dx$$

**Optimal.** Leaf size=86

$$\frac{5x \sec^2(x)}{16a\sqrt{a \sec^4(x)}} + \frac{5 \tan(x)}{16a\sqrt{a \sec^4(x)}} + \frac{\sin(x) \cos^3(x)}{6a\sqrt{a \sec^4(x)}} + \frac{5 \sin(x) \cos(x)}{24a\sqrt{a \sec^4(x)}}$$

[Out] (5\*x\*Sec[x]^2)/(16\*a\*Sqrt[a\*Sec[x]^4]) + (5\*Cos[x]\*Sin[x])/(24\*a\*Sqrt[a\*Sec[x]^4]) + (Cos[x]^3\*Sin[x])/(6\*a\*Sqrt[a\*Sec[x]^4]) + (5\*Tan[x])/(16\*a\*Sqrt[a\*Sec[x]^4])

**Rubi [A]** time = 0.0319133, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {4123, 2635, 8}

$$\frac{5x \sec^2(x)}{16a\sqrt{a \sec^4(x)}} + \frac{5 \tan(x)}{16a\sqrt{a \sec^4(x)}} + \frac{\sin(x) \cos^3(x)}{6a\sqrt{a \sec^4(x)}} + \frac{5 \sin(x) \cos(x)}{24a\sqrt{a \sec^4(x)}}$$

Antiderivative was successfully verified.

[In] Int[(a\*Sec[x]^4)^(-3/2), x]

[Out] (5\*x\*Sec[x]^2)/(16\*a\*Sqrt[a\*Sec[x]^4]) + (5\*Cos[x]\*Sin[x])/(24\*a\*Sqrt[a\*Sec[x]^4]) + (Cos[x]^3\*Sin[x])/(6\*a\*Sqrt[a\*Sec[x]^4]) + (5\*Tan[x])/(16\*a\*Sqrt[a\*Sec[x]^4])

### Rule 4123

Int[((b\_.)\*((c\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_))^(p\_), x\_Symbol] := Dist[(b^IntPart[p]\*(b\*(c\*Sec[e + f\*x])^n)^FracPart[p])/(c\*Sec[e + f\*x]^(n\*FracPart[p])), Int[(c\*Sec[e + f\*x])^(n\*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

**Rule 8**

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

**Rubi steps**

$$\begin{aligned}
 \int \frac{1}{(a \sec^4(x))^{3/2}} dx &= \frac{\sec^2(x) \int \cos^6(x) dx}{a \sqrt{a} \sec^4(x)} \\
 &= \frac{\cos^3(x) \sin(x)}{6a \sqrt{a} \sec^4(x)} + \frac{(5 \sec^2(x)) \int \cos^4(x) dx}{6a \sqrt{a} \sec^4(x)} \\
 &= \frac{5 \cos(x) \sin(x)}{24a \sqrt{a} \sec^4(x)} + \frac{\cos^3(x) \sin(x)}{6a \sqrt{a} \sec^4(x)} + \frac{(5 \sec^2(x)) \int \cos^2(x) dx}{8a \sqrt{a} \sec^4(x)} \\
 &= \frac{5 \cos(x) \sin(x)}{24a \sqrt{a} \sec^4(x)} + \frac{\cos^3(x) \sin(x)}{6a \sqrt{a} \sec^4(x)} + \frac{5 \tan(x)}{16a \sqrt{a} \sec^4(x)} + \frac{(5 \sec^2(x)) \int 1 dx}{16a \sqrt{a} \sec^4(x)} \\
 &= \frac{5x \sec^2(x)}{16a \sqrt{a} \sec^4(x)} + \frac{5 \cos(x) \sin(x)}{24a \sqrt{a} \sec^4(x)} + \frac{\cos^3(x) \sin(x)}{6a \sqrt{a} \sec^4(x)} + \frac{5 \tan(x)}{16a \sqrt{a} \sec^4(x)}
 \end{aligned}$$

**Mathematica [A]** time = 0.0388284, size = 38, normalized size = 0.44

$$\frac{(60x + 45 \sin(2x) + 9 \sin(4x) + \sin(6x)) \sec^6(x)}{192 (a \sec^4(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*Sec[x]^4)^(-3/2), x]

[Out] (Sec[x]^6\*(60\*x + 45\*Sin[2\*x] + 9\*Sin[4\*x] + Sin[6\*x]))/(192\*(a\*Sec[x]^4)^(-3/2))

**Maple [A]** time = 0.104, size = 41, normalized size = 0.5

$$\frac{8 (\cos(x))^5 \sin(x) + 10 (\cos(x))^3 \sin(x) + 15 \cos(x) \sin(x) + 15x \left( \frac{a}{(\cos(x))^4} \right)^{-\frac{3}{2}}}{48 (\cos(x))^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a*sec(x)^4)^(3/2),x)`

[Out]  $\frac{1}{48} \frac{(8 \cos(x)^5 \sin(x) + 10 \cos(x)^3 \sin(x) + 15 \cos(x) \sin(x) + 15x)}{\cos(x)^6} \frac{1}{(a/\cos(x)^4)^{3/2}}$

**Maxima [A]** time = 1.64573, size = 78, normalized size = 0.91

$$\frac{15 \tan(x)^5 + 40 \tan(x)^3 + 33 \tan(x)}{48 \left( a^{\frac{3}{2}} \tan(x)^6 + 3 a^{\frac{3}{2}} \tan(x)^4 + 3 a^{\frac{3}{2}} \tan(x)^2 + a^{\frac{3}{2}} \right)} + \frac{5x}{16 a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*sec(x)^4)^(3/2),x, algorithm="maxima")`

[Out]  $\frac{1}{48} \frac{(15 \tan(x)^5 + 40 \tan(x)^3 + 33 \tan(x))}{(a^{3/2} \tan(x)^6 + 3 a^{3/2} \tan(x)^4 + 3 a^{3/2} \tan(x)^2 + a^{3/2})} + \frac{5}{16} \frac{x}{a^{3/2}}$

**Fricas [A]** time = 1.46067, size = 126, normalized size = 1.47

$$\frac{(15x \cos(x)^2 + (8 \cos(x)^7 + 10 \cos(x)^5 + 15 \cos(x)^3) \sin(x)) \sqrt{\frac{a}{\cos(x)^4}}}{48 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*sec(x)^4)^(3/2),x, algorithm="fricas")`

[Out]  $\frac{1}{48} \frac{(15x \cos(x)^2 + (8 \cos(x)^7 + 10 \cos(x)^5 + 15 \cos(x)^3) \sin(x)) \sqrt{a/\cos(x)^4}}{a^2}$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a \sec^4(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sec(x)**4)**(-3/2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.67 \quad \int \frac{1}{(a \sec^4(x))^{5/2}} dx$$

**Optimal.** Leaf size=132

$$\frac{63x \sec^2(x)}{256a^2 \sqrt{a \sec^4(x)}} + \frac{63 \tan(x)}{256a^2 \sqrt{a \sec^4(x)}} + \frac{\sin(x) \cos^7(x)}{10a^2 \sqrt{a \sec^4(x)}} + \frac{9 \sin(x) \cos^5(x)}{80a^2 \sqrt{a \sec^4(x)}} + \frac{21 \sin(x) \cos^3(x)}{160a^2 \sqrt{a \sec^4(x)}} + \frac{21 \sin(x) \cos(x)}{128a^2 \sqrt{a \sec^4(x)}}$$

[Out] (63\*x\*Sec[x]^2)/(256\*a^2\*Sqrt[a\*Sec[x]^4]) + (21\*Cos[x]\*Sin[x])/(128\*a^2\*Sqrt[a\*Sec[x]^4]) + (21\*Cos[x]^3\*Sin[x])/(160\*a^2\*Sqrt[a\*Sec[x]^4]) + (9\*Cos[x]^5\*Sin[x])/(80\*a^2\*Sqrt[a\*Sec[x]^4]) + (Cos[x]^7\*Sin[x])/(10\*a^2\*Sqrt[a\*Sec[x]^4]) + (63\*Tan[x])/(256\*a^2\*Sqrt[a\*Sec[x]^4])

**Rubi [A]** time = 0.0508577, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {4123, 2635, 8}

$$\frac{63x \sec^2(x)}{256a^2 \sqrt{a \sec^4(x)}} + \frac{63 \tan(x)}{256a^2 \sqrt{a \sec^4(x)}} + \frac{\sin(x) \cos^7(x)}{10a^2 \sqrt{a \sec^4(x)}} + \frac{9 \sin(x) \cos^5(x)}{80a^2 \sqrt{a \sec^4(x)}} + \frac{21 \sin(x) \cos^3(x)}{160a^2 \sqrt{a \sec^4(x)}} + \frac{21 \sin(x) \cos(x)}{128a^2 \sqrt{a \sec^4(x)}}$$

Antiderivative was successfully verified.

[In] Int[(a\*Sec[x]^4)^(-5/2), x]

[Out] (63\*x\*Sec[x]^2)/(256\*a^2\*Sqrt[a\*Sec[x]^4]) + (21\*Cos[x]\*Sin[x])/(128\*a^2\*Sqrt[a\*Sec[x]^4]) + (21\*Cos[x]^3\*Sin[x])/(160\*a^2\*Sqrt[a\*Sec[x]^4]) + (9\*Cos[x]^5\*Sin[x])/(80\*a^2\*Sqrt[a\*Sec[x]^4]) + (Cos[x]^7\*Sin[x])/(10\*a^2\*Sqrt[a\*Sec[x]^4]) + (63\*Tan[x])/(256\*a^2\*Sqrt[a\*Sec[x]^4])

#### Rule 4123

Int[((b\_.)\*((c\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_))^(p\_), x\_Symbol] := Dist[(b^IntPart[p]\*(b\*(c\*Sec[e + f\*x])^n)^FracPart[p])/(c\*Sec[e + f\*x])^(n\*FracPart[p]), Int[(c\*Sec[e + f\*x])^(n\*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p]

#### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x])\*(b\*Sin[c + d\*x])^(n - 1)/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

]

**Rule 8**

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a \sec^4(x))^{5/2}} dx &= \frac{\sec^2(x) \int \cos^{10}(x) dx}{a^2 \sqrt{a \sec^4(x)}} \\
&= \frac{\cos^7(x) \sin(x)}{10a^2 \sqrt{a \sec^4(x)}} + \frac{(9 \sec^2(x)) \int \cos^8(x) dx}{10a^2 \sqrt{a \sec^4(x)}} \\
&= \frac{9 \cos^5(x) \sin(x)}{80a^2 \sqrt{a \sec^4(x)}} + \frac{\cos^7(x) \sin(x)}{10a^2 \sqrt{a \sec^4(x)}} + \frac{(63 \sec^2(x)) \int \cos^6(x) dx}{80a^2 \sqrt{a \sec^4(x)}} \\
&= \frac{21 \cos^3(x) \sin(x)}{160a^2 \sqrt{a \sec^4(x)}} + \frac{9 \cos^5(x) \sin(x)}{80a^2 \sqrt{a \sec^4(x)}} + \frac{\cos^7(x) \sin(x)}{10a^2 \sqrt{a \sec^4(x)}} + \frac{(21 \sec^2(x)) \int \cos^4(x) dx}{32a^2 \sqrt{a \sec^4(x)}} \\
&= \frac{21 \cos(x) \sin(x)}{128a^2 \sqrt{a \sec^4(x)}} + \frac{21 \cos^3(x) \sin(x)}{160a^2 \sqrt{a \sec^4(x)}} + \frac{9 \cos^5(x) \sin(x)}{80a^2 \sqrt{a \sec^4(x)}} + \frac{\cos^7(x) \sin(x)}{10a^2 \sqrt{a \sec^4(x)}} + \frac{(63 \sec^2(x)) \int \cos^2(x) dx}{128a^2 \sqrt{a \sec^4(x)}} \\
&= \frac{21 \cos(x) \sin(x)}{128a^2 \sqrt{a \sec^4(x)}} + \frac{21 \cos^3(x) \sin(x)}{160a^2 \sqrt{a \sec^4(x)}} + \frac{9 \cos^5(x) \sin(x)}{80a^2 \sqrt{a \sec^4(x)}} + \frac{\cos^7(x) \sin(x)}{10a^2 \sqrt{a \sec^4(x)}} + \frac{63 \tan(x)}{256a^2 \sqrt{a \sec^4(x)}} + \frac{63x \sec^2(x)}{256a^2 \sqrt{a \sec^4(x)}} \\
&= \frac{63x \sec^2(x)}{256a^2 \sqrt{a \sec^4(x)}} + \frac{21 \cos(x) \sin(x)}{128a^2 \sqrt{a \sec^4(x)}} + \frac{21 \cos^3(x) \sin(x)}{160a^2 \sqrt{a \sec^4(x)}} + \frac{9 \cos^5(x) \sin(x)}{80a^2 \sqrt{a \sec^4(x)}} + \frac{\cos^7(x) \sin(x)}{10a^2 \sqrt{a \sec^4(x)}} + \frac{63 \tan(x)}{256a^2 \sqrt{a \sec^4(x)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0824578, size = 55, normalized size = 0.42

$$\frac{(2520x + 2100 \sin(2x) + 600 \sin(4x) + 150 \sin(6x) + 25 \sin(8x) + 2 \sin(10x)) \cos^2(x) \sqrt{a \sec^4(x)}}{10240a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*Sec[x]^4)^(-5/2), x]

[Out] (Cos[x]^2\*Sqrt[a\*Sec[x]^4]\*(2520\*x + 2100\*Sin[2\*x] + 600\*Sin[4\*x] + 150\*Sin[6\*x] + 25\*Sin[8\*x] + 2\*Sin[10\*x]))/(10240\*a^3)

**Maple [A]** time = 0.221, size = 57, normalized size = 0.4

$$\frac{128 (\cos(x))^9 \sin(x) + 144 (\cos(x))^7 \sin(x) + 168 (\cos(x))^5 \sin(x) + 210 (\cos(x))^3 \sin(x) + 315 \cos(x) \sin(x) + 315}{1280 (\cos(x))^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a\*sec(x)^4)^(5/2),x)

[Out] 1/1280\*(128\*cos(x)^9\*sin(x)+144\*cos(x)^7\*sin(x)+168\*cos(x)^5\*sin(x)+210\*cos(x)^3\*sin(x)+315\*cos(x)\*sin(x)+315\*x)/cos(x)^10/(a/cos(x)^4)^(5/2)

**Maxima [A]** time = 1.71142, size = 119, normalized size = 0.9

$$\frac{315 \tan(x)^9 + 1470 \tan(x)^7 + 2688 \tan(x)^5 + 2370 \tan(x)^3 + 965 \tan(x)}{1280 \left( a^{\frac{5}{2}} \tan(x)^{10} + 5 a^{\frac{5}{2}} \tan(x)^8 + 10 a^{\frac{5}{2}} \tan(x)^6 + 10 a^{\frac{5}{2}} \tan(x)^4 + 5 a^{\frac{5}{2}} \tan(x)^2 + a^{\frac{5}{2}} \right)} + \frac{63 x}{256 a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)^4)^(5/2),x, algorithm="maxima")

[Out] 1/1280\*(315\*tan(x)^9 + 1470\*tan(x)^7 + 2688\*tan(x)^5 + 2370\*tan(x)^3 + 965\*tan(x))/(a^(5/2)\*tan(x)^10 + 5\*a^(5/2)\*tan(x)^8 + 10\*a^(5/2)\*tan(x)^6 + 10\*a^(5/2)\*tan(x)^4 + 5\*a^(5/2)\*tan(x)^2 + a^(5/2)) + 63/256\*x/a^(5/2)

**Fricas [A]** time = 1.45152, size = 177, normalized size = 1.34

$$\frac{(315 x \cos(x)^2 + (128 \cos(x)^{11} + 144 \cos(x)^9 + 168 \cos(x)^7 + 210 \cos(x)^5 + 315 \cos(x)^3) \sin(x)) \sqrt{\frac{a}{\cos(x)^4}}}{1280 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)^4)^(5/2),x, algorithm="fricas")

[Out] 1/1280\*(315\*x\*cos(x)^2 + (128\*cos(x)^11 + 144\*cos(x)^9 + 168\*cos(x)^7 + 210\*cos(x)^5 + 315\*cos(x)^3)\*sin(x))\*sqrt(a/cos(x)^4)/a^3

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a \sec^4(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)\*\*4)\*\*(5/2),x)

[Out] Integral((a\*sec(x)\*\*4)\*\*(-5/2), x)

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a\*sec(x)^4)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError



### 3.68 $\int ((b \sec(c + dx))^p)^n dx$

**Optimal.** Leaf size=81

$$\frac{\sin(c + dx) \cos(c + dx) ((b \sec(c + dx))^p)^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(1 - np), \frac{1}{2}(3 - np), \cos^2(c + dx)\right)}{d(1 - np)\sqrt{\sin^2(c + dx)}}$$

[Out] -((Cos[c + d\*x]\*Hypergeometric2F1[1/2, (1 - n\*p)/2, (3 - n\*p)/2, Cos[c + d\*x]^2]\*((b\*Sec[c + d\*x])^p)^n\*Sin[c + d\*x])/(d\*(1 - n\*p)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0532608, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {4123, 3772, 2643}

$$\frac{\sin(c + dx) \cos(c + dx) ((b \sec(c + dx))^p)^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(3 - np); \cos^2(c + dx)\right)}{d(1 - np)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b\*Sec[c + d\*x])^p)^n, x]

[Out] -((Cos[c + d\*x]\*Hypergeometric2F1[1/2, (1 - n\*p)/2, (3 - n\*p)/2, Cos[c + d\*x]^2]\*((b\*Sec[c + d\*x])^p)^n\*Sin[c + d\*x])/(d\*(1 - n\*p)\*Sqrt[Sin[c + d\*x]^2])

#### Rule 4123

Int[((b\_.)\*((c\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.))^(p\_), x\_Symbol] := Dist[(b^IntPart[p]\*(b\*(c\*Sec[e + f\*x])^n)^FracPart[p])/(c\*Sec[e + f\*x])^(n\*FracPart[p]), Int[(c\*Sec[e + f\*x])^(n\*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] & !IntegerQ[p]

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/((Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int ((b \sec(c + dx))^p)^n dx &= ((b \sec(c + dx))^{-np} ((b \sec(c + dx))^p)^n) \int (b \sec(c + dx))^{np} dx \\ &= \left( \left( \frac{\cos(c + dx)}{b} \right)^{np} ((b \sec(c + dx))^p)^n \right) \int \left( \frac{\cos(c + dx)}{b} \right)^{-np} dx \\ &= -\frac{\cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(3 - np); \cos^2(c + dx)\right) ((b \sec(c + dx))^p)^n \sin(c + dx)}{d(1 - np)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0937918, size = 69, normalized size = 0.85

$$\frac{\sqrt{-\tan^2(c + dx) \cot(c + dx)} ((b \sec(c + dx))^p)^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{np}{2}, \frac{1}{2}(np + 2), \sec^2(c + dx)\right)}{dnp}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((b*Sec[c + d*x])^p)^n,x]
```

```
[Out] (Cot[c + d*x]*Hypergeometric2F1[1/2, (n*p)/2, (2 + n*p)/2, Sec[c + d*x]^2]*
((b*Sec[c + d*x])^p)^n*Sqrt[-Tan[c + d*x]^2])/(d*n*p)
```

**Maple [F]** time = 0.444, size = 0, normalized size = 0.

$$\int ((b \sec(dx + c))^p)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*sec(d*x+c))^p)^n,x)
```

[Out] `int(((b*sec(d*x+c))^p)^n,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int ((b \sec(dx + c))^p)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((b*sec(d*x+c))^p)^n,x, algorithm="maxima")`

[Out] `integrate(((b*sec(d*x + c))^p)^n, x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left((b \sec(dx + c))^p\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((b*sec(d*x+c))^p)^n,x, algorithm="fricas")`

[Out] `integral(((b*sec(d*x + c))^p)^n, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int ((b \sec(c + dx))^p)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((b*sec(d*x+c))**p)**n,x)`

[Out] `Integral(((b*sec(c + d*x))**p)**n, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*sec(d*x+c))^p)^n,x, algorithm="giac")
```

```
[Out] integrate(((b*sec(d*x + c))^p)^n, x)
```

### 3.69 $\int (a(b \sec(c + dx))^p)^n dx$

**Optimal.** Leaf size=83

$$\frac{\sin(c + dx) \cos(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(1 - np), \frac{1}{2}(3 - np), \cos^2(c + dx)\right) (a(b \sec(c + dx))^p)^n}{d(1 - np) \sqrt{\sin^2(c + dx)}}$$

[Out] -((Cos[c + d\*x]\*Hypergeometric2F1[1/2, (1 - n\*p)/2, (3 - n\*p)/2, Cos[c + d\*x]^2]\*(a\*(b\*Sec[c + d\*x])^p)^n\*Sin[c + d\*x])/(d\*(1 - n\*p)\*Sqrt[Sin[c + d\*x]^2]))

**Rubi [A]** time = 0.0495264, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {4123, 3772, 2643}

$$\frac{\sin(c + dx) \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(3 - np); \cos^2(c + dx)\right) (a(b \sec(c + dx))^p)^n}{d(1 - np) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(a\*(b\*Sec[c + d\*x])^p)^n, x]

[Out] -((Cos[c + d\*x]\*Hypergeometric2F1[1/2, (1 - n\*p)/2, (3 - n\*p)/2, Cos[c + d\*x]^2]\*(a\*(b\*Sec[c + d\*x])^p)^n\*Sin[c + d\*x])/(d\*(1 - n\*p)\*Sqrt[Sin[c + d\*x]^2]))

#### Rule 4123

Int[((b\_.)\*((c\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_))^(p\_), x\_Symbol] :> Dist[(b^IntPart[p]\*(b\*(c\*Sec[e + f\*x])^n)^FracPart[p])/(c\*Sec[e + f\*x])^(n\*FracPart[p]), Int[(c\*Sec[e + f\*x])^(n\*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] && !IntegerQ[p]

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)])\*(b\_.)^(n\_), x\_Symbol] :> Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/((Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x])*
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int (a(b \sec(c + dx))^p)^n dx &= ((b \sec(c + dx))^{-np} (a(b \sec(c + dx))^p)^n) \int (b \sec(c + dx))^{np} dx \\ &= \left( \left( \frac{\cos(c + dx)}{b} \right)^{np} (a(b \sec(c + dx))^p)^n \right) \int \left( \frac{\cos(c + dx)}{b} \right)^{-np} dx \\ &= -\frac{\cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - np); \frac{1}{2}(3 - np); \cos^2(c + dx)\right) (a(b \sec(c + dx))^p)^n \sin(c + dx)}{d(1 - np)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.072802, size = 71, normalized size = 0.86

$$\frac{\sqrt{-\tan^2(c + dx)} \cot(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{np}{2}, \frac{1}{2}(np + 2), \sec^2(c + dx)\right) (a(b \sec(c + dx))^p)^n}{dnp}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*(b*Sec[c + d*x])^p)^n,x]
```

```
[Out] (Cot[c + d*x]*Hypergeometric2F1[1/2, (n*p)/2, (2 + n*p)/2, Sec[c + d*x]^2]*
(a*(b*Sec[c + d*x])^p)^n*Sqrt[-Tan[c + d*x]^2])/(d*n*p)
```

**Maple [F]** time = 0.197, size = 0, normalized size = 0.

$$\int (a(b \sec(dx + c))^p)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*sec(d*x+c))^p)^n,x)
```

[Out] `int((a*(b*sec(d*x+c))^p)^n,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( (b \sec(dx + c))^p a \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*(b*sec(d*x+c))^p)^n,x, algorithm="maxima")`

[Out] `integrate(((b*sec(d*x + c))^p*a)^n, x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left((b \sec(dx + c))^p a\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*(b*sec(d*x+c))^p)^n,x, algorithm="fricas")`

[Out] `integral(((b*sec(d*x + c))^p*a)^n, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( a (b \sec(c + dx))^p \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*(b*sec(d*x+c))**p)**n,x)`

[Out] `Integral((a*(b*sec(c + d*x))**p)**n, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( (b \sec(dx + c))^p a \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sec(d*x+c))^p)^n,x, algorithm="giac")
```

```
[Out] integrate(((b*sec(d*x + c))^p*a)^n, x)
```



### 3.70 $\int \sec^4(c + dx) \sqrt{b \sec(c + dx)} dx$

**Optimal.** Leaf size=97

$$\frac{10\sqrt{\cos(c + dx)}\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)\sqrt{b \sec(c + dx)}}{21d} + \frac{2 \sin(c + dx)(b \sec(c + dx))^{7/2}}{7b^3d} + \frac{10 \sin(c + dx)(b \sec(c + dx))}{21bd}$$

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*d) + (10\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(21\*b\*d) + (2\*(b\*Sec[c + d\*x])^(7/2)\*Sin[c + d\*x])/(7\*b^3\*d)

**Rubi [A]** time = 0.0604263, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2641}

$$\frac{2 \sin(c + dx)(b \sec(c + dx))^{7/2}}{7b^3d} + \frac{10 \sin(c + dx)(b \sec(c + dx))^{3/2}}{21bd} + \frac{10\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)\sqrt{b \sec(c + dx)}}{21d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^4\*Sqrt[b\*Sec[c + d\*x]],x]

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*d) + (10\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(21\*b\*d) + (2\*(b\*Sec[c + d\*x])^(7/2)\*Sin[c + d\*x])/(7\*b^3\*d)

#### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3768

Int[(csc[(c\_.) + (d\_)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] :> -Simp[(b\*Csc[c + d\*x]\*(b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \sec^4(c + dx) \sqrt{b \sec(c + dx)} dx &= \frac{\int (b \sec(c + dx))^{9/2} dx}{b^4} \\
&= \frac{2(b \sec(c + dx))^{7/2} \sin(c + dx)}{7b^3 d} + \frac{5 \int (b \sec(c + dx))^{5/2} dx}{7b^2} \\
&= \frac{10(b \sec(c + dx))^{3/2} \sin(c + dx)}{21bd} + \frac{2(b \sec(c + dx))^{7/2} \sin(c + dx)}{7b^3 d} + \frac{5}{21} \int \sqrt{b \sec(c + dx)} dx \\
&= \frac{10(b \sec(c + dx))^{3/2} \sin(c + dx)}{21bd} + \frac{2(b \sec(c + dx))^{7/2} \sin(c + dx)}{7b^3 d} + \frac{1}{21} (5\sqrt{\cos(c + dx)} + \dots) \\
&= \frac{10\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{21d} + \frac{10(b \sec(c + dx))^{3/2} \sin(c + dx)}{21bd} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.21158, size = 69, normalized size = 0.71

$$\frac{\sec^2(c + dx) \sqrt{b \sec(c + dx)} \left( 10 \cos^{\frac{5}{2}}(c + dx) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 5 \sin(2(c + dx)) + 6 \tan(c + dx) \right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[b*Sec[c + d*x]], x]
```

```
[Out] (Sec[c + d*x]^2*Sqrt[b*Sec[c + d*x]]*(10*Cos[c + d*x]^(5/2)*EllipticF[(c +
d*x)/2, 2] + 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

**Maple [C]** time = 0.224, size = 152, normalized size = 1.6

$$-\frac{2(\cos(dx + c) + 1)^2(-1 + \cos(dx + c))}{21d(\sin(dx + c))^3(\cos(dx + c))^3} \left( 5i(\cos(dx + c))^3 \sin(dx + c) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x)`

[Out] 
$$-2/21/d*(\cos(d*x+c)+1)^2*(-1+\cos(d*x+c))*(5*I*\cos(d*x+c)^3*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*EllipticF(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-5*\cos(d*x+c)^3+5*\cos(d*x+c)^2-3*\cos(d*x+c)+3)*(b/\cos(d*x+c))^{1/2}/\sin(d*x+c)^3/\cos(d*x+c)^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^4, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec(dx + c)} \sec(dx + c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^4, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(c + dx)} \sec^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**4*(b*sec(d*x+c))**(1/2),x)`

```
[Out] Integral(sqrt(b*sec(c + d*x))*sec(c + d*x)**4, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^4, x)
```

### 3.71 $\int \sec^3(c + dx)\sqrt{b \sec(c + dx)} dx$

**Optimal.** Leaf size=95

$$\frac{2 \sin(c + dx)(b \sec(c + dx))^{5/2}}{5b^2d} + \frac{6 \sin(c + dx)\sqrt{b \sec(c + dx)}}{5d} - \frac{6bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

[Out]  $(-6*b*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (6*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*b^2*d)$

**Rubi [A]** time = 0.0606493, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2639}

$$\frac{2 \sin(c + dx)(b \sec(c + dx))^{5/2}}{5b^2d} + \frac{6 \sin(c + dx)\sqrt{b \sec(c + dx)}}{5d} - \frac{6bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sec}[c + d*x]^3*\text{Sqrt}[b*\text{Sec}[c + d*x]], x]$

[Out]  $(-6*b*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (6*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*b^2*d)$

#### Rule 16

$\text{Int}[(u_*)*(v_)^(m_)*((b_)*(v_))^(n_), x\_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^(m+n), x], x] /; \text{FreeQ}\{b, n\}, x \ \&\& \ \text{IntegerQ}[m]$

#### Rule 3768

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_)]*(b_))^(n_), x\_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Csc}[c + d*x])^(n-1)/(d*(n-1)), x] + \text{Dist}[(b^2*(n-2))/(n-1), \text{Int}[(b*\text{Csc}[c + d*x])^(n-2), x], x] /; \text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \sec^3(c + dx) \sqrt{b \sec(c + dx)} dx &= \frac{\int (b \sec(c + dx))^{7/2} dx}{b^3} \\
&= \frac{2(b \sec(c + dx))^{5/2} \sin(c + dx)}{5b^2d} + \frac{3 \int (b \sec(c + dx))^{3/2} dx}{5b} \\
&= \frac{6\sqrt{b \sec(c + dx)} \sin(c + dx)}{5d} + \frac{2(b \sec(c + dx))^{5/2} \sin(c + dx)}{5b^2d} - \frac{1}{5}(3b) \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\
&= \frac{6\sqrt{b \sec(c + dx)} \sin(c + dx)}{5d} + \frac{2(b \sec(c + dx))^{5/2} \sin(c + dx)}{5b^2d} - \frac{(3b) \int \sqrt{\cos(c + dx)}}{5\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} dx \\
&= -\frac{6bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} + \frac{6\sqrt{b \sec(c + dx)} \sin(c + dx)}{5d} + \frac{2(b \sec(c + dx))^{5/2}}{5b^2d}
\end{aligned}$$

**Mathematica [A]** time = 0.165085, size = 69, normalized size = 0.73

$$\frac{\sec^2(c + dx) \sqrt{b \sec(c + dx)} \left( 7 \sin(c + dx) + 3 \sin(3(c + dx)) - 12 \cos^{\frac{5}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[b*Sec[c + d*x]], x]
```

```
[Out] (Sec[c + d*x]^2*Sqrt[b*Sec[c + d*x]]*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c +
d*x)/2, 2] + 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*d)
```

**Maple [C]** time = 0.208, size = 356, normalized size = 3.8

$$\frac{2 (\cos(dx + c) + 1)^2 (-1 + \cos(dx + c))^2}{5d (\sin(dx + c))^5 (\cos(dx + c))^2} \left( 3i \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} (\cos(dx + c))^3 \text{EllipticE}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x)`

[Out]  $2/5/d*(\cos(d*x+c)+1)^2*(-1+\cos(d*x+c))^2*(3*I*(1/(\cos(d*x+c)+1))^(1/2)*(\cos(d*x+c)/(\cos(d*x+c)+1))^(1/2)*\cos(d*x+c)^3*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)-3*I*(1/(\cos(d*x+c)+1))^(1/2)*(\cos(d*x+c)/(\cos(d*x+c)+1))^(1/2)*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)^3*\sin(d*x+c)+3*I*(1/(\cos(d*x+c)+1))^(1/2)*(\cos(d*x+c)/(\cos(d*x+c)+1))^(1/2)*\cos(d*x+c)^2*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)-3*I*(1/(\cos(d*x+c)+1))^(1/2)*(\cos(d*x+c)/(\cos(d*x+c)+1))^(1/2)*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)^2*\sin(d*x+c)-3*\cos(d*x+c)^3+2*\cos(d*x+c)^2+1)*(b/\cos(d*x+c))^(1/2)/\sin(d*x+c)^5/\cos(d*x+c)^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^3, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec(dx + c)} \sec(dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(c + dx)} \sec^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*3\*(b\*sec(d\*x+c))\*\*(1/2),x)

[Out] Integral(sqrt(b\*sec(c + d\*x))\*sec(c + d\*x)\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^3\*(b\*sec(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b\*sec(d\*x + c))\*sec(d\*x + c)^3, x)



### 3.72 $\int \sec^2(c + dx) \sqrt{b \sec(c + dx)} dx$

**Optimal.** Leaf size=69

$$\frac{2\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{b \sec(c + dx)}}{3d} + \frac{2 \sin(c + dx) (b \sec(c + dx))^{3/2}}{3bd}$$

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*d) + (2\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(3\*b\*d)

**Rubi [A]** time = 0.0420639, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2641}

$$\frac{2 \sin(c + dx) (b \sec(c + dx))^{3/2}}{3bd} + \frac{2\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^2\*Sqrt[b\*Sec[c + d\*x]],x]

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*d) + (2\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(3\*b\*d)

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> -Simp[(b\*Csc[c + d\*x]\*(b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(c + dx) \sqrt{b \sec(c + dx)} dx &= \frac{\int (b \sec(c + dx))^{5/2} dx}{b^2} \\
&= \frac{2(b \sec(c + dx))^{3/2} \sin(c + dx)}{3bd} + \frac{1}{3} \int \sqrt{b \sec(c + dx)} dx \\
&= \frac{2(b \sec(c + dx))^{3/2} \sin(c + dx)}{3bd} + \frac{1}{3} \left( \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{3d} + \frac{2(b \sec(c + dx))^{3/2} \sin(c + dx)}{3bd}
\end{aligned}$$

**Mathematica [A]** time = 0.0836222, size = 51, normalized size = 0.74

$$\frac{2(b \sec(c + dx))^{3/2} \left( \cos^{\frac{3}{2}}(c + dx) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \sin(c + dx) \right)}{3bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^2\*Sqrt[b\*Sec[c + d\*x]],x]

```
[Out] (2*(b*Sec[c + d*x])^(3/2)*(Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + Sin[c + d*x]))/(3*b*d)
```

**Maple [C]** time = 0.155, size = 130, normalized size = 1.9

$$-\frac{(-2 + 2 \cos(dx + c)) (\cos(dx + c) + 1)^2}{3d (\sin(dx + c))^3 \cos(dx + c)} \sqrt{\frac{b}{\cos(dx + c)}} \left( i \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x)`

[Out] 
$$-2/3/d*(b/\cos(d*x+c))^{1/2}*(-1+\cos(d*x+c))*(I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)*\sin(d*x+c)-\cos(d*x+c)+1)*(\cos(d*x+c)+1)^2/\sin(d*x+c)^3/\cos(d*x+c)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec(dx + c)} \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(c + dx)} \sec^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(1/2),x)`

[Out] `Integral(sqrt(b*sec(c + d*x))*sec(c + d*x)**2, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^2, x)
```

### 3.73 $\int \sec(c + dx) \sqrt{b \sec(c + dx)} dx$

Optimal. Leaf size=63

$$\frac{2 \sin(c + dx) \sqrt{b \sec(c + dx)}}{d} - \frac{2bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}}$$

[Out]  $(-2*b*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d$

**Rubi [A]** time = 0.0407368, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {16, 3768, 3771, 2639}

$$\frac{2 \sin(c + dx) \sqrt{b \sec(c + dx)}}{d} - \frac{2bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]\*Sqrt[b\*Sec[c + d\*x]], x]

[Out]  $(-2*b*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d$

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec(c + dx) \sqrt{b \sec(c + dx)} dx &= \frac{\int (b \sec(c + dx))^{3/2} dx}{b} \\
&= \frac{2\sqrt{b \sec(c + dx)} \sin(c + dx)}{d} - b \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\
&= \frac{2\sqrt{b \sec(c + dx)} \sin(c + dx)}{d} - \frac{b \int \sqrt{\cos(c + dx)} dx}{\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} \\
&= -\frac{2bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2\sqrt{b \sec(c + dx)} \sin(c + dx)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.0511457, size = 47, normalized size = 0.75

$$\frac{2\sqrt{b \sec(c + dx)} \left( \sin(c + dx) - \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]\*Sqrt[b\*Sec[c + d\*x]],x]

```
[Out] (2*Sqrt[b*Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
Sin[c + d*x]))/d
```

**Maple [C]** time = 0.214, size = 316, normalized size = 5.

$$2 \frac{(\cos(dx + c) + 1)^2 (-1 + \cos(dx + c))^2}{d (\sin(dx + c))^5} \sqrt{\frac{b}{\cos(dx + c)}} \left( i \cos(dx + c) \sin(dx + c) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x)`

[Out]  $2/d*(b/\cos(d*x+c))^{1/2}*(\cos(d*x+c)+1)^2*(-1+\cos(d*x+c))^2*(I*\cos(d*x+c)*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)*\sin(d*x+c)+I*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-I*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})*\sin(d*x+c)-\cos(d*x+c)+1)/\sin(d*x+c)^5$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx+c)} \sec(dx+c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sec(d*x + c))*sec(d*x + c), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{b \sec(dx+c)} \sec(dx+c), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*sec(d*x + c), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(c+dx)} \sec(c+dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*sec(c + d*x), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c), x)
```



### 3.74 $\int \sqrt{b \sec(c + dx)} dx$

**Optimal.** Leaf size=38

$$\frac{2\sqrt{\cos(c + dx)}\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)\sqrt{b \sec(c + dx)}}{d}$$

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/d

**Rubi [A]** time = 0.0202235, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3771, 2641}

$$\frac{2\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)\sqrt{b \sec(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b\*Sec[c + d\*x]],x]

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/d

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] :> Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \sqrt{b \sec(c + dx)} dx &= (\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\ &= \frac{2\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)\sqrt{b \sec(c + dx)}}{d} \end{aligned}$$

**Mathematica [A]** time = 0.0202658, size = 38, normalized size = 1.

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b\*Sec[c + d\*x]], x]

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/d

**Maple [C]** time = 0.141, size = 98, normalized size = 2.6

$$\frac{-2i(-1 + \cos(dx + c))(\cos(dx + c) + 1)^2}{d(\sin(dx + c))^2} \sqrt{\frac{b}{\cos(dx + c)}} \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(1/2), x)

[Out] -2\*I/d\*(b/cos(d\*x+c))^(1/2)\*(-1+cos(d\*x+c))\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*(cos(d\*x+c)+1)^2/sin(d\*x+c)^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(b\*sec(d\*x + c)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c)), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))**(1/2),x)`

[Out] `Integral(sqrt(b*sec(c + d*x)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(b*sec(d*x + c)), x)`

### 3.75 $\int \cos(c + dx) \sqrt{b \sec(c + dx)} dx$

**Optimal.** Leaf size=39

$$\frac{2bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

[Out] (2\*b\*EllipticE[(c + d\*x)/2, 2])/(d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0296558, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {16, 3771, 2639}

$$\frac{2bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]\*Sqrt[b\*Sec[c + d\*x]],x]

[Out] (2\*b\*EllipticE[(c + d\*x)/2, 2])/(d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \cos(c + dx) \sqrt{b \sec(c + dx)} dx &= b \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\ &= \frac{b \int \sqrt{\cos(c + dx)} dx}{\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} \\ &= \frac{2bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0348944, size = 39, normalized size = 1.

$$\frac{2bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d\*x]\*Sqrt[b\*Sec[c + d\*x]],x]

[Out] (2\*b\*EllipticE[(c + d\*x)/2, 2])/(d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

**Maple [C]** time = 0.149, size = 303, normalized size = 7.8

$$2 \frac{1}{d \sin(dx + c)} \left( i \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \cos(dx + c) \sin(dx + c) - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)\*(b\*sec(d\*x+c))^(1/2),x)

[Out] 2/d\*(I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*cos(d\*x+c)\*sin(d\*x+c)-I\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*cos(d\*x+c)\*sin(d\*x+c)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)+I\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*sin(d\*x+c)-I\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*sin(d\*x+c)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)-cos(d\*x+c)^2+cos(d\*x+c))\*(b/cos(d\*x+c))^(1/2)/sin(d\*x+c)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*(b\*sec(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b\*sec(d\*x + c))\*cos(d\*x + c), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec(dx + c)} \cos(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*(b\*sec(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b\*sec(d\*x + c))\*cos(d\*x + c), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(c + dx)} \cos(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*(b\*sec(d\*x+c))\*\*(1/2),x)

[Out] Integral(sqrt(b\*sec(c + d\*x))\*cos(c + d\*x), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c), x)
```

### 3.76 $\int \cos^2(c + dx) \sqrt{b \sec(c + dx)} dx$

**Optimal.** Leaf size=67

$$\frac{2\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{b \sec(c + dx)}}{3d} + \frac{2b \sin(c + dx)}{3d \sqrt{b \sec(c + dx)}}$$

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*d) + (2\*b\*Sin[c + d\*x])/(3\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0516895, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2641}

$$\frac{2b \sin(c + dx)}{3d \sqrt{b \sec(c + dx)}} + \frac{2\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^2\*Sqrt[b\*Sec[c + d\*x]],x]

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*d) + (2\*b\*Sin[c + d\*x])/(3\*d\*Sqrt[b\*Sec[c + d\*x]])

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&



EqQ[n^2, 1/4]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int \cos^2(c + dx) \sqrt{b \sec(c + dx)} dx &= b^2 \int \frac{1}{(b \sec(c + dx))^{3/2}} dx \\
 &= \frac{2b \sin(c + dx)}{3d \sqrt{b \sec(c + dx)}} + \frac{1}{3} \int \sqrt{b \sec(c + dx)} dx \\
 &= \frac{2b \sin(c + dx)}{3d \sqrt{b \sec(c + dx)}} + \frac{1}{3} \left( \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &= \frac{2\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{3d} + \frac{2b \sin(c + dx)}{3d \sqrt{b \sec(c + dx)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0513573, size = 51, normalized size = 0.76

$$\frac{\sqrt{b \sec(c + dx)} \left( 2\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \sin(2(c + dx)) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d\*x]^2\*Sqrt[b\*Sec[c + d\*x]], x]

```
[Out] (Sqrt[b*Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin
[2*(c + d*x)]))/(3*d)
```

**Maple [C]** time = 0.164, size = 123, normalized size = 1.8

$$-\frac{(-2 + 2 \cos(dx + c)) (\cos(dx + c) + 1)^2}{3d (\sin(dx + c))^3} \left( i \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)^2
+cos(d*x+c))*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(1/2)/sin(d*x+c)^3
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^2, x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{b \sec(dx + c)} \cos(dx + c)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^2, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(c + dx)} \cos^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*cos(c + d*x)**2, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^2\*(b\*sec(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b\*sec(d\*x + c))\*cos(d\*x + c)^2, x)

### 3.77 $\int \cos^3(c + dx) \sqrt{b \sec(c + dx)} dx$

**Optimal.** Leaf size=70

$$\frac{2b^2 \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}} + \frac{6bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

[Out] (6\*b\*EllipticE[(c + d\*x)/2, 2])/(5\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b^2\*Sin[c + d\*x])/(5\*d\*(b\*Sec[c + d\*x])^(3/2))

**Rubi [A]** time = 0.0514325, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2639}

$$\frac{2b^2 \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}} + \frac{6bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^3\*Sqrt[b\*Sec[c + d\*x]],x]

[Out] (6\*b\*EllipticE[(c + d\*x)/2, 2])/(5\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b^2\*Sin[c + d\*x])/(5\*d\*(b\*Sec[c + d\*x])^(3/2))

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c + dx) \sqrt{b \sec(c + dx)} dx &= b^3 \int \frac{1}{(b \sec(c + dx))^{5/2}} dx \\
&= \frac{2b^2 \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}} + \frac{1}{5}(3b) \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\
&= \frac{2b^2 \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}} + \frac{(3b) \int \sqrt{\cos(c + dx)} dx}{5\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} \\
&= \frac{6bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} + \frac{2b^2 \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0594578, size = 57, normalized size = 0.81

$$\frac{\sqrt{b \sec(c + dx)} \left( \sin(c + dx) + \sin(3(c + dx)) + 12\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{10d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d\*x]^3\*Sqrt[b\*Sec[c + d\*x]], x]

```
[Out] (Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Sin[c + d*x] + Sin[3*(c + d*x)]))/(10*d)
```

**Maple [C]** time = 0.208, size = 315, normalized size = 4.5

$$\frac{2}{5d \sin(dx + c)} \sqrt{\frac{b}{\cos(dx + c)}} \left( 3i \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \cos(dx + c) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/5/d*(b/cos(d*x+c))^(1/2)*(3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-3*I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+3*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-cos(d*x+c)^4-2*cos(d*x+c)^2+3*cos(d*x+c))/sin(d*x+c)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \cos(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^3, x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{b \sec(dx + c)} \cos(dx + c)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^3, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \cos(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^3, x)
```

### 3.78 $\int \cos^4(c + dx) \sqrt{b \sec(c + dx)} dx$

**Optimal.** Leaf size=95

$$\frac{10\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{b \sec(c + dx)}}{21d} + \frac{2b^3 \sin(c + dx)}{7d(b \sec(c + dx))^{5/2}} + \frac{10b \sin(c + dx)}{21d\sqrt{b \sec(c + dx)}}$$

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*d) + (2\*b^3\*Sin[c + d\*x])/(7\*d\*(b\*Sec[c + d\*x])^(5/2)) + (10\*b\*Sin[c + d\*x])/(21\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0701063, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2641}

$$\frac{2b^3 \sin(c + dx)}{7d(b \sec(c + dx))^{5/2}} + \frac{10b \sin(c + dx)}{21d\sqrt{b \sec(c + dx)}} + \frac{10\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{21d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^4\*Sqrt[b\*Sec[c + d\*x]],x]

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*d) + (2\*b^3\*Sin[c + d\*x])/(7\*d\*(b\*Sec[c + d\*x])^(5/2)) + (10\*b\*Sin[c + d\*x])/(21\*d\*Sqrt[b\*Sec[c + d\*x]])

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771



```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
 \int \cos^4(c + dx) \sqrt{b \sec(c + dx)} dx &= b^4 \int \frac{1}{(b \sec(c + dx))^{7/2}} dx \\
 &= \frac{2b^3 \sin(c + dx)}{7d(b \sec(c + dx))^{5/2}} + \frac{1}{7} (5b^2) \int \frac{1}{(b \sec(c + dx))^{3/2}} dx \\
 &= \frac{2b^3 \sin(c + dx)}{7d(b \sec(c + dx))^{5/2}} + \frac{10b \sin(c + dx)}{21d\sqrt{b \sec(c + dx)}} + \frac{5}{21} \int \sqrt{b \sec(c + dx)} dx \\
 &= \frac{2b^3 \sin(c + dx)}{7d(b \sec(c + dx))^{5/2}} + \frac{10b \sin(c + dx)}{21d\sqrt{b \sec(c + dx)}} + \frac{1}{21} (5\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}) \int dx \\
 &= \frac{10\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{21d} + \frac{2b^3 \sin(c + dx)}{7d(b \sec(c + dx))^{5/2}} + \frac{10b \sin(c + dx)}{21d\sqrt{b \sec(c + dx)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.089581, size = 63, normalized size = 0.66

$$\frac{\sqrt{b \sec(c + dx)} \left( 40\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 26 \sin(2(c + dx)) + 3 \sin(4(c + dx)) \right)}{84d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[b*Sec[c + d*x]], x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26
*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*d)
```

**Maple [C]** time = 0.188, size = 145, normalized size = 1.5

$$\frac{(-2 + 2 \cos(dx + c)) (\cos(dx + c) + 1)^2}{21 d (\sin(dx + c))^3} \left( 5 i \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x)`

[Out] 
$$-2/21/d*(-1+\cos(dx+c))*(5*I*EllipticF(I*(-1+\cos(dx+c))/\sin(dx+c),I)*(1/(\cos(dx+c)+1))^{1/2}*(\cos(dx+c)/(\cos(dx+c)+1))^{1/2}*\sin(dx+c)-3*\cos(dx+c)^4+3*\cos(dx+c)^3-5*\cos(dx+c)^2+5*\cos(dx+c))*(\cos(dx+c)+1)^2*(b/\cos(dx+c))^{1/2}/\sin(dx+c)^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx+c)} \cos(dx+c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^4, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec(dx+c)} \cos(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^4, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**4*(b*sec(d*x+c))**(1/2),x)`

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="giac")`

[Out] Timed out

### 3.79 $\int \cos^5(c + dx) \sqrt{b \sec(c + dx)} dx$

**Optimal.** Leaf size=98

$$\frac{2b^4 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14b^2 \sin(c + dx)}{45d(b \sec(c + dx))^{3/2}} + \frac{14bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

[Out] (14\*b\*EllipticE[(c + d\*x)/2, 2])/(15\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b^4\*Sin[c + d\*x])/(9\*d\*(b\*Sec[c + d\*x])^(7/2)) + (14\*b^2\*Sin[c + d\*x])/(45\*d\*(b\*Sec[c + d\*x])^(3/2))

---

**Rubi [A]** time = 0.0693045, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2639}

$$\frac{2b^4 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14b^2 \sin(c + dx)}{45d(b \sec(c + dx))^{3/2}} + \frac{14bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^5\*Sqrt[b\*Sec[c + d\*x]],x]

[Out] (14\*b\*EllipticE[(c + d\*x)/2, 2])/(15\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b^4\*Sin[c + d\*x])/(9\*d\*(b\*Sec[c + d\*x])^(7/2)) + (14\*b^2\*Sin[c + d\*x])/(45\*d\*(b\*Sec[c + d\*x])^(3/2))

#### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \cos^5(c + dx) \sqrt{b \sec(c + dx)} dx &= b^5 \int \frac{1}{(b \sec(c + dx))^{9/2}} dx \\
&= \frac{2b^4 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{1}{9} (7b^3) \int \frac{1}{(b \sec(c + dx))^{5/2}} dx \\
&= \frac{2b^4 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14b^2 \sin(c + dx)}{45d(b \sec(c + dx))^{3/2}} + \frac{1}{15} (7b) \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\
&= \frac{2b^4 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14b^2 \sin(c + dx)}{45d(b \sec(c + dx))^{3/2}} + \frac{(7b) \int \sqrt{\cos(c + dx)} dx}{15\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} \\
&= \frac{14bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} + \frac{2b^4 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14b^2 \sin(c + dx)}{45d(b \sec(c + dx))^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.243606, size = 71, normalized size = 0.72

$$\frac{\sqrt{b \sec(c + dx)} \left( (33 \sin(c + dx) + 5 \sin(3(c + dx))) \cos^2(c + dx) + 84 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{90d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sqrt[b*Sec[c + d*x]], x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(84*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Co
s[c + d*x]^2*(33*Sin[c + d*x] + 5*Sin[3*(c + d*x)])))/(90*d)
```

**Maple [C]** time = 0.181, size = 325, normalized size = 3.3

$$-\frac{2}{45d \sin(dx + c)} \sqrt{\frac{b}{\cos(dx + c)}} \left( 21 i \cos(dx + c) \sin(dx + c) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticE}\left(\frac{i(-1)}{\dots}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x)`

[Out]  $-2/45/d*(b/\cos(d*x+c))^{1/2}*(21*I*\cos(d*x+c)*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-21*I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)*\sin(d*x+c)+5*\cos(d*x+c)^6+21*I*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-21*I*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\sin(d*x+c)+2*\cos(d*x+c)^4+14*\cos(d*x+c)^2-21*\cos(d*x+c))/\sin(d*x+c)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \cos(dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^5, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{b \sec(dx + c)} \cos(dx + c)^5, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^5, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**5*(b*sec(d*x+c))**(1/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \cos(dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^5, x)`

### 3.80 $\int \sec^3(c + dx)(b \sec(c + dx))^{3/2} dx$

**Optimal.** Leaf size=95

$$\frac{10b\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^2d} + \frac{10\sin(c+dx)(b\sec(c+dx))^{5/2}}{21d}$$

[Out] (10\*b\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*d) + (10\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(21\*d) + (2\*(b\*Sec[c + d\*x])^(7/2)\*Sin[c + d\*x])/(7\*b^2\*d)

**Rubi [A]** time = 0.0594839, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^2d} + \frac{10\sin(c+dx)(b\sec(c+dx))^{3/2}}{21d} + \frac{10b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^3\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] (10\*b\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*d) + (10\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(21\*d) + (2\*(b\*Sec[c + d\*x])^(7/2)\*Sin[c + d\*x])/(7\*b^2\*d)

#### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := -Simp[(b\*Csc[c + d\*x]\*(b\*Csc[c + d\*x])^(n-1))/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*Csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771



```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
 \int \sec^3(c + dx)(b \sec(c + dx))^{3/2} dx &= \frac{\int (b \sec(c + dx))^{9/2} dx}{b^3} \\
 &= \frac{2(b \sec(c + dx))^{7/2} \sin(c + dx)}{7b^2d} + \frac{5 \int (b \sec(c + dx))^{5/2} dx}{7b} \\
 &= \frac{10(b \sec(c + dx))^{3/2} \sin(c + dx)}{21d} + \frac{2(b \sec(c + dx))^{7/2} \sin(c + dx)}{7b^2d} + \frac{1}{21}(5b) \int \sqrt{b} \\
 &= \frac{10(b \sec(c + dx))^{3/2} \sin(c + dx)}{21d} + \frac{2(b \sec(c + dx))^{7/2} \sin(c + dx)}{7b^2d} + \frac{1}{21} (5b\sqrt{\cos(c + dx)}) \\
 &= \frac{10b\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{21d} + \frac{10(b \sec(c + dx))^{3/2} \sin(c + dx)}{21d}
 \end{aligned}$$

**Mathematica [A]** time = 0.183115, size = 64, normalized size = 0.67

$$\frac{(b \sec(c + dx))^{5/2} \left( 10 \cos^{\frac{5}{2}}(c + dx) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 5 \sin(2(c + dx)) + 6 \tan(c + dx) \right)}{21bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(b*Sec[c + d*x])^(3/2), x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*b*d)
```

**Maple [C]** time = 0.195, size = 152, normalized size = 1.6

$$\frac{2(\cos(dx + c) + 1)^2(-1 + \cos(dx + c))}{21d(\cos(dx + c))^2(\sin(dx + c))^3} \left( 5i(\cos(dx + c))^3 \sin(dx + c) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 5 \sin(2(c + dx)) + 6 \tan(c + dx) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x)`

[Out] 
$$-2/21/d*(\cos(d*x+c)+1)^2*(-1+\cos(d*x+c))*(5*I*\cos(d*x+c)^3*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*EllipticF(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-5*\cos(d*x+c)^3+5*\cos(d*x+c)^2-3*\cos(d*x+c)+3)*(b/\cos(d*x+c))^{3/2}/\cos(d*x+c)^2/\sin(d*x+c)^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^3, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{b \sec(dx + c)} b \sec(dx + c)^4, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*b*sec(d*x + c)^4, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**3*(b*sec(d*x+c))**(3/2),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="giac")`

[Out] `integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^3, x)`

### 3.81 $\int \sec^2(c + dx)(b \sec(c + dx))^{3/2} dx$

**Optimal.** Leaf size=98

$$-\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5bd} + \frac{6b\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}$$

[Out]  $(-6*b^2*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (6*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*Sec[c + d*x])^{5/2}*Sin[c + d*x])/(5*b*d)$

**Rubi [A]** time = 0.0591151, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2639}

$$-\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5bd} + \frac{6b\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^2\*(b\*Sec[c + d\*x])^(3/2), x]

[Out]  $(-6*b^2*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (6*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*Sec[c + d*x])^{5/2}*Sin[c + d*x])/(5*b*d)$

#### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := -Simp[(b\*Csc[c + d\*x]\*(b\*Csc[c + d\*x])^(n-1))/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*Csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \sec^2(c + dx)(b \sec(c + dx))^{3/2} dx &= \frac{\int (b \sec(c + dx))^{7/2} dx}{b^2} \\
&= \frac{2(b \sec(c + dx))^{5/2} \sin(c + dx)}{5bd} + \frac{3}{5} \int (b \sec(c + dx))^{3/2} dx \\
&= \frac{6b\sqrt{b \sec(c + dx)} \sin(c + dx)}{5d} + \frac{2(b \sec(c + dx))^{5/2} \sin(c + dx)}{5bd} - \frac{1}{5} (3b^2) \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\
&= \frac{6b\sqrt{b \sec(c + dx)} \sin(c + dx)}{5d} + \frac{2(b \sec(c + dx))^{5/2} \sin(c + dx)}{5bd} - \frac{(3b^2) \int \sqrt{\cos(c + dx)}}{5\sqrt{\cos(c + dx)}} dx \\
&= \frac{6b^2 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} + \frac{6b\sqrt{b \sec(c + dx)} \sin(c + dx)}{5d} + \frac{2(b \sec(c + dx))^{5/2} \sin(c + dx)}{5bd}
\end{aligned}$$

**Mathematica [A]** time = 0.174518, size = 64, normalized size = 0.65

$$\frac{(b \sec(c + dx))^{5/2} \left( 7 \sin(c + dx) + 3 \sin(3(c + dx)) - 12 \cos^2(c + dx) E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{10bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(3/2), x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] +
7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*b*d)
```

**Maple [C]** time = 0.19, size = 356, normalized size = 3.6

$$\frac{2 (\cos(dx + c) + 1)^2 (-1 + \cos(dx + c))^2}{5d (\sin(dx + c))^5 \cos(dx + c)} \left( 3i \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} (\cos(dx + c))^3 \text{EllipticE}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)} \middle| 2\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x)`

[Out]  $2/5/d*(\cos(d*x+c)+1)^2*(-1+\cos(d*x+c))^2*(3*I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\cos(d*x+c)^3*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)-3*I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)^3*\sin(d*x+c)+3*I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\cos(d*x+c)^2*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)-3*I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)^2*\sin(d*x+c)-3*\cos(d*x+c)^3+2*\cos(d*x+c)^2+1)*(b/\cos(d*x+c))^{3/2}/\sin(d*x+c)^5/\cos(d*x+c)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{b \sec(dx + c)} b \sec(dx + c)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*b*sec(d*x + c)^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(c + dx))^{\frac{3}{2}} \sec^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(3/2),x)`

[Out] `Integral((b*sec(c + d*x))**(3/2)*sec(c + d*x)**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="giac")`

[Out] `integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^2, x)`

### 3.82 $\int \sec(c + dx)(b \sec(c + dx))^{3/2} dx$

**Optimal.** Leaf size=67

$$\frac{2b\sqrt{\cos(c + dx)}\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)\sqrt{b\sec(c + dx)}}{3d} + \frac{2\sin(c + dx)(b\sec(c + dx))^{3/2}}{3d}$$

[Out] (2\*b\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*d) + (2\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(3\*d)

**Rubi [A]** time = 0.0407184, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sin(c + dx)(b\sec(c + dx))^{3/2}}{3d} + \frac{2b\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b\sec(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] (2\*b\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*d) + (2\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(3\*d)

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Csc[c + d\*x]\*(b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&



EqQ[n^2, 1/4]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \sec(c + dx)(b \sec(c + dx))^{3/2} dx &= \frac{\int (b \sec(c + dx))^{5/2} dx}{b} \\ &= \frac{2(b \sec(c + dx))^{3/2} \sin(c + dx)}{3d} + \frac{1}{3}b \int \sqrt{b \sec(c + dx)} dx \\ &= \frac{2(b \sec(c + dx))^{3/2} \sin(c + dx)}{3d} + \frac{1}{3} (b\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\ &= \frac{2b\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)\sqrt{b \sec(c + dx)}}{3d} + \frac{2(b \sec(c + dx))^{3/2} \sin(c + dx)}{3d} \end{aligned}$$

**Mathematica [A]** time = 0.0721231, size = 49, normalized size = 0.73

$$\frac{2b\sqrt{b \sec(c + dx)} \left( \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \tan(c + dx) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] (2\*b\*Sqrt[b\*Sec[c + d\*x]]\*(Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2] + Tan[c + d\*x]))/(3\*d)

**Maple [C]** time = 0.146, size = 122, normalized size = 1.8

$$\frac{2(\cos(dx + c) + 1)^2(-1 + \cos(dx + c))}{3d(\sin(dx + c))^3} \left( i\sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-cos(d*x+c)+1)*(b/cos(d*x+c))^(3/2)/sin(d*x+c)^3
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{b \sec(dx + c)} b \sec(dx + c)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b*sec(d*x + c)^2, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(c + dx))^{\frac{3}{2}} \sec(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(3/2)*sec(c + d*x), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")`

[Out] `integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c), x)`

### 3.83 $\int (b \sec(c + dx))^{3/2} dx$

**Optimal.** Leaf size=66

$$\frac{2b \sin(c + dx) \sqrt{b \sec(c + dx)}}{d} - \frac{2b^2 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}}$$

[Out]  $(-2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d$

**Rubi [A]** time = 0.0335228, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3768, 3771, 2639}

$$\frac{2b \sin(c + dx) \sqrt{b \sec(c + dx)}}{d} - \frac{2b^2 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(3/2), x]

[Out]  $(-2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d$

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x] \*(b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - P  
i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned} \int (b \sec(c + dx))^{3/2} dx &= \frac{2b\sqrt{b \sec(c + dx)} \sin(c + dx)}{d} - b^2 \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\ &= \frac{2b\sqrt{b \sec(c + dx)} \sin(c + dx)}{d} - \frac{b^2 \int \sqrt{\cos(c + dx)} dx}{\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} \\ &= -\frac{2b^2 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2b\sqrt{b \sec(c + dx)} \sin(c + dx)}{d} \end{aligned}$$

**Mathematica [A]** time = 0.0370968, size = 48, normalized size = 0.73

$$\frac{2b\sqrt{b \sec(c + dx)} \left( \sin(c + dx) - \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Sec[c + d\*x])^(3/2),x]

[Out] (2\*b\*Sqrt[b\*Sec[c + d\*x]]\*(-(Sqrt[Cos[c + d\*x]]\*EllipticE[(c + d\*x)/2, 2])  
+ Sin[c + d\*x]))/d

**Maple [C]** time = 0.199, size = 320, normalized size = 4.9

$$-2 \frac{(\cos(dx + c) + 1)^2 (-1 + \cos(dx + c))^2 \cos(dx + c)}{d (\sin(dx + c))^5} \left( i \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, I\right) \cos(dx + c) \sin(dx + c) - I \text{EllipticE}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, I\right) \cos(dx + c) \sin(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(3/2),x)

[Out] -2/d\*(cos(d\*x+c)+1)^2\*(-1+cos(d\*x+c))^2\*(I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*cos(d\*x+c)\*sin(d\*x+c)-I\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*cos(d\*x+c)\*sin(d\*x+c))

$d*x+c)*(1/(\cos(d*x+c)+1))^{(1/2)}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}+I*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*(1/(\cos(d*x+c)+1))^{(1/2)}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}*\sin(d*x+c)-I*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{(1/2)}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{(1/2)}+\cos(d*x+c)-1)*\cos(d*x+c)*(b/\cos(d*x+c))^{(3/2)}/\sin(d*x+c)^5$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{b \sec(dx + c)} b \sec(dx + c), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b\*sec(d\*x + c))\*b\*sec(d\*x + c), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(c + dx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(3/2),x)

[Out] Integral((b\*sec(c + d\*x))\*\*(3/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(3/2), x)

$$3.84 \quad \int \cos(c + dx)(b \sec(c + dx))^{3/2} dx$$

**Optimal.** Leaf size=39

$$\frac{2b\sqrt{\cos(c + dx)}\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)\sqrt{b\sec(c + dx)}}{d}$$

[Out] (2\*b\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/d

**Rubi [A]** time = 0.030127, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {16, 3771, 2641}

$$\frac{2b\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)\sqrt{b\sec(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] (2\*b\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/d

### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

### Rubi steps



$$\begin{aligned}
\int \cos(c + dx)(b \sec(c + dx))^{3/2} dx &= b \int \sqrt{b \sec(c + dx)} dx \\
&= (b \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2b \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.0193133, size = 39, normalized size = 1.

$$\frac{2b \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{b \sec(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d\*x]\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] (2\*b\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/d

**Maple [C]** time = 0.136, size = 98, normalized size = 2.5

$$\frac{-2i(-1 + \cos(dx + c))(\cos(dx + c) + 1)^3}{d(\sin(dx + c))^2} \left(\frac{b}{\cos(dx + c)}\right)^{\frac{3}{2}} \left(\frac{\cos(dx + c)}{\cos(dx + c) + 1}\right)^{\frac{3}{2}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)\*(b\*sec(d\*x+c))^(3/2), x)

[Out] -2\*I/d\*(b/cos(d\*x+c))^(3/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(3/2)\*(-1+cos(d\*x+c))\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*(cos(d\*x+c)+1)^3\*(1/(cos(d\*x+c)+1))^(1/2)/sin(d\*x+c)^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec(dx + c)} b \cos(dx + c) \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*b*cos(d*x + c)*sec(d*x + c), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(b*sec(d*x+c))**(3/2),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")`

[Out] `integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c), x)`

$$3.85 \quad \int \cos^2(c + dx)(b \sec(c + dx))^{3/2} dx$$

Optimal. Leaf size=41

$$\frac{2b^2 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

[Out] (2\*b^2\*EllipticE[(c + d\*x)/2, 2])/(d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0378043, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16, 3771, 2639}

$$\frac{2b^2 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^2\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] (2\*b^2\*EllipticE[(c + d\*x)/2, 2])/(d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

#### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3771

Int[(csc[(c\_.) + (d\_)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \cos^2(c + dx)(b \sec(c + dx))^{3/2} dx &= b^2 \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\ &= \frac{b^2 \int \sqrt{\cos(c + dx)} dx}{\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} \\ &= \frac{2b^2 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0235292, size = 41, normalized size = 1.

$$\frac{2b^2 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d\*x]^2\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] (2\*b^2\*EllipticE[(c + d\*x)/2, 2])/(d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

**Maple [C]** time = 0.147, size = 309, normalized size = 7.5

$$2 \frac{\cos(dx + c)}{d \sin(dx + c)} \left( i \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \cos(dx + c) \sin(dx + c) - i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)^2\*(b\*sec(d\*x+c))^(3/2), x)

[Out] 2/d\*(I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*cos(d\*x+c)\*sin(d\*x+c)-I\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*cos(d\*x+c)\*sin(d\*x+c)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)+I\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*sin(d\*x+c)-I\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*sin(d\*x+c)\*(1/(cos(d\*x+c)+1))^(1/2)\*

$\cos(dx+c)/(\cos(dx+c)+1)^{(1/2)} - \cos(dx+c)^2 + \cos(dx+c) * \cos(dx+c) * (b/\cos(dx+c))^{(3/2)}/\sin(dx+c)$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx+c))^{\frac{3}{2}} \cos(dx+c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^2\*(b\*sec(dx+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b\*sec(dx + c))^(3/2)\*cos(dx + c)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec(dx+c)} b \cos(dx+c)^2 \sec(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^2\*(b\*sec(dx+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b\*sec(dx + c))\*b\*cos(dx + c)^2\*sec(dx + c), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)\*\*2\*(b\*sec(dx+c))\*\*(3/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^2, x)
```

### 3.86 $\int \cos^3(c + dx)(b \sec(c + dx))^{3/2} dx$

**Optimal.** Leaf size=70

$$\frac{2b\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2b^2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

[Out] (2\*b\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*d) + (2\*b^2\*Sin[c + d\*x])/(3\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0542509, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2641}

$$\frac{2b^2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}} + \frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^3\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] (2\*b\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*d) + (2\*b^2\*Sin[c + d\*x])/(3\*d\*Sqrt[b\*Sec[c + d\*x]])

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n+1))/(b\*d^n), x] + Dist[(n+1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int \cos^3(c + dx)(b \sec(c + dx))^{3/2} dx &= b^3 \int \frac{1}{(b \sec(c + dx))^{3/2}} dx \\
 &= \frac{2b^2 \sin(c + dx)}{3d \sqrt{b \sec(c + dx)}} + \frac{1}{3} b \int \sqrt{b \sec(c + dx)} dx \\
 &= \frac{2b^2 \sin(c + dx)}{3d \sqrt{b \sec(c + dx)}} + \frac{1}{3} (b \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &= \frac{2b \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{3d} + \frac{2b^2 \sin(c + dx)}{3d \sqrt{b \sec(c + dx)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0493521, size = 52, normalized size = 0.74

$$\frac{b \sqrt{b \sec(c + dx)} \left( 2 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \sin(2(c + dx)) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d\*x]^3\*(b\*Sec[c + d\*x])^(3/2), x]

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + S
in[2*(c + d*x)]))/(3*d)
```

**Maple [C]** time = 0.16, size = 129, normalized size = 1.8

$$-\frac{2(\cos(dx + c) + 1)^2(-1 + \cos(dx + c))\cos(dx + c)}{3d(\sin(dx + c))^3} \left(\frac{b}{\cos(dx + c)}\right)^{\frac{3}{2}} \left(i \operatorname{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \sqrt{\cos(dx + c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] int(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(3/2)*(-1+cos(d*x+c))*cos(d*x+c)*(I*
EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)^2+cos(d*x+c))/sin(d*x+c)^3
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \cos(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^3, x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec(dx + c)} b \cos(dx + c)^3 \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b*cos(d*x + c)^3*sec(d*x + c), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \cos(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^3, x)
```

### 3.87 $\int \cos^4(c + dx)(b \sec(c + dx))^{3/2} dx$

Optimal. Leaf size=72

$$\frac{2b^3 \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}} + \frac{6b^2 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

[Out] (6\*b^2\*EllipticE[(c + d\*x)/2, 2])/(5\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b^3\*Sin[c + d\*x])/(5\*d\*(b\*Sec[c + d\*x])^(3/2))

**Rubi [A]** time = 0.0541705, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2639}

$$\frac{2b^3 \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}} + \frac{6b^2 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^4\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] (6\*b^2\*EllipticE[(c + d\*x)/2, 2])/(5\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b^3\*Sin[c + d\*x])/(5\*d\*(b\*Sec[c + d\*x])^(3/2))

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c + dx)(b \sec(c + dx))^{3/2} dx &= b^4 \int \frac{1}{(b \sec(c + dx))^{5/2}} dx \\
&= \frac{2b^3 \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}} + \frac{1}{5} (3b^2) \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\
&= \frac{2b^3 \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}} + \frac{(3b^2) \int \sqrt{\cos(c + dx)} dx}{5\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} \\
&= \frac{6b^2 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} + \frac{2b^3 \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0491233, size = 58, normalized size = 0.81

$$\frac{b\sqrt{b \sec(c + dx)} \left( \sin(c + dx) + \sin(3(c + dx)) + 12\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{10d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d\*x]^4\*(b\*Sec[c + d\*x])^(3/2), x]

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
Sin[c + d*x] + Sin[3*(c + d*x)]))/(10*d)
```

**Maple [C]** time = 0.196, size = 321, normalized size = 4.5

$$\frac{2 \cos(dx + c)}{5d \sin(dx + c)} \left( 3i \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \cos(dx + c) \sin(dx + c) - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x)`

[Out]  $\frac{2}{5}d \cdot (3I \cdot (1/\cos(dx+c)+1))^{1/2} \cdot (\cos(dx+c)/(\cos(dx+c)+1))^{1/2} \cdot \text{EllipticF}(I \cdot (-1+\cos(dx+c))/\sin(dx+c), I) \cdot \cos(dx+c) \cdot \sin(dx+c) - 3I \cdot \cos(dx+c) \cdot \sin(dx+c) \cdot (1/\cos(dx+c)+1)^{1/2} \cdot (\cos(dx+c)/(\cos(dx+c)+1))^{1/2} \cdot \text{EllipticE}(I \cdot (-1+\cos(dx+c))/\sin(dx+c), I) + 3I \cdot \text{EllipticF}(I \cdot (-1+\cos(dx+c))/\sin(dx+c), I) \cdot (1/\cos(dx+c)+1)^{1/2} \cdot (\cos(dx+c)/(\cos(dx+c)+1))^{1/2} \cdot \sin(dx+c) - 3I \cdot \sin(dx+c) \cdot (1/\cos(dx+c)+1)^{1/2} \cdot (\cos(dx+c)/(\cos(dx+c)+1))^{1/2} \cdot \text{EllipticE}(I \cdot (-1+\cos(dx+c))/\sin(dx+c), I) - \cos(dx+c)^4 - 2 \cdot \cos(dx+c)^2 + 3 \cdot \cos(dx+c) \cdot \cos(dx+c) \cdot (b/\cos(dx+c))^{3/2} / \sin(dx+c)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx+c))^{\frac{3}{2}} \cos(dx+c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^4, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{b \sec(dx+c)} b \cos(dx+c)^4 \sec(dx+c), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*b*cos(d*x + c)^4*sec(d*x + c), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \cos(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^4, x)
```

### 3.88 $\int \cos^5(c + dx)(b \sec(c + dx))^{3/2} dx$

**Optimal.** Leaf size=98

$$\frac{10b\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{2b^4 \sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b^2 \sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}$$

[Out] (10\*b\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*d) + (2\*b^4\*Sin[c + d\*x])/(7\*d\*(b\*Sec[c + d\*x])^(5/2)) + (10\*b^2\*Sin[c + d\*x])/(21\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0730224, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2641}

$$\frac{2b^4 \sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b^2 \sin(c+dx)}{21d\sqrt{b\sec(c+dx)}} + \frac{10b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^5\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] (10\*b\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*d) + (2\*b^4\*Sin[c + d\*x])/(7\*d\*(b\*Sec[c + d\*x])^(5/2)) + (10\*b^2\*Sin[c + d\*x])/(21\*d\*Sqrt[b\*Sec[c + d\*x]])

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n+1))/(b\*d\*n), x] + Dist[(n+1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \cos^5(c + dx)(b \sec(c + dx))^{3/2} dx &= b^5 \int \frac{1}{(b \sec(c + dx))^{7/2}} dx \\
&= \frac{2b^4 \sin(c + dx)}{7d(b \sec(c + dx))^{5/2}} + \frac{1}{7} (5b^3) \int \frac{1}{(b \sec(c + dx))^{3/2}} dx \\
&= \frac{2b^4 \sin(c + dx)}{7d(b \sec(c + dx))^{5/2}} + \frac{10b^2 \sin(c + dx)}{21d\sqrt{b \sec(c + dx)}} + \frac{1}{21} (5b) \int \sqrt{b \sec(c + dx)} dx \\
&= \frac{2b^4 \sin(c + dx)}{7d(b \sec(c + dx))^{5/2}} + \frac{10b^2 \sin(c + dx)}{21d\sqrt{b \sec(c + dx)}} + \frac{1}{21} (5b\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}) \int \\
&= \frac{10b\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)\sqrt{b \sec(c + dx)}}{21d} + \frac{2b^4 \sin(c + dx)}{7d(b \sec(c + dx))^{5/2}} + \frac{10b^2 \sin(c + dx)}{21d\sqrt{b \sec(c + dx)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0779546, size = 64, normalized size = 0.65

$$\frac{b\sqrt{b \sec(c + dx)} \left( 40\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 26 \sin(2(c + dx)) + 3 \sin(4(c + dx)) \right)}{84d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(b*Sec[c + d*x])^(3/2), x]
```

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
26*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*d)
```

**Maple [C]** time = 0.184, size = 151, normalized size = 1.5

$$-\frac{2(\cos(dx + c) + 1)^2(-1 + \cos(dx + c))\cos(dx + c)}{21d(\sin(dx + c))^3} \left( \frac{b}{\cos(dx + c)} \right)^{\frac{3}{2}} \left( 5i \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \sqrt{(\cos(dx + c) + 1)^2} \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x)`

[Out] 
$$-2/21/d*(\cos(dx+c)+1)^2*(b/\cos(dx+c))^{3/2}*(-1+\cos(dx+c))*\cos(dx+c)*(5*I*EllipticF(I*(-1+\cos(dx+c))/\sin(dx+c),I)*(1/(\cos(dx+c)+1))^{1/2}*(\cos(dx+c)/(\cos(dx+c)+1))^{1/2}*\sin(dx+c)-3*\cos(dx+c)^4+3*\cos(dx+c)^3-5*\cos(dx+c)^2+5*\cos(dx+c))/\sin(dx+c)^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \cos(dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^5, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec(dx + c)} b \cos(dx + c)^5 \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*b*cos(d*x + c)^5*sec(d*x + c), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**5*(b*sec(d*x+c))**(3/2),x)`

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x, algorithm="giac")`

[Out] Timed out

### 3.89 $\int \cos^6(c + dx)(b \sec(c + dx))^{3/2} dx$

Optimal. Leaf size=100

$$\frac{2b^5 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14b^3 \sin(c + dx)}{45d(b \sec(c + dx))^{3/2}} + \frac{14b^2 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

[Out] (14\*b^2\*EllipticE[(c + d\*x)/2, 2])/(15\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b^5\*Sin[c + d\*x])/(9\*d\*(b\*Sec[c + d\*x])^(7/2)) + (14\*b^3\*Sin[c + d\*x])/(45\*d\*(b\*Sec[c + d\*x])^(3/2))

**Rubi [A]** time = 0.0747851, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2639}

$$\frac{2b^5 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14b^3 \sin(c + dx)}{45d(b \sec(c + dx))^{3/2}} + \frac{14b^2 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^6\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] (14\*b^2\*EllipticE[(c + d\*x)/2, 2])/(15\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b^5\*Sin[c + d\*x])/(9\*d\*(b\*Sec[c + d\*x])^(7/2)) + (14\*b^3\*Sin[c + d\*x])/(45\*d\*(b\*Sec[c + d\*x])^(3/2))

#### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \cos^6(c + dx)(b \sec(c + dx))^{3/2} dx &= b^6 \int \frac{1}{(b \sec(c + dx))^{9/2}} dx \\
&= \frac{2b^5 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{1}{9} (7b^4) \int \frac{1}{(b \sec(c + dx))^{5/2}} dx \\
&= \frac{2b^5 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14b^3 \sin(c + dx)}{45d(b \sec(c + dx))^{3/2}} + \frac{1}{15} (7b^2) \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\
&= \frac{2b^5 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14b^3 \sin(c + dx)}{45d(b \sec(c + dx))^{3/2}} + \frac{(7b^2) \int \sqrt{\cos(c + dx)} dx}{15\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} \\
&= \frac{14b^2 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} + \frac{2b^5 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14b^3 \sin(c + dx)}{45d(b \sec(c + dx))^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.160356, size = 72, normalized size = 0.72

$$\frac{b\sqrt{b \sec(c + dx)} \left( (33 \sin(c + dx) + 5 \sin(3(c + dx))) \cos^2(c + dx) + 84\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{90d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(b*Sec[c + d*x])^(3/2), x]
```

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*(84*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
Cos[c + d*x]^2*(33*Sin[c + d*x] + 5*Sin[3*(c + d*x)])))/(90*d)
```

**Maple [C]** time = 0.182, size = 331, normalized size = 3.3

$$-\frac{2 \cos(dx + c)}{45 d \sin(dx + c)} \left( 21 i \cos(dx + c) \sin(dx + c) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticE}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}\right), \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x)`

[Out] 
$$-2/45/d*(21*I*\cos(d*x+c)*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*EllipticE(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-21*I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*EllipticF(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)*\sin(d*x+c)+5*\cos(d*x+c)^6+21*I*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*EllipticE(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-21*I*EllipticF(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\sin(d*x+c)+2*\cos(d*x+c)^4+14*\cos(d*x+c)^2-21*\cos(d*x+c))*\cos(d*x+c)*(b/\cos(d*x+c))^{3/2}/\sin(d*x+c)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \cos(dx + c)^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^6, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec(dx + c)} b \cos(dx + c)^6 \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*b*cos(d*x + c)^6*sec(d*x + c), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**6*(b*sec(d*x+c))**(3/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \cos(dx + c)^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x, algorithm="giac")`

[Out] `integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^6, x)`

### 3.90 $\int \sec^2(c + dx)(b \sec(c + dx))^{5/2} dx$

**Optimal.** Leaf size=98

$$\frac{10b^2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7bd} + \frac{10b\sin(c+dx)(b\sec(c+dx))^{3/2}}{21d}$$

[Out] (10\*b^2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*d) + (10\*b\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(21\*d) + (2\*(b\*Sec[c + d\*x])^(7/2)\*Sin[c + d\*x])/(7\*b\*d)

**Rubi [A]** time = 0.0588525, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2641}

$$\frac{10b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7bd} + \frac{10b\sin(c+dx)(b\sec(c+dx))^{3/2}}{21d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^2\*(b\*Sec[c + d\*x])^(5/2), x]

[Out] (10\*b^2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*d) + (10\*b\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(21\*d) + (2\*(b\*Sec[c + d\*x])^(7/2)\*Sin[c + d\*x])/(7\*b\*d)

#### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3768

Int[(csc[(c\_.) + (d\_)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] :> -Simp[(b\*Csc[c + d\*x]\*(b\*Csc[c + d\*x])^(n-1))/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*Csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
 \int \sec^2(c + dx)(b \sec(c + dx))^{5/2} dx &= \frac{\int (b \sec(c + dx))^{9/2} dx}{b^2} \\
 &= \frac{2(b \sec(c + dx))^{7/2} \sin(c + dx)}{7bd} + \frac{5}{7} \int (b \sec(c + dx))^{5/2} dx \\
 &= \frac{10b(b \sec(c + dx))^{3/2} \sin(c + dx)}{21d} + \frac{2(b \sec(c + dx))^{7/2} \sin(c + dx)}{7bd} + \frac{1}{21} (5b^2) \int \sqrt{\cos(c + dx)} dx \\
 &= \frac{10b(b \sec(c + dx))^{3/2} \sin(c + dx)}{21d} + \frac{2(b \sec(c + dx))^{7/2} \sin(c + dx)}{7bd} + \frac{1}{21} (5b^2 \sqrt{\cos(c + dx)}) \\
 &= \frac{10b^2 \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{21d} + \frac{10b(b \sec(c + dx))^{3/2} \sin(c + dx)}{21d}
 \end{aligned}$$

**Mathematica [A]** time = 0.179666, size = 61, normalized size = 0.62

$$\frac{(b \sec(c + dx))^{5/2} \left( 10 \cos^{\frac{5}{2}}(c + dx) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 5 \sin(2(c + dx)) + 6 \tan(c + dx) \right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(5/2), x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

**Maple [C]** time = 0.191, size = 152, normalized size = 1.6

$$\frac{2 (\cos(dx + c) + 1)^2 (-1 + \cos(dx + c))}{21 d \cos(dx + c) (\sin(dx + c))^3} \left( 5 i (\cos(dx + c))^3 \sin(dx + c) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 5 \sin(2(c + dx)) + 6 \tan(c + dx) \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x)`

[Out] 
$$-2/21/d*(\cos(d*x+c)+1)^2*(-1+\cos(d*x+c))*(5*I*\cos(d*x+c)^3*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*EllipticF(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-5*\cos(d*x+c)^3+5*\cos(d*x+c)^2-3*\cos(d*x+c)+3)*(b/\cos(d*x+c))^{5/2}/\cos(d*x+c)/\sin(d*x+c)^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c)^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{b \sec(dx + c)} b^2 \sec(dx + c)^4, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*b^2*sec(d*x + c)^4, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(5/2),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^2\*(b\*sec(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(5/2)\*sec(d\*x + c)^2, x)

### 3.91 $\int \sec(c + dx)(b \sec(c + dx))^{5/2} dx$

**Optimal.** Leaf size=97

$$\frac{6b^2 \sin(c + dx) \sqrt{b \sec(c + dx)}}{5d} - \frac{6b^3 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2 \sin(c + dx) (b \sec(c + dx))^{5/2}}{5d}$$

[Out]  $(-6*b^3*EllipticE[(c + d*x)/2, 2])/(5*d*sqrt[Cos[c + d*x]]*sqrt[b*Sec[c + d*x]]) + (6*b^2*sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d)$

**Rubi [A]** time = 0.0567826, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {16, 3768, 3771, 2639}

$$\frac{6b^2 \sin(c + dx) \sqrt{b \sec(c + dx)}}{5d} - \frac{6b^3 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2 \sin(c + dx) (b \sec(c + dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sec}[c + d*x]*(b*\text{Sec}[c + d*x])^{5/2}, x]$

[Out]  $(-6*b^3*EllipticE[(c + d*x)/2, 2])/(5*d*sqrt[Cos[c + d*x]]*sqrt[b*Sec[c + d*x]]) + (6*b^2*sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d)$

#### Rule 16

$\text{Int}[(u_*)*(v_)^{(m_*)}*((b_)*(v_))^{(n_)}, x\_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /;$  FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3768

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_))^{(n_)}, x\_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Csc}[c + d*x])^{(n-1)})/(d*(n-1)), x] + \text{Dist}[(b^2*(n-2))/(n-1), \text{Int}[(b*\text{Csc}[c + d*x])^{(n-2)}, x], x] /;$  FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \sec(c + dx)(b \sec(c + dx))^{5/2} dx &= \frac{\int (b \sec(c + dx))^{7/2} dx}{b} \\
&= \frac{2(b \sec(c + dx))^{5/2} \sin(c + dx)}{5d} + \frac{1}{5}(3b) \int (b \sec(c + dx))^{3/2} dx \\
&= \frac{6b^2 \sqrt{b \sec(c + dx)} \sin(c + dx)}{5d} + \frac{2(b \sec(c + dx))^{5/2} \sin(c + dx)}{5d} - \frac{1}{5}(3b^3) \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\
&= \frac{6b^2 \sqrt{b \sec(c + dx)} \sin(c + dx)}{5d} + \frac{2(b \sec(c + dx))^{5/2} \sin(c + dx)}{5d} - \frac{(3b^3) \int \sqrt{\cos(c + dx)}}{5\sqrt{\cos(c + dx)}\sqrt{b}} \\
&= -\frac{6b^3 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} + \frac{6b^2 \sqrt{b \sec(c + dx)} \sin(c + dx)}{5d} + \frac{2(b \sec(c + dx))^{5/2} \sin(c + dx)}{5d}
\end{aligned}$$

**Mathematica [A]** time = 0.164128, size = 61, normalized size = 0.63

$$\frac{(b \sec(c + dx))^{5/2} \left( 7 \sin(c + dx) + 3 \sin(3(c + dx)) - 12 \cos^2(c + dx) E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(5/2), x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] +
7*Sin[c + d*x] + 3*Sin[3*(c + d*x)])/(10*d)
```

**Maple [C]** time = 0.204, size = 348, normalized size = 3.6

$$\frac{2 (\cos(dx + c) + 1)^2 (-1 + \cos(dx + c))^2}{5d (\sin(dx + c))^5} \left( 3i \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} (\cos(dx + c))^3 \text{EllipticE}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x)`

[Out]  $2/5/d*(\cos(d*x+c)+1)^2*(-1+\cos(d*x+c))^2*(3*I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\cos(d*x+c)^3*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)-3*I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)^3*\sin(d*x+c)+3*I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\cos(d*x+c)^2*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)-3*I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)^2*\sin(d*x+c)-3*\cos(d*x+c)^3+2*\cos(d*x+c)^2+1)*(b/\cos(d*x+c))^{5/2}/\sin(d*x+c)^5$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{b \sec(dx + c)} b^2 \sec(dx + c)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*b^2*sec(d*x + c)^3, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(b*sec(d*x+c))**(5/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")`

[Out] `integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c), x)`

### 3.92 $\int (b \sec(c + dx))^{5/2} dx$

**Optimal.** Leaf size=70

$$\frac{2b^2 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{b \sec(c + dx)}}{3d} + \frac{2b \sin(c + dx) (b \sec(c + dx))^{3/2}}{3d}$$

[Out] (2\*b^2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*d) + (2\*b\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(3\*d)

**Rubi [A]** time = 0.0323627, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3768, 3771, 2641}

$$\frac{2b^2 \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{3d} + \frac{2b \sin(c + dx) (b \sec(c + dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(5/2), x]

[Out] (2\*b^2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*d) + (2\*b\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(3\*d)

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Csc[c + d\*x])\*(b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n \* Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (b \sec(c + dx))^{5/2} dx &= \frac{2b(b \sec(c + dx))^{3/2} \sin(c + dx)}{3d} + \frac{1}{3} b^2 \int \sqrt{b \sec(c + dx)} dx \\
&= \frac{2b(b \sec(c + dx))^{3/2} \sin(c + dx)}{3d} + \frac{1}{3} (b^2 \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2b^2 \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{3d} + \frac{2b(b \sec(c + dx))^{3/2} \sin(c + dx)}{3d}
\end{aligned}$$

**Mathematica [A]** time = 0.0169936, size = 51, normalized size = 0.73

$$\frac{2b^2 \sqrt{b \sec(c + dx)} \left( \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \tan(c + dx) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Sec[c + d\*x])^(5/2), x]

[Out] (2\*b^2\*Sqrt[b\*Sec[c + d\*x]]\*(Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2] + Tan[c + d\*x]))/(3\*d)

**Maple [C]** time = 0.159, size = 128, normalized size = 1.8

$$-\frac{(-2 + 2 \cos(dx + c)) \cos(dx + c) (\cos(dx + c) + 1)^2}{3d (\sin(dx + c))^3} \left( i \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, 2\right) + \tan(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(5/2), x)

[Out] -2/3/d\*(-1+cos(d\*x+c))\*(I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*cos(d\*x+c)\*sin(d\*x+c)-cos(d\*x+c)+1)\*cos(d\*x+c)\*(cos(d\*x+c)+1)^2\*(b/cos(d\*x+c))^(5/2)/sin(d\*x+c)^3



**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^(5/2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec(dx + c)} b^2 \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b\*sec(d\*x + c))\*b^2\*sec(d\*x + c)^2, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(5/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2), x)
```

### 3.93 $\int \cos(c + dx)(b \sec(c + dx))^{5/2} dx$

**Optimal.** Leaf size=68

$$\frac{2b^2 \sin(c + dx) \sqrt{b \sec(c + dx)}}{d} - \frac{2b^3 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}}$$

[Out]  $(-2*b^3*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*b^2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d$

**Rubi [A]** time = 0.0445958, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {16, 3768, 3771, 2639}

$$\frac{2b^2 \sin(c + dx) \sqrt{b \sec(c + dx)}}{d} - \frac{2b^3 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cos}[c + d*x]*(b*\text{Sec}[c + d*x])^{(5/2)}, x]$

[Out]  $(-2*b^3*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*b^2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d$

#### Rule 16

$\text{Int}[(u_.)*(v_)^{(m_.)}*((b_.)*(v_))^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n, x\} \ \&\& \ \text{IntegerQ}[m]$

#### Rule 3768

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_)]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Csc}[c + d*x])^{(n-1)}]/(d*(n-1)), x] + \text{Dist}[(b^2*(n-2))/(n-1), \text{Int}[(b*\text{Csc}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d, x\} \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

#### Rule 3771

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_)]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^{(n)}*\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}\{b, c, d, x\} \ \&\&$

EqQ[n^2, 1/4]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos(c + dx)(b \sec(c + dx))^{5/2} dx &= b \int (b \sec(c + dx))^{3/2} dx \\
&= \frac{2b^2 \sqrt{b \sec(c + dx)} \sin(c + dx)}{d} - b^3 \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\
&= \frac{2b^2 \sqrt{b \sec(c + dx)} \sin(c + dx)}{d} - \frac{b^3 \int \sqrt{\cos(c + dx)} dx}{\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} \\
&= -\frac{2b^3 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2b^2 \sqrt{b \sec(c + dx)} \sin(c + dx)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.0312653, size = 50, normalized size = 0.74

$$\frac{2b^2 \sqrt{b \sec(c + dx)} \left( \sin(c + dx) - \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d\*x]\*(b\*Sec[c + d\*x])^(5/2),x]

```
[Out] (2*b^2*Sqrt[b*Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
) + Sin[c + d*x]))/d
```

**Maple [C]** time = 0.218, size = 324, normalized size = 4.8

$$2 \frac{(\cos(dx + c) + 1)^2 (-1 + \cos(dx + c))^2 (\cos(dx + c))^2}{d (\sin(dx + c))^5} \left( i \cos(dx + c) \sin(dx + c) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x)`

[Out]  $2/d*(\cos(d*x+c)+1)^2*(-1+\cos(d*x+c))^{2*(I*\cos(d*x+c)*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*EllipticE(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2})^{1/2}*EllipticF(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)*\sin(d*x+c)+I*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*EllipticE(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-I*EllipticF(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\sin(d*x+c)-\cos(d*x+c)+1)*\cos(d*x+c)^2*(b/\cos(d*x+c))^{5/2}/\sin(d*x+c)^5$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec(dx + c)} b^2 \cos(dx + c) \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*b^2*cos(d*x + c)*sec(d*x + c)^2, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c), x)
```

### 3.94 $\int \cos^2(c + dx)(b \sec(c + dx))^{5/2} dx$

**Optimal.** Leaf size=41

$$\frac{2b^2 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{b \sec(c + dx)}}{d}$$

[Out]  $(2*b^2*\sqrt{\cos[c + d*x]}*\operatorname{EllipticF}[(c + d*x)/2, 2]*\sqrt{b*\sec[c + d*x]})/d$

**Rubi [A]** time = 0.0379056, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16, 3771, 2641}

$$\frac{2b^2 \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\cos[c + d*x]^2*(b*\sec[c + d*x])^{5/2}, x]$

[Out]  $(2*b^2*\sqrt{\cos[c + d*x]}*\operatorname{EllipticF}[(c + d*x)/2, 2]*\sqrt{b*\sec[c + d*x]})/d$

#### Rule 16

$\operatorname{Int}[(u_*)*(v_)^{(m_*)}*((b_*)*(v_))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[1/b^m, \operatorname{Int}[u*(b*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{b, n, x\} \ \&\& \ \operatorname{IntegerQ}[m]$

#### Rule 3771

$\operatorname{Int}[(\operatorname{csc}[c_*) + (d_*)*(x_*)]^{(b_*)}*(v_))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[(b*\operatorname{Csc}[c + d*x])^n*\operatorname{Sin}[c + d*x]^n, \operatorname{Int}[1/\operatorname{Sin}[c + d*x]^n, x], x] /; \operatorname{FreeQ}\{b, c, d, x\} \ \&\& \ \operatorname{EqQ}[n^2, 1/4]$

#### Rule 2641

$\operatorname{Int}[1/\sqrt{\sin[(c_*) + (d_*)*(x_*)]}, x\_Symbol] \rightarrow \operatorname{Simp}[(2*\operatorname{EllipticF}[(1*(c - \operatorname{Pi}/2 + d*x))/2, 2])/d, x] /; \operatorname{FreeQ}\{c, d, x\}$

#### Rubi steps

$$\begin{aligned}
\int \cos^2(c + dx)(b \sec(c + dx))^{5/2} dx &= b^2 \int \sqrt{b \sec(c + dx)} dx \\
&= (b^2 \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2b^2 \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.0159645, size = 41, normalized size = 1.

$$\frac{2b^2 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{b \sec(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d\*x]^2\*(b\*Sec[c + d\*x])^(5/2), x]

[Out] (2\*b^2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/d

**Maple [C]** time = 0.153, size = 98, normalized size = 2.4

$$\frac{-2i(-1 + \cos(dx + c))(\cos(dx + c) + 1)^5}{d(\sin(dx + c))^2} \left(\frac{b}{\cos(dx + c)}\right)^{\frac{5}{2}} \left(\frac{\cos(dx + c)}{\cos(dx + c) + 1}\right)^{\frac{5}{2}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) ((\cos(dx + c) + 1))^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)^2\*(b\*sec(d\*x+c))^(5/2), x)

[Out] -2\*I/d\*(b/cos(d\*x+c))^(5/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(5/2)\*(-1+cos(d\*x+c))\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*(cos(d\*x+c)+1)^5\*(1/(cos(d\*x+c)+1))^(3/2)/sin(d\*x+c)^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \cos(dx + c)^2 dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^2, x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec(dx + c)} b^2 \cos(dx + c)^2 \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*b^2*cos(d*x + c)^2*sec(d*x + c)^2, x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(5/2),x)`

[Out] Timed out

**Giac** [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="giac")`

[Out] `integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^2, x)`

### 3.95 $\int \cos^3(c + dx)(b \sec(c + dx))^{5/2} dx$

Optimal. Leaf size=41

$$\frac{2b^3 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

[Out] (2\*b^3\*EllipticE[(c + d\*x)/2, 2])/(d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0367042, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16, 3771, 2639}

$$\frac{2b^3 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^3\*(b\*Sec[c + d\*x])^(5/2), x]

[Out] (2\*b^3\*EllipticE[(c + d\*x)/2, 2])/(d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \cos^3(c + dx)(b \sec(c + dx))^{5/2} dx &= b^3 \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\ &= \frac{b^3 \int \sqrt{\cos(c + dx)} dx}{\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} \\ &= \frac{2b^3 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0433608, size = 38, normalized size = 0.93

$$\frac{2 \cos^{\frac{5}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \middle| 2\right) (b \sec(c + dx))^{5/2}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d\*x]^3\*(b\*Sec[c + d\*x])^(5/2), x]

[Out] (2\*Cos[c + d\*x]^(5/2)\*EllipticE[(c + d\*x)/2, 2]\*(b\*Sec[c + d\*x])^(5/2))/d

**Maple [C]** time = 0.158, size = 311, normalized size = 7.6

$$2 \frac{(\cos(dx + c))^2}{d \sin(dx + c)} \left( i \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \cos(dx + c) \sin(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)^3\*(b\*sec(d\*x+c))^(5/2), x)

[Out] 2/d\*(I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*cos(d\*x+c)\*sin(d\*x+c)-I\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*cos(d\*x+c)\*sin(d\*x+c)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)+I\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*sin(d\*x+c)-I\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*sin(d\*x+c)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)-cos(d\*x+c)^2+cos(d\*x+c))\*cos(d\*x+c)^2\*(b/c

$\cos(dx+c)^{5/2}/\sin(dx+c)$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx+c))^{\frac{5}{2}} \cos(dx+c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^3\*(b\*sec(dx+c))^(5/2),x, algorithm="maxima")

[Out] integrate((b\*sec(dx + c))^(5/2)\*cos(dx + c)^3, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{b \sec(dx+c)} b^2 \cos(dx+c)^3 \sec(dx+c)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^3\*(b\*sec(dx+c))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b\*sec(dx + c))\*b^2\*cos(dx + c)^3\*sec(dx + c)^2, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)\*\*3\*(b\*sec(dx+c))\*\*(5/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx+c))^{\frac{5}{2}} \cos(dx+c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^3, x)
```

### 3.96 $\int \cos^4(c + dx)(b \sec(c + dx))^{5/2} dx$

**Optimal.** Leaf size=72

$$\frac{2b^2 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{b \sec(c + dx)}}{3d} + \frac{2b^3 \sin(c + dx)}{3d \sqrt{b \sec(c + dx)}}$$

[Out] (2\*b^2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*d) + (2\*b^3\*Sin[c + d\*x])/(3\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0546094, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2641}

$$\frac{2b^3 \sin(c + dx)}{3d \sqrt{b \sec(c + dx)}} + \frac{2b^2 \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^4\*(b\*Sec[c + d\*x])^(5/2), x]

[Out] (2\*b^2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*d) + (2\*b^3\*Sin[c + d\*x])/(3\*d\*Sqrt[b\*Sec[c + d\*x]])

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int \cos^4(c + dx)(b \sec(c + dx))^{5/2} dx &= b^4 \int \frac{1}{(b \sec(c + dx))^{3/2}} dx \\
 &= \frac{2b^3 \sin(c + dx)}{3d \sqrt{b \sec(c + dx)}} + \frac{1}{3} b^2 \int \sqrt{b \sec(c + dx)} dx \\
 &= \frac{2b^3 \sin(c + dx)}{3d \sqrt{b \sec(c + dx)}} + \frac{1}{3} (b^2 \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &= \frac{2b^2 \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{3d} + \frac{2b^3 \sin(c + dx)}{3d \sqrt{b \sec(c + dx)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0494816, size = 54, normalized size = 0.75

$$\frac{b^2 \sqrt{b \sec(c + dx)} \left( 2 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \sin(2(c + dx)) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d\*x]^4\*(b\*Sec[c + d\*x])^(5/2), x]

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)]))/(3*d)
```

**Maple [C]** time = 0.167, size = 131, normalized size = 1.8

$$\frac{2 (\cos(dx + c) + 1)^2 (-1 + \cos(dx + c)) (\cos(dx + c))^2}{3d (\sin(dx + c))^3} \left( \frac{b}{\cos(dx + c)} \right)^{\frac{5}{2}} \left( i \operatorname{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \sqrt{\cos(dx + c)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x)`

[Out] 
$$-2/3/d*(\cos(dx+c)+1)^2*(b/\cos(dx+c))^{5/2}*(-1+\cos(dx+c))*\cos(dx+c)^2*(I*\text{EllipticF}(I*(-1+\cos(dx+c))/\sin(dx+c),I)*(1/(\cos(dx+c)+1))^{1/2}*(\cos(dx+c)/(\cos(dx+c)+1))^{1/2}*\sin(dx+c)-\cos(dx+c)^2+\cos(dx+c))/\sin(dx+c)^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx+c))^{\frac{5}{2}} \cos(dx+c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^4, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{b \sec(dx+c)} b^2 \cos(dx+c)^4 \sec(dx+c)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*b^2*cos(d*x + c)^4*sec(d*x + c)^2, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**4*(b*sec(d*x+c))**(5/2),x)`



[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \cos(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^4\*(b\*sec(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(5/2)\*cos(d\*x + c)^4, x)

### 3.97 $\int \cos^5(c + dx)(b \sec(c + dx))^{5/2} dx$

**Optimal.** Leaf size=72

$$\frac{2b^4 \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}} + \frac{6b^3 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

[Out] (6\*b^3\*EllipticE[(c + d\*x)/2, 2])/(5\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b^4\*Sin[c + d\*x])/(5\*d\*(b\*Sec[c + d\*x])^(3/2))

**Rubi [A]** time = 0.0537524, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2639}

$$\frac{2b^4 \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}} + \frac{6b^3 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^5\*(b\*Sec[c + d\*x])^(5/2), x]

[Out] (6\*b^3\*EllipticE[(c + d\*x)/2, 2])/(5\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b^4\*Sin[c + d\*x])/(5\*d\*(b\*Sec[c + d\*x])^(3/2))

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(c + dx)(b \sec(c + dx))^{5/2} dx &= b^5 \int \frac{1}{(b \sec(c + dx))^{5/2}} dx \\
&= \frac{2b^4 \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}} + \frac{1}{5} (3b^3) \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\
&= \frac{2b^4 \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}} + \frac{(3b^3) \int \sqrt{\cos(c + dx)} dx}{5\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} \\
&= \frac{6b^3 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} + \frac{2b^4 \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0513129, size = 60, normalized size = 0.83

$$\frac{b^2 \sqrt{b \sec(c + dx)} \left( \sin(c + dx) + \sin(3(c + dx)) + 12 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{10d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d\*x]^5\*(b\*Sec[c + d\*x])^(5/2), x]

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
+ Sin[c + d*x] + Sin[3*(c + d*x)]))/(10*d)
```

**Maple [C]** time = 0.201, size = 321, normalized size = 4.5

$$-\frac{2(\cos(dx + c))^2}{5d \sin(dx + c)} \left( \frac{b}{\cos(dx + c)} \right)^{\frac{5}{2}} \left( 3i \cos(dx + c) \sin(dx + c) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticE}\left(\frac{i(-}{\right)}
\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/5/d*(b/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(3*I*cos(d*x+c)*sin(d*x+c)*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x
+c))/sin(d*x+c),I)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)+3*I*
sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellip
ticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+
c)+cos(d*x+c)^4+2*cos(d*x+c)^2-3*cos(d*x+c))/sin(d*x+c)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \cos(dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^5, x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec(dx + c)} b^2 \cos(dx + c)^5 \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b^2*cos(d*x + c)^5*sec(d*x + c)^2, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \cos(dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^5, x)
```

### 3.98 $\int \cos^6(c + dx)(b \sec(c + dx))^{5/2} dx$

**Optimal.** Leaf size=100

$$\frac{10b^2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{2b^5\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b^3\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}$$

[Out] (10\*b^2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*d) + (2\*b^5\*Sin[c + d\*x])/(7\*d\*(b\*Sec[c + d\*x])^(5/2)) + (10\*b^3\*Sin[c + d\*x])/(21\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0714372, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2641}

$$\frac{2b^5\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b^3\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}} + \frac{10b^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^6\*(b\*Sec[c + d\*x])^(5/2), x]

[Out] (10\*b^2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*d) + (2\*b^5\*Sin[c + d\*x])/(7\*d\*(b\*Sec[c + d\*x])^(5/2)) + (10\*b^3\*Sin[c + d\*x])/(21\*d\*Sqrt[b\*Sec[c + d\*x]])

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n+1))/(b\*d\*n), x] + Dist[(n+1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \cos^6(c + dx)(b \sec(c + dx))^{5/2} dx &= b^6 \int \frac{1}{(b \sec(c + dx))^{7/2}} dx \\
&= \frac{2b^5 \sin(c + dx)}{7d(b \sec(c + dx))^{5/2}} + \frac{1}{7} (5b^4) \int \frac{1}{(b \sec(c + dx))^{3/2}} dx \\
&= \frac{2b^5 \sin(c + dx)}{7d(b \sec(c + dx))^{5/2}} + \frac{10b^3 \sin(c + dx)}{21d\sqrt{b \sec(c + dx)}} + \frac{1}{21} (5b^2) \int \sqrt{b \sec(c + dx)} dx \\
&= \frac{2b^5 \sin(c + dx)}{7d(b \sec(c + dx))^{5/2}} + \frac{10b^3 \sin(c + dx)}{21d\sqrt{b \sec(c + dx)}} + \frac{1}{21} (5b^2 \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}) \\
&= \frac{10b^2 \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{21d} + \frac{2b^5 \sin(c + dx)}{7d(b \sec(c + dx))^{5/2}} + \frac{10b^3}{21d\sqrt{b \sec(c + dx)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0732741, size = 66, normalized size = 0.66

$$\frac{b^2 \sqrt{b \sec(c + dx)} \left( 40 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 26 \sin(2(c + dx)) + 3 \sin(4(c + dx)) \right)}{84d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(b*Sec[c + d*x])^(5/2), x]
```

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
+ 26*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*d)
```

**Maple [C]** time = 0.188, size = 153, normalized size = 1.5

$$\frac{2 (\cos(dx + c) + 1)^2 (-1 + \cos(dx + c)) (\cos(dx + c))^2}{21 d (\sin(dx + c))^3} \left( \frac{b}{\cos(dx + c)} \right)^{\frac{5}{2}} \left( -5 i \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} E
\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x)`

[Out]  $2/21/d*(\cos(dx+c)+1)^2*(b/\cos(dx+c))^{5/2}*(-1+\cos(dx+c))*\cos(dx+c)^2*(-5*I*(1/(\cos(dx+c)+1))^{1/2}*(\cos(dx+c)/(\cos(dx+c)+1))^{1/2}*EllipticF(I*(-1+\cos(dx+c))/\sin(dx+c),I)*\sin(dx+c)+3*\cos(dx+c)^4-3*\cos(dx+c)^3+5*\cos(dx+c)^2-5*\cos(dx+c))/\sin(dx+c)^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx+c))^{\frac{5}{2}} \cos(dx+c)^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^6, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{b \sec(dx+c)} b^2 \cos(dx+c)^6 \sec(dx+c)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*b^2*cos(d*x + c)^6*sec(d*x + c)^2, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**6*(b*sec(d*x+c))**(5/2),x)`



[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \cos(dx + c)^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^6\*(b\*sec(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(5/2)\*cos(d\*x + c)^6, x)

### 3.99 $\int \cos^7(c + dx)(b \sec(c + dx))^{5/2} dx$

**Optimal.** Leaf size=100

$$\frac{2b^6 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14b^4 \sin(c + dx)}{45d(b \sec(c + dx))^{3/2}} + \frac{14b^3 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

[Out] (14\*b^3\*EllipticE[(c + d\*x)/2, 2])/(15\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b^6\*Sin[c + d\*x])/(9\*d\*(b\*Sec[c + d\*x])^(7/2)) + (14\*b^4\*Sin[c + d\*x])/(45\*d\*(b\*Sec[c + d\*x])^(3/2))

**Rubi [A]** time = 0.0740449, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2639}

$$\frac{2b^6 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14b^4 \sin(c + dx)}{45d(b \sec(c + dx))^{3/2}} + \frac{14b^3 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^7\*(b\*Sec[c + d\*x])^(5/2), x]

[Out] (14\*b^3\*EllipticE[(c + d\*x)/2, 2])/(15\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b^6\*Sin[c + d\*x])/(9\*d\*(b\*Sec[c + d\*x])^(7/2)) + (14\*b^4\*Sin[c + d\*x])/(45\*d\*(b\*Sec[c + d\*x])^(3/2))

#### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \cos^7(c + dx)(b \sec(c + dx))^{5/2} dx &= b^7 \int \frac{1}{(b \sec(c + dx))^{9/2}} dx \\
&= \frac{2b^6 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{1}{9} (7b^5) \int \frac{1}{(b \sec(c + dx))^{5/2}} dx \\
&= \frac{2b^6 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14b^4 \sin(c + dx)}{45d(b \sec(c + dx))^{3/2}} + \frac{1}{15} (7b^3) \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\
&= \frac{2b^6 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14b^4 \sin(c + dx)}{45d(b \sec(c + dx))^{3/2}} + \frac{(7b^3) \int \sqrt{\cos(c + dx)} dx}{15\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} \\
&= \frac{14b^3 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} + \frac{2b^6 \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14b^4 \sin(c + dx)}{45d(b \sec(c + dx))^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.166259, size = 74, normalized size = 0.74

$$\frac{b^2 \sqrt{b \sec(c + dx)} \left( (33 \sin(c + dx) + 5 \sin(3(c + dx))) \cos^2(c + dx) + 84 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{90d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(b*Sec[c + d*x])^(5/2), x]
```

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*(84*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
+ Cos[c + d*x]^2*(33*Sin[c + d*x] + 5*Sin[3*(c + d*x)])))/(90*d)
```

**Maple [C]** time = 0.186, size = 333, normalized size = 3.3

$$\frac{2 (\cos(dx + c))^2}{45 d \sin(dx + c)} \left( \frac{b}{\cos(dx + c)} \right)^{\frac{5}{2}} \left( 21 i \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \sin(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/45/d*(b/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(21*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)-21*I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-5*cos(d*x+c)^6+21*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-21*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-2*cos(d*x+c)^4-14*cos(d*x+c)^2+21*cos(d*x+c))/sin(d*x+c)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \cos(dx + c)^7 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^7, x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec(dx + c)} b^2 \cos(dx + c)^7 \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b^2*cos(d*x + c)^7*sec(d*x + c)^2, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**7*(b*sec(d*x+c))**(5/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \cos(dx + c)^7 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x, algorithm="giac")`

[Out] `integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^7, x)`

### 3.100 $\int (b \sec(c + dx))^{7/2} dx$

**Optimal.** Leaf size=98

$$\frac{6b^3 \sin(c + dx) \sqrt{b \sec(c + dx)}}{5d} - \frac{6b^4 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2b \sin(c + dx) (b \sec(c + dx))^{5/2}}{5d}$$

[Out]  $(-6*b^4*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (6*b^3*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b*(b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d)$

**Rubi [A]** time = 0.0484491, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3768, 3771, 2639}

$$\frac{6b^3 \sin(c + dx) \sqrt{b \sec(c + dx)}}{5d} - \frac{6b^4 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2b \sin(c + dx) (b \sec(c + dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(7/2), x]

[Out]  $(-6*b^4*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (6*b^3*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b*(b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d)$

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> -Simp[(b\*Csc[c + d\*x]\*(b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - P  
i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned}
 \int (b \sec(c + dx))^{7/2} dx &= \frac{2b(b \sec(c + dx))^{5/2} \sin(c + dx)}{5d} + \frac{1}{5} (3b^2) \int (b \sec(c + dx))^{3/2} dx \\
 &= \frac{6b^3 \sqrt{b \sec(c + dx)} \sin(c + dx)}{5d} + \frac{2b(b \sec(c + dx))^{5/2} \sin(c + dx)}{5d} - \frac{1}{5} (3b^4) \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\
 &= \frac{6b^3 \sqrt{b \sec(c + dx)} \sin(c + dx)}{5d} + \frac{2b(b \sec(c + dx))^{5/2} \sin(c + dx)}{5d} - \frac{(3b^4) \int \sqrt{\cos(c + dx)} dx}{5\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} \\
 &= -\frac{6b^4 E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{6b^3 \sqrt{b \sec(c + dx)} \sin(c + dx)}{5d} + \frac{2b(b \sec(c + dx))^{5/2} \sin(c + dx)}{5d}
 \end{aligned}$$

**Mathematica [A]** time = 0.0847728, size = 62, normalized size = 0.63

$$\frac{b(b \sec(c + dx))^{5/2} \left(7 \sin(c + dx) + 3 \sin(3(c + dx)) - 12 \cos^2(c + dx) E\left(\frac{1}{2}(c + dx) \middle| 2\right)\right)}{10d}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Sec[c + d\*x])^(7/2), x]

[Out] (b\*(b\*Sec[c + d\*x])^(5/2)\*(-12\*Cos[c + d\*x]^(5/2)\*EllipticE[(c + d\*x)/2, 2] + 7\*Sin[c + d\*x] + 3\*Sin[3\*(c + d\*x)]))/(10\*d)

**Maple [C]** time = 0.222, size = 354, normalized size = 3.6

$$\frac{2(-1 + \cos(dx + c))^2 \cos(dx + c) (\cos(dx + c) + 1)^2}{5d (\sin(dx + c))^5} \left( 3i \operatorname{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) (\cos(dx + c))^3 \sqrt{(\cos(dx + c) + 1)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(7/2), x)

```
[Out] -2/5/d*(-1+cos(d*x+c))^2*(3*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos
(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)-3*I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^3*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*I*EllipticF(I*
(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-3*I*EllipticE(I*(-1+cos(d*x+c))/sin(d
*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)^2*sin(d*x+c)+3*cos(d*x+c)^3-2*cos(d*x+c)^2-1)*cos(d*x+c)*(cos(d*x+c)+1
)^2*(b/cos(d*x+c))^(7/2)/sin(d*x+c)^5
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(7/2), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{b \sec(dx + c)} b^3 \sec(dx + c)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*b^3*sec(d*x + c)^3, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(7/2),x)
```



[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(7/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(7/2), x)

$$3.101 \quad \int \frac{\sec^5(c+dx)}{\sqrt{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=100

$$\frac{10\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b \sec(c+dx)}}{21bd} + \frac{2 \sin(c+dx)(b \sec(c+dx))^{7/2}}{7b^4d} + \frac{10 \sin(c+dx)(b \sec(c+dx))^3}{21b^2d}$$

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*b\*d) + (10\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(21\*b^2\*d) + (2\*(b\*Sec[c + d\*x])^(7/2)\*Sin[c + d\*x])/(7\*b^4\*d)

**Rubi [A]** time = 0.0574047, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2641}

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^{7/2}}{7b^4d} + \frac{10 \sin(c+dx)(b \sec(c+dx))^{3/2}}{21b^2d} + \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx) \middle| 2\right)\sqrt{b \sec(c+dx)}}{21bd}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^5/Sqrt[b\*Sec[c + d\*x]], x]

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*b\*d) + (10\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(21\*b^2\*d) + (2\*(b\*Sec[c + d\*x])^(7/2)\*Sin[c + d\*x])/(7\*b^4\*d)

### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := -Simp[(b\*Csc[c + d\*x]\*(b\*Csc[c + d\*x])^(n-1))/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*Csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(c+dx)}{\sqrt{b \sec(c+dx)}} dx &= \frac{\int (b \sec(c+dx))^{9/2} dx}{b^5} \\
&= \frac{2(b \sec(c+dx))^{7/2} \sin(c+dx)}{7b^4 d} + \frac{5 \int (b \sec(c+dx))^{5/2} dx}{7b^3} \\
&= \frac{10(b \sec(c+dx))^{3/2} \sin(c+dx)}{21b^2 d} + \frac{2(b \sec(c+dx))^{7/2} \sin(c+dx)}{7b^4 d} + \frac{5 \int \sqrt{b \sec(c+dx)} dx}{21b} \\
&= \frac{10(b \sec(c+dx))^{3/2} \sin(c+dx)}{21b^2 d} + \frac{2(b \sec(c+dx))^{7/2} \sin(c+dx)}{7b^4 d} + \frac{(5\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)})}{21b} \\
&= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \sec(c+dx)}}{21bd} + \frac{10(b \sec(c+dx))^{3/2} \sin(c+dx)}{21b^2 d} + \frac{2(b \sec(c+dx))^{7/2} \sin(c+dx)}{7b^4 d}
\end{aligned}$$

**Mathematica [A]** time = 0.0827893, size = 69, normalized size = 0.69

$$\frac{\sec^3(c+dx) \left( 10 \cos^{\frac{5}{2}}(c+dx) \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + 5 \sin(2(c+dx)) + 6 \tan(c+dx) \right)}{21d\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[b*Sec[c + d*x]], x]
```

```
[Out] (Sec[c + d*x]^3*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d*Sqrt[b*Sec[c + d*x]])
```

**Maple [C]** time = 0.23, size = 152, normalized size = 1.5

$$\frac{(-2 + 2 \cos(dx + c)) (\cos(dx + c) + 1)^2}{21 d (\sin(dx + c))^3 (\cos(dx + c))^4} \left( 5 i (\cos(dx + c))^3 \sin(dx + c) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^5/(b\*sec(d\*x+c))^(1/2),x)

[Out] -2/21/d\*(-1+cos(d\*x+c))\*(5\*I\*cos(d\*x+c)^3\*sin(d\*x+c)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)-5\*cos(d\*x+c)^3+5\*cos(d\*x+c)^2-3\*cos(d\*x+c)+3)\*(cos(d\*x+c)+1)^2/sin(d\*x+c)^3/cos(d\*x+c)^4/(b/cos(d\*x+c))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^5}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^5/(b\*sec(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(d\*x + c)^5/sqrt(b\*sec(d\*x + c)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx + c)} \sec(dx + c)^4}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^5/(b\*sec(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b\*sec(d\*x + c))\*sec(d\*x + c)^4/b, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*5/(b\*sec(d\*x+c))\*\*(1/2), x)

[Out] Integral(sec(c + d\*x)\*\*5/sqrt(b\*sec(c + d\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^5}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^5/(b\*sec(d\*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(sec(d\*x + c)^5/sqrt(b\*sec(d\*x + c)), x)

### 3.102 $\int \frac{\sec^4(c+dx)}{\sqrt{b \sec(c+dx)}} dx$

**Optimal.** Leaf size=97

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^{5/2}}{5b^3d} + \frac{6 \sin(c+dx)\sqrt{b \sec(c+dx)}}{5bd} - \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

[Out] (-6\*EllipticE[(c + d\*x)/2, 2])/(5\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (6\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(5\*b\*d) + (2\*(b\*Sec[c + d\*x])^(5/2)\*Sin[c + d\*x])/(5\*b^3\*d)

**Rubi [A]** time = 0.0586433, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2639}

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^{5/2}}{5b^3d} + \frac{6 \sin(c+dx)\sqrt{b \sec(c+dx)}}{5bd} - \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^4/Sqrt[b\*Sec[c + d\*x]],x]

[Out] (-6\*EllipticE[(c + d\*x)/2, 2])/(5\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (6\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(5\*b\*d) + (2\*(b\*Sec[c + d\*x])^(5/2)\*Sin[c + d\*x])/(5\*b^3\*d)

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Csc[c + d\*x]\*(b\*Csc[c + d\*x])^(n-1))/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*Csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c+dx)}{\sqrt{b \sec(c+dx)}} dx &= \frac{\int (b \sec(c+dx))^{7/2} dx}{b^4} \\
&= \frac{2(b \sec(c+dx))^{5/2} \sin(c+dx)}{5b^3 d} + \frac{3 \int (b \sec(c+dx))^{3/2} dx}{5b^2} \\
&= \frac{6\sqrt{b \sec(c+dx)} \sin(c+dx)}{5bd} + \frac{2(b \sec(c+dx))^{5/2} \sin(c+dx)}{5b^3 d} - \frac{3}{5} \int \frac{1}{\sqrt{b \sec(c+dx)}} dx \\
&= \frac{6\sqrt{b \sec(c+dx)} \sin(c+dx)}{5bd} + \frac{2(b \sec(c+dx))^{5/2} \sin(c+dx)}{5b^3 d} - \frac{3 \int \sqrt{\cos(c+dx)} dx}{5\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}} \\
&= -\frac{6E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}} + \frac{6\sqrt{b \sec(c+dx)} \sin(c+dx)}{5bd} + \frac{2(b \sec(c+dx))^{5/2} \sin(c+dx)}{5b^3 d}
\end{aligned}$$

**Mathematica [A]** time = 0.225734, size = 61, normalized size = 0.63

$$\frac{2 \tan(c+dx) \left( \sec^2(c+dx) + 3 \right) - \frac{6E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{\sqrt{\cos(c+dx)}}}{5d\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[b*Sec[c + d*x]], x]
```

```
[Out] ((-6*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(3 + Sec[c + d*x]^2)
*Tan[c + d*x])/(5*d*Sqrt[b*Sec[c + d*x]])
```

**Maple [C]** time = 0.236, size = 356, normalized size = 3.7

$$\frac{2(-1 + \cos(dx + c))^2 (\cos(dx + c) + 1)^2}{5d(\cos(dx + c))^3 (\sin(dx + c))^5} \left( 3i\sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} (\cos(dx + c))^3 \operatorname{EllipticE}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^4/(b\*sec(d\*x+c))^(1/2),x)

[Out] 2/5/d\*(-1+cos(d\*x+c))^2\*(3\*I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*cos(d\*x+c)^3\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*sin(d\*x+c)-3\*I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*cos(d\*x+c)^3\*sin(d\*x+c)+3\*I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*cos(d\*x+c)^2\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*sin(d\*x+c)-3\*I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*cos(d\*x+c)^2\*sin(d\*x+c)-3\*cos(d\*x+c)^3+2\*cos(d\*x+c)^2+1\*(cos(d\*x+c)+1)^2/cos(d\*x+c)^3/(b/cos(d\*x+c))^(1/2)/sin(d\*x+c)^5

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^4}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^4/(b\*sec(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(d\*x + c)^4/sqrt(b\*sec(d\*x + c)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx + c)} \sec(dx + c)^3}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^4/(b\*sec(d\*x+c))^(1/2),x, algorithm="fricas")



[Out] `integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^3/b, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**4/(b*sec(d*x+c))**(1/2), x)`

[Out] `Integral(sec(c + d*x)**4/sqrt(b*sec(c + d*x)), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^4}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(1/2), x, algorithm="giac")`

[Out] `integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c)), x)`

### 3.103 $\int \frac{\sec^3(c+dx)}{\sqrt{b \sec(c+dx)}} dx$

**Optimal.** Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b \sec(c+dx)}}{3bd} + \frac{2 \sin(c+dx)(b \sec(c+dx))^{3/2}}{3b^2d}$$

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*b\*d) + (2\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(3\*b^2\*d)

**Rubi [A]** time = 0.0394617, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2641}

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^{3/2}}{3b^2d} + \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx) \middle| 2\right)\sqrt{b \sec(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^3/Sqrt[b\*Sec[c + d\*x]],x]

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*b\*d) + (2\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(3\*b^2\*d)

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Csc[c + d\*x]\*(b\*Csc[c + d\*x])^(n-1))/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*Csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{\sqrt{b \sec(c+dx)}} dx &= \frac{\int (b \sec(c+dx))^{5/2} dx}{b^3} \\
&= \frac{2(b \sec(c+dx))^{3/2} \sin(c+dx)}{3b^2d} + \frac{\int \sqrt{b \sec(c+dx)} dx}{3b} \\
&= \frac{2(b \sec(c+dx))^{3/2} \sin(c+dx)}{3b^2d} + \frac{(\sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3b} \\
&= \frac{2\sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \sec(c+dx)}}{3bd} + \frac{2(b \sec(c+dx))^{3/2} \sin(c+dx)}{3b^2d}
\end{aligned}$$

**Mathematica [A]** time = 0.0744825, size = 51, normalized size = 0.71

$$\frac{2\sqrt{b \sec(c+dx)} \left( \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + \tan(c+dx) \right)}{3bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^3/Sqrt[b\*Sec[c + d\*x]], x]

[Out] (2\*Sqrt[b\*Sec[c + d\*x]]\*(Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2] + Tan[c + d\*x]))/(3\*b\*d)

**Maple [C]** time = 0.181, size = 130, normalized size = 1.8

$$-\frac{(-2 + 2 \cos(dx + c)) (\cos(dx + c) + 1)^2}{3d (\cos(dx + c))^2 (\sin(dx + c))^3} \left( i \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \operatorname{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right), i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x)`

[Out] 
$$-2/3/d*(-1+\cos(d*x+c))*(I*(1/(\cos(d*x+c)+1))^(1/2)*(\cos(d*x+c)/(\cos(d*x+c)+1))^(1/2)*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)*\sin(d*x+c)-\cos(d*x+c)+1)*(\cos(d*x+c)+1)^2/\cos(d*x+c)^2/(b/\cos(d*x+c))^(1/2)/\sin(d*x+c)^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{\sqrt{b}\sec(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b}\sec(dx+c)\sec(dx+c)^2}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^2/b, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{\sqrt{b}\sec(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(b*sec(c + d*x)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^3}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c)), x)
```

$$3.104 \quad \int \frac{\sec^2(c+dx)}{\sqrt{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=65

$$\frac{2 \sin(c+dx) \sqrt{b \sec(c+dx)}}{bd} - \frac{2E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{d \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}}$$

[Out] (-2\*EllipticE[(c + d\*x)/2, 2])/(d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(b\*d)

**Rubi [A]** time = 0.0401477, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2639}

$$\frac{2 \sin(c+dx) \sqrt{b \sec(c+dx)}}{bd} - \frac{2E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{d \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^2/Sqrt[b\*Sec[c + d\*x]],x]

[Out] (-2\*EllipticE[(c + d\*x)/2, 2])/(d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(b\*d)

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x] \* (b\*Csc[c + d\*x])^(n-1))/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*Csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n \* Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c + dx)}{\sqrt{b \sec(c + dx)}} dx &= \frac{\int (b \sec(c + dx))^{3/2} dx}{b^2} \\
&= \frac{2\sqrt{b \sec(c + dx)} \sin(c + dx)}{bd} - \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\
&= \frac{2\sqrt{b \sec(c + dx)} \sin(c + dx)}{bd} - \frac{\int \sqrt{\cos(c + dx)} dx}{\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} \\
&= -\frac{2E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2\sqrt{b \sec(c + dx)} \sin(c + dx)}{bd}
\end{aligned}$$

**Mathematica [A]** time = 0.0782271, size = 48, normalized size = 0.74

$$\frac{2 \tan(c + dx) - \frac{2E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{\sqrt{\cos(c+dx)}}}{d\sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^2/Sqrt[b\*Sec[c + d\*x]], x]

```
[Out] ((-2*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*Tan[c + d*x])/(d*Sqr
t[b*Sec[c + d*x]])
```

**Maple [C]** time = 0.218, size = 319, normalized size = 4.9

$$2 \frac{(\cos(dx + c) + 1)^2 (-1 + \cos(dx + c))^2}{db (\sin(dx + c))^5} \left( i \cos(dx + c) \sin(dx + c) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticE} \left( \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)+I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)*(b/cos(d*x+c))^(1/2)/b/sin(d*x+c)^5
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{\sqrt{b \sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c)), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)} \sec(dx+c)}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c))*sec(d*x + c)/b, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{\sqrt{b \sec(c+dx)}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2/(b*sec(d*x+c))**(1/2),x)`

[Out] `Integral(sec(c + d*x)**2/sqrt(b*sec(c + d*x)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{\sqrt{b \sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c)), x)`

$$3.105 \quad \int \frac{\sec(c+dx)}{\sqrt{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=41

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b \sec(c+dx)}}{bd}$$

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(b\*d)

**Rubi [A]** time = 0.0212964, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {16, 3771, 2641}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \sec(c+dx)}}{bd}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]/Sqrt[b\*Sec[c + d\*x]],x]

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(b\*d)

### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned} \int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}} dx &= \frac{\int \sqrt{b\sec(c+dx)} dx}{b} \\ &= \frac{(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{b} \\ &= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{bd} \end{aligned}$$

**Mathematica [A]** time = 0.0163518, size = 41, normalized size = 1.

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b\sec(c+dx)}}{bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]/Sqrt[b\*Sec[c + d\*x]], x]

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(b\*d)

**Maple [C]** time = 0.158, size = 98, normalized size = 2.4

$$\frac{-2i(-1 + \cos(dx+c))(\cos(dx+c)+1)^2}{d(\sin(dx+c))^2} \left((\cos(dx+c)+1)^{-1}\right)^{\frac{3}{2}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx+c))}{\sin(dx+c)}, i\right) \frac{1}{\sqrt{\frac{b}{\cos(dx+c)}}} \frac{1}{\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)/(b\*sec(d\*x+c))^(1/2), x)

[Out] -2\*I/d\*(1/(cos(d\*x+c)+1))^(3/2)\*(-1+cos(d\*x+c))\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*(cos(d\*x+c)+1)^2/(b/cos(d\*x+c))^(1/2)/(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)/sin(d\*x+c)^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{\sqrt{b\sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sec(d*x + c)/sqrt(b*sec(d*x + c)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx + c)}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))/b, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)/(b*sec(d*x+c))**(1/2),x)`

[Out] `Integral(sec(c + d*x)/sqrt(b*sec(c + d*x)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate(sec(d*x + c)/sqrt(b*sec(d*x + c)), x)`

$$3.106 \quad \int \frac{1}{\sqrt{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=38

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

[Out] (2\*EllipticE[(c + d\*x)/2, 2])/(d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0190712, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3771, 2639}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[b\*Sec[c + d\*x]],x]

[Out] (2\*EllipticE[(c + d\*x)/2, 2])/(d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] :> Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\int \frac{1}{\sqrt{b \sec(c + dx)}} dx = \frac{\int \sqrt{\cos(c + dx)} dx}{\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} \\ = \frac{2E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}}$$

**Mathematica [A]** time = 0.014566, size = 38, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[b\*Sec[c + d\*x]],x]

[Out] (2\*EllipticE[(c + d\*x)/2, 2])/(d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

**Maple [C]** time = 0.157, size = 306, normalized size = 8.1

$$2 \frac{1}{d \sin(dx + c) b} \left( i \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \cos(dx + c) \sin(dx + c) - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*sec(d\*x+c))^(1/2),x)

[Out] 2/d\*(I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*cos(d\*x+c)\*sin(d\*x+c)-I\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*cos(d\*x+c)\*sin(d\*x+c)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)+I\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*sin(d\*x+c)-I\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*sin(d\*x+c)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)-cos(d\*x+c)^2+cos(d\*x+c))\*(b/cos(d\*x+c))^(1/2)/sin(d\*x+c)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*sec(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b\*sec(d\*x + c)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx + c)}}{b \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*sec(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b\*sec(d\*x + c))/(b\*sec(d\*x + c)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*sec(d\*x+c))\*\*(1/2),x)

[Out] Integral(1/sqrt(b\*sec(c + d\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(d*x + c)), x)
```



$$3.107 \quad \int \frac{\cos(c+dx)}{\sqrt{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=69

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b \sec(c+dx)}}{3bd} + \frac{2 \sin(c+dx)}{3d\sqrt{b \sec(c+dx)}}$$

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*b\*d) + (2\*Sin[c + d\*x])/(3\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0434153, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {16, 3769, 3771, 2641}

$$\frac{2 \sin(c+dx)}{3d\sqrt{b \sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \sec(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]/Sqrt[b\*Sec[c + d\*x]],x]

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*b\*d) + (2\*Sin[c + d\*x])/(3\*d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n+1))/(b\*d^n), x] + Dist[(n+1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\cos(c+dx)}{\sqrt{b \sec(c+dx)}} dx &= b \int \frac{1}{(b \sec(c+dx))^{3/2}} dx \\
 &= \frac{2 \sin(c+dx)}{3d \sqrt{b \sec(c+dx)}} + \frac{\int \sqrt{b \sec(c+dx)} dx}{3b} \\
 &= \frac{2 \sin(c+dx)}{3d \sqrt{b \sec(c+dx)}} + \frac{(\sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3b} \\
 &= \frac{2 \sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \sec(c+dx)}}{3bd} + \frac{2 \sin(c+dx)}{3d \sqrt{b \sec(c+dx)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0675756, size = 60, normalized size = 0.87

$$\frac{b \sec^2(c+dx) \left( 2 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + \sin(2(c+dx)) \right)}{3d(b \sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d\*x]/Sqrt[b\*Sec[c + d\*x]], x]

```
[Out] (b*Sec[c + d*x]^2*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(
  c + d*x)]))/(3*d*(b*Sec[c + d*x])^(3/2))
```

**Maple [C]** time = 0.16, size = 126, normalized size = 1.8

$$\frac{(-2 + 2 \cos(dx+c)) (\cos(dx+c) + 1)^2}{3db (\sin(dx+c))^3} \left( i \operatorname{EllipticF}\left(\frac{i(-1 + \cos(dx+c))}{\sin(dx+c)}, i\right) \sqrt{(\cos(dx+c) + 1)^{-1}} \sqrt{\frac{\cos(dx+c)}{\cos(dx+c) + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x)`

[Out] 
$$-2/3/d*(-1+\cos(dx+c))*(I*\text{EllipticF}(I*(-1+\cos(dx+c))/\sin(dx+c),I))*(1/(\cos(dx+c)+1))^{1/2}*(\cos(dx+c)/(\cos(dx+c)+1))^{1/2}*\sin(dx+c)-\cos(dx+c)^2+\cos(dx+c))*(\cos(dx+c)+1)^2*(b/\cos(dx+c))^{1/2}/b/\sin(dx+c)^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{\sqrt{b \sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(cos(d*x + c)/sqrt(b*sec(d*x + c)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)} \cos(dx+c)}{b \sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*cos(d*x + c)/(b*sec(d*x + c)), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{\sqrt{b \sec(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)/(b*sec(d*x+c))**(1/2),x)`

[Out] Integral(cos(c + d\*x)/sqrt(b\*sec(c + d\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c)}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)/(b\*sec(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(cos(d\*x + c)/sqrt(b\*sec(d\*x + c)), x)

$$3.108 \quad \int \frac{\cos^2(c+dx)}{\sqrt{b \sec(c+dx)}} dx$$

Optimal. Leaf size=67

$$\frac{2b \sin(c+dx)}{5d(b \sec(c+dx))^{3/2}} + \frac{6E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

[Out] (6\*EllipticE[(c + d\*x)/2, 2])/(5\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b\*Sin[c + d\*x])/(5\*d\*(b\*Sec[c + d\*x])^(3/2))

**Rubi [A]** time = 0.0507569, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2639}

$$\frac{2b \sin(c+dx)}{5d(b \sec(c+dx))^{3/2}} + \frac{6E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^2/Sqrt[b\*Sec[c + d\*x]],x]

[Out] (6\*EllipticE[(c + d\*x)/2, 2])/(5\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b\*Sin[c + d\*x])/(5\*d\*(b\*Sec[c + d\*x])^(3/2))

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c + dx)}{\sqrt{b \sec(c + dx)}} dx &= b^2 \int \frac{1}{(b \sec(c + dx))^{5/2}} dx \\
&= \frac{2b \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}} + \frac{3}{5} \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\
&= \frac{2b \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}} + \frac{3 \int \sqrt{\cos(c + dx)} dx}{5\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} \\
&= \frac{6E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} + \frac{2b \sin(c + dx)}{5d(b \sec(c + dx))^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0817869, size = 60, normalized size = 0.9

$$\frac{\sqrt{b \sec(c + dx)} \left( \sin(c + dx) + \sin(3(c + dx)) + 12\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{10bd}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d\*x]^2/Sqrt[b\*Sec[c + d\*x]],x]

```
[Out] (Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Si
n[c + d*x] + Sin[3*(c + d*x)]))/(10*b*d)
```

**Maple [C]** time = 0.215, size = 316, normalized size = 4.7

$$-\frac{2}{5d \sin(dx + c)b} \left( 3i \cos(dx + c) \sin(dx + c) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticE}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}\right), \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x)`

[Out] 
$$-2/5/d*(3*I*\cos(d*x+c)*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-3*I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)*\cos(d*x+c)+3*I*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-3*I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)+\cos(d*x+c)^4+2*\cos(d*x+c)^2-3*\cos(d*x+c))*(b/\cos(d*x+c))^{1/2}/\sin(d*x+c)/b$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{\sqrt{b \sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)} \cos(dx+c)^2}{b \sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^2/(b*sec(d*x + c)), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{\sqrt{b \sec(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(1/2),x)`

[Out] `Integral(cos(c + d*x)**2/sqrt(b*sec(c + d*x)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c)^2}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c)), x)`



$$3.109 \quad \int \frac{\cos^3(c+dx)}{\sqrt{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=97

$$\frac{10\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b \sec(c+dx)}}{21bd} + \frac{2b^2 \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}} + \frac{10 \sin(c+dx)}{21d\sqrt{b \sec(c+dx)}}$$

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*b\*d) + (2\*b^2\*Sin[c + d\*x])/(7\*d\*(b\*Sec[c + d\*x])^(5/2)) + (10\*Sin[c + d\*x])/(21\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0690577, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2641}

$$\frac{2b^2 \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}} + \frac{10 \sin(c+dx)}{21d\sqrt{b \sec(c+dx)}} + \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx) \middle| 2\right)\sqrt{b \sec(c+dx)}}{21bd}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^3/Sqrt[b\*Sec[c + d\*x]], x]

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*b\*d) + (2\*b^2\*Sin[c + d\*x])/(7\*d\*(b\*Sec[c + d\*x])^(5/2)) + (10\*Sin[c + d\*x])/(21\*d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n+1))/(b\*d\*n), x] + Dist[(n+1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n_], x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{\sqrt{b \sec(c+dx)}} dx &= b^3 \int \frac{1}{(b \sec(c+dx))^{7/2}} dx \\
&= \frac{2b^2 \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}} + \frac{1}{7}(5b) \int \frac{1}{(b \sec(c+dx))^{3/2}} dx \\
&= \frac{2b^2 \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}} + \frac{10 \sin(c+dx)}{21d\sqrt{b \sec(c+dx)}} + \frac{5 \int \sqrt{b \sec(c+dx)} dx}{21b} \\
&= \frac{2b^2 \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}} + \frac{10 \sin(c+dx)}{21d\sqrt{b \sec(c+dx)}} + \frac{(5\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{21b} \\
&= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx) \middle| 2\right)\sqrt{b \sec(c+dx)}}{21bd} + \frac{2b^2 \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}} + \frac{10 \sin(c+dx)}{21d\sqrt{b \sec(c+dx)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0748665, size = 66, normalized size = 0.68

$$\frac{\sqrt{b \sec(c+dx)} \left( 40\sqrt{\cos(c+dx)} \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + 26 \sin(2(c+dx)) + 3 \sin(4(c+dx)) \right)}{84bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26
*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*b*d)
```

**Maple [C]** time = 0.204, size = 148, normalized size = 1.5

$$-\frac{2(\cos(dx+c)+1)^2(-1+\cos(dx+c))}{21db(\sin(dx+c))^3} \left( 5i \operatorname{EllipticF} \left( \frac{i(-1+\cos(dx+c))}{\sin(dx+c)}, i \right) \sqrt{(\cos(dx+c)+1)^{-1}} \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x)`

[Out] `-2/21/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(5*I*EllipticF(I*(-1+cos(d*x+c)))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*cos(d*x+c)^4+3*cos(d*x+c)^3-5*cos(d*x+c)^2+5*cos(d*x+c))*(b/cos(d*x+c))^(1/2)/b/sin(d*x+c)^3`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{\sqrt{b \sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(cos(d*x + c)^3/sqrt(b*sec(d*x + c)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{\sqrt{b \sec(dx+c)} \cos(dx+c)^3}{b \sec(dx+c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^3/(b*sec(d*x + c)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*\*3/(b\*sec(d\*x+c))\*\*(1/2), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c)^3}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^3/(b\*sec(d\*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(cos(d\*x + c)^3/sqrt(b\*sec(d\*x + c)), x)

$$3.110 \quad \int \frac{\cos^4(c+dx)}{\sqrt{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=95

$$\frac{2b^3 \sin(c+dx)}{9d(b \sec(c+dx))^{7/2}} + \frac{14b \sin(c+dx)}{45d(b \sec(c+dx))^{3/2}} + \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

[Out] (14\*EllipticE[(c + d\*x)/2, 2])/(15\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b^3\*Sin[c + d\*x])/(9\*d\*(b\*Sec[c + d\*x])^(7/2)) + (14\*b\*Sin[c + d\*x])/(45\*d\*(b\*Sec[c + d\*x])^(3/2))

**Rubi [A]** time = 0.069369, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2639}

$$\frac{2b^3 \sin(c+dx)}{9d(b \sec(c+dx))^{7/2}} + \frac{14b \sin(c+dx)}{45d(b \sec(c+dx))^{3/2}} + \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^4/Sqrt[b\*Sec[c + d\*x]],x]

[Out] (14\*EllipticE[(c + d\*x)/2, 2])/(15\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b^3\*Sin[c + d\*x])/(9\*d\*(b\*Sec[c + d\*x])^(7/2)) + (14\*b\*Sin[c + d\*x])/(45\*d\*(b\*Sec[c + d\*x])^(3/2))

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n+1))/(b\*d\*n), x] + Dist[(n+1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)}{\sqrt{b \sec(c+dx)}} dx &= b^4 \int \frac{1}{(b \sec(c+dx))^{9/2}} dx \\
&= \frac{2b^3 \sin(c+dx)}{9d(b \sec(c+dx))^{7/2}} + \frac{1}{9} (7b^2) \int \frac{1}{(b \sec(c+dx))^{5/2}} dx \\
&= \frac{2b^3 \sin(c+dx)}{9d(b \sec(c+dx))^{7/2}} + \frac{14b \sin(c+dx)}{45d(b \sec(c+dx))^{3/2}} + \frac{7}{15} \int \frac{1}{\sqrt{b \sec(c+dx)}} dx \\
&= \frac{2b^3 \sin(c+dx)}{9d(b \sec(c+dx))^{7/2}} + \frac{14b \sin(c+dx)}{45d(b \sec(c+dx))^{3/2}} + \frac{7 \int \sqrt{\cos(c+dx)} dx}{15 \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}} \\
&= \frac{14E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{15d \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}} + \frac{2b^3 \sin(c+dx)}{9d(b \sec(c+dx))^{7/2}} + \frac{14b \sin(c+dx)}{45d(b \sec(c+dx))^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.189403, size = 70, normalized size = 0.74

$$\frac{4(33 \sin(c+dx) + 5 \sin(3(c+dx))) \cos(c+dx) + \frac{336E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{\sqrt{\cos(c+dx)}}}{360d \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] ((336*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 4*Cos[c + d*x]*(33*Si
n[c + d*x] + 5*Sin[3*(c + d*x)]))/(360*d*Sqrt[b*Sec[c + d*x]])
```

**Maple [C]** time = 0.202, size = 328, normalized size = 3.5

$$\frac{2}{45 d \sin(dx + c) b} \left( 21 i \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \text{EllipticF} \left( \frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i \right) \sin(dx + c) \cos(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)^4/(b\*sec(d\*x+c))^(1/2),x)

[Out] 2/45/d\*(21\*I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*sin(d\*x+c)\*cos(d\*x+c)-21\*I\*cos(d\*x+c)\*sin(d\*x+c)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)-5\*cos(d\*x+c)^6+21\*I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*sin(d\*x+c)-21\*I\*sin(d\*x+c)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)-2\*cos(d\*x+c)^4-14\*cos(d\*x+c)^2+21\*cos(d\*x+c))\*(b/cos(d\*x+c))^(1/2)/sin(d\*x+c)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c)^4}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^4/(b\*sec(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d\*x + c)^4/sqrt(b\*sec(d\*x + c)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\sqrt{b \sec(dx + c)} \cos(dx + c)^4}{b \sec(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^4/(b\*sec(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] `integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^4/(b*sec(d*x + c)), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**4/(b*sec(d*x+c))**(1/2), x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c)^4}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^4/(b*sec(d*x+c))^(1/2), x, algorithm="giac")`

[Out] `integrate(cos(d*x + c)^4/sqrt(b*sec(d*x + c)), x)`



$$3.111 \quad \int \frac{\sec^6(c+dx)}{(b \sec(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=100

$$\frac{10\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b \sec(c+dx)}}{21b^2d} + \frac{2 \sin(c+dx)(b \sec(c+dx))^{7/2}}{7b^5d} + \frac{10 \sin(c+dx)(b \sec(c+dx))^{3/2}}{21b^3d}$$

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*b^2\*d) + (10\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(21\*b^3\*d) + (2\*(b\*Sec[c + d\*x])^(7/2)\*Sin[c + d\*x])/(7\*b^5\*d)

**Rubi [A]** time = 0.0586986, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2641}

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^{7/2}}{7b^5d} + \frac{10 \sin(c+dx)(b \sec(c+dx))^{3/2}}{21b^3d} + \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx) \middle| 2\right)\sqrt{b \sec(c+dx)}}{21b^2d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^6/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*b^2\*d) + (10\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(21\*b^3\*d) + (2\*(b\*Sec[c + d\*x])^(7/2)\*Sin[c + d\*x])/(7\*b^5\*d)

### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3768

Int[(csc[(c\_)+(d\_)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x])\*(b\*Csc[c + d\*x])^(n-1)/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*Csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{\sec^6(c+dx)}{(b \sec(c+dx))^{3/2}} dx &= \frac{\int (b \sec(c+dx))^{9/2} dx}{b^6} \\ &= \frac{2(b \sec(c+dx))^{7/2} \sin(c+dx)}{7b^5d} + \frac{5 \int (b \sec(c+dx))^{5/2} dx}{7b^4} \\ &= \frac{10(b \sec(c+dx))^{3/2} \sin(c+dx)}{21b^3d} + \frac{2(b \sec(c+dx))^{7/2} \sin(c+dx)}{7b^5d} + \frac{5 \int \sqrt{b \sec(c+dx)} dx}{21b^2} \\ &= \frac{10(b \sec(c+dx))^{3/2} \sin(c+dx)}{21b^3d} + \frac{2(b \sec(c+dx))^{7/2} \sin(c+dx)}{7b^5d} + \frac{(5\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)})}{21b^2} \\ &= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \sec(c+dx)}}{21b^2d} + \frac{10(b \sec(c+dx))^{3/2} \sin(c+dx)}{21b^3d} + \frac{2(b \sec(c+dx))^{7/2} \sin(c+dx)}{7b^5d} \end{aligned}$$

**Mathematica [A]** time = 0.166195, size = 69, normalized size = 0.69

$$\frac{\sec^4(c+dx) \left( 10 \cos^{\frac{5}{2}}(c+dx) \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + 5 \sin(2(c+dx)) + 6 \tan(c+dx) \right)}{21d(b \sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(b*Sec[c + d*x])^(3/2), x]
```

```
[Out] (Sec[c + d*x]^4*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d*(b*Sec[c + d*x])^(3/2))
```

**Maple [C]** time = 0.219, size = 152, normalized size = 1.5

$$-\frac{2(\cos(dx+c)+1)^2(-1+\cos(dx+c))}{21d(\cos(dx+c))^5(\sin(dx+c))^3} \left( 5i(\cos(dx+c))^3 \sin(dx+c) \sqrt{(\cos(dx+c)+1)^{-1}} \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}} \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + 5 \sin(2(c+dx)) + 6 \tan(c+dx) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x)`

[Out] 
$$-2/21/d*(\cos(d*x+c)+1)^2*(-1+\cos(d*x+c))*(5*I*\cos(d*x+c)^3*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*EllipticF(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-5*\cos(d*x+c)^3+5*\cos(d*x+c)^2-3*\cos(d*x+c)+3)/\cos(d*x+c)^5/(b/\cos(d*x+c))^{3/2}/\sin(d*x+c)^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^6}{(b \sec(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate(sec(d*x + c)^6/(b*sec(d*x + c))^(3/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)} \sec(dx+c)^4}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^4/b^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^6(c+dx)}{(b \sec(c+dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*6/(b\*sec(d\*x+c))\*\*(3/2),x)

[Out] Integral(sec(c + d\*x)\*\*6/(b\*sec(c + d\*x))\*\*(3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^6}{(b \sec(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^6/(b\*sec(d\*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(sec(d\*x + c)^6/(b\*sec(d\*x + c))^(3/2), x)

$$3.112 \quad \int \frac{\sec^5(c+dx)}{(b \sec(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=100

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^{5/2}}{5b^4d} + \frac{6 \sin(c+dx)\sqrt{b \sec(c+dx)}}{5b^2d} - \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

[Out] (-6\*EllipticE[(c + d\*x)/2, 2])/(5\*b\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (6\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(5\*b^2\*d) + (2\*(b\*Sec[c + d\*x])^(5/2)\*Sin[c + d\*x])/(5\*b^4\*d)

**Rubi [A]** time = 0.0576487, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2639}

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^{5/2}}{5b^4d} + \frac{6 \sin(c+dx)\sqrt{b \sec(c+dx)}}{5b^2d} - \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^5/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (-6\*EllipticE[(c + d\*x)/2, 2])/(5\*b\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (6\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(5\*b^2\*d) + (2\*(b\*Sec[c + d\*x])^(5/2)\*Sin[c + d\*x])/(5\*b^4\*d)

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x])\*(b\*Csc[c + d\*x])^(n-1)/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*Csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(c+dx)}{(b \sec(c+dx))^{3/2}} dx &= \frac{\int (b \sec(c+dx))^{7/2} dx}{b^5} \\
&= \frac{2(b \sec(c+dx))^{5/2} \sin(c+dx)}{5b^4 d} + \frac{3 \int (b \sec(c+dx))^{3/2} dx}{5b^3} \\
&= \frac{6\sqrt{b \sec(c+dx)} \sin(c+dx)}{5b^2 d} + \frac{2(b \sec(c+dx))^{5/2} \sin(c+dx)}{5b^4 d} - \frac{3 \int \frac{1}{\sqrt{b \sec(c+dx)}} dx}{5b} \\
&= \frac{6\sqrt{b \sec(c+dx)} \sin(c+dx)}{5b^2 d} + \frac{2(b \sec(c+dx))^{5/2} \sin(c+dx)}{5b^4 d} - \frac{3 \int \sqrt{\cos(c+dx)} dx}{5b\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}} \\
&= -\frac{6E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}} + \frac{6\sqrt{b \sec(c+dx)} \sin(c+dx)}{5b^2 d} + \frac{2(b \sec(c+dx))^{5/2} \sin(c+dx)}{5b^4 d}
\end{aligned}$$

**Mathematica [A]** time = 0.0534337, size = 64, normalized size = 0.64

$$\frac{2 \tan(c+dx) \left( \sec^2(c+dx) + 3 \right) - \frac{6E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{\sqrt{\cos(c+dx)}}}{5bd\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(b*Sec[c + d*x])^(3/2), x]
```

```
[Out] ((-6*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(3 + Sec[c + d*x]^2)
*Tan[c + d*x])/(5*b*d*Sqrt[b*Sec[c + d*x]])
```

**Maple [C]** time = 0.223, size = 356, normalized size = 3.6

$$\frac{2 (\cos(dx+c)+1)^2 (-1+\cos(dx+c))^2}{5 d (\sin(dx+c))^5 (\cos(dx+c))^4} \left( 3 i \sqrt{(\cos(dx+c)+1)^{-1}} \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}} (\cos(dx+c))^3 \operatorname{EllipticE}\left(\frac{i(-1+\sin(dx+c))}{\sin(dx+c)}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^5/(b\*sec(d\*x+c))^(3/2),x)

[Out] 2/5/d\*(cos(d\*x+c)+1)^2\*(-1+cos(d\*x+c))^2\*(3\*I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*cos(d\*x+c)^3\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*sin(d\*x+c)-3\*I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*cos(d\*x+c)^3\*sin(d\*x+c)+3\*I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*cos(d\*x+c)^2\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*sin(d\*x+c)-3\*I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*cos(d\*x+c)^2\*sin(d\*x+c)-3\*cos(d\*x+c)^3+2\*cos(d\*x+c)^2+1)/sin(d\*x+c)^5/cos(d\*x+c)^4/(b/cos(d\*x+c))^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^5}{(b \sec(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^5/(b\*sec(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(sec(d\*x + c)^5/(b\*sec(d\*x + c))^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{b \sec(dx+c)} \sec(dx+c)^3}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^5/(b\*sec(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] `integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^3/b^2, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5(c + dx)}{(b \sec(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**5/(b*sec(d*x+c))**(3/2), x)`

[Out] `Integral(sec(c + d*x)**5/(b*sec(c + d*x))**(3/2), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^5}{(b \sec(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(3/2), x, algorithm="giac")`

[Out] `integrate(sec(d*x + c)^5/(b*sec(d*x + c))^(3/2), x)`



$$3.113 \quad \int \frac{\sec^4(c+dx)}{(b \sec(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b \sec(c+dx)}}{3b^2d} + \frac{2 \sin(c+dx)(b \sec(c+dx))^{3/2}}{3b^3d}$$

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*b^2\*d) + (2\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(3\*b^3\*d)

**Rubi [A]** time = 0.0377128, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2641}

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^{3/2}}{3b^3d} + \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx) \middle| 2\right)\sqrt{b \sec(c+dx)}}{3b^2d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^4/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*b^2\*d) + (2\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(3\*b^3\*d)

### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x])\*(b\*Csc[c + d\*x])^(n-1)/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*Csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\sec^4(c+dx)}{(b \sec(c+dx))^{3/2}} dx &= \frac{\int (b \sec(c+dx))^{5/2} dx}{b^4} \\
 &= \frac{2(b \sec(c+dx))^{3/2} \sin(c+dx)}{3b^3d} + \frac{\int \sqrt{b \sec(c+dx)} dx}{3b^2} \\
 &= \frac{2(b \sec(c+dx))^{3/2} \sin(c+dx)}{3b^3d} + \frac{(\sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3b^2} \\
 &= \frac{2\sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \sec(c+dx)}}{3b^2d} + \frac{2(b \sec(c+dx))^{3/2} \sin(c+dx)}{3b^3d}
 \end{aligned}$$

**Mathematica [A]** time = 0.0968243, size = 56, normalized size = 0.78

$$\frac{2 \sec^3(c+dx) \left( \cos^2(c+dx) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + \sin(c+dx) \right)}{3d(b \sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(b*Sec[c + d*x])^(3/2), x]
```

```
[Out] (2*Sec[c + d*x]^3*(Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + Sin[c + d
*x]))/(3*d*(b*Sec[c + d*x])^(3/2))
```

**Maple [C]** time = 0.17, size = 125, normalized size = 1.7

$$-\frac{(-2 + 2 \cos(dx+c))(\cos(dx+c)+1)^2}{3db^3(\sin(dx+c))^3} \left( i \sqrt{(\cos(dx+c)+1)^{-1}} \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}} \operatorname{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}, i\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x)`

[Out] 
$$-2/3/d*(-1+\cos(d*x+c))*(I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)*\sin(d*x+c)-\cos(d*x+c)+1)*(\cos(d*x+c)+1)^2*(b/\cos(d*x+c))^{3/2}/b^3/\sin(d*x+c)^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{(b \sec(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate(sec(d*x + c)^4/(b*sec(d*x + c))^(3/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)} \sec(dx+c)^2}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^2/b^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{(b \sec(c+dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**4/(b*sec(d*x+c))**(3/2),x)`

[Out] Integral(sec(c + d\*x)\*\*4/(b\*sec(c + d\*x))\*\*(3/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^4}{(b \sec(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^4/(b\*sec(d\*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(sec(d\*x + c)^4/(b\*sec(d\*x + c))^(3/2), x)

$$3.114 \quad \int \frac{\sec^3(c+dx)}{(b \sec(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=68

$$\frac{2 \sin(c+dx) \sqrt{b \sec(c+dx)}}{b^2 d} - \frac{2E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{bd \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}}$$

[Out] (-2\*EllipticE[(c + d\*x)/2, 2])/(b\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(b^2\*d)

**Rubi [A]** time = 0.0395441, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2639}

$$\frac{2 \sin(c+dx) \sqrt{b \sec(c+dx)}}{b^2 d} - \frac{2E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{bd \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^3/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (-2\*EllipticE[(c + d\*x)/2, 2])/(b\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(b^2\*d)

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x])\*(b\*Csc[c + d\*x])^(n-1)/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*Csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{(b \sec(c+dx))^{3/2}} dx &= \frac{\int (b \sec(c+dx))^{3/2} dx}{b^3} \\
&= \frac{2\sqrt{b \sec(c+dx)} \sin(c+dx)}{b^2 d} - \frac{\int \frac{1}{\sqrt{b \sec(c+dx)}} dx}{b} \\
&= \frac{2\sqrt{b \sec(c+dx)} \sin(c+dx)}{b^2 d} - \frac{\int \sqrt{\cos(c+dx)} dx}{b\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}} \\
&= -\frac{2E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}} + \frac{2\sqrt{b \sec(c+dx)} \sin(c+dx)}{b^2 d}
\end{aligned}$$

**Mathematica [A]** time = 0.0570341, size = 51, normalized size = 0.75

$$\frac{2 \tan(c+dx) - \frac{2E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{\sqrt{\cos(c+dx)}}}{bd\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^3/(b\*Sec[c + d\*x])^(3/2), x]

```
[Out] ((-2*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*Tan[c + d*x])/(b*d*S
qrt[b*Sec[c + d*x]])
```

**Maple [C]** time = 0.224, size = 322, normalized size = 4.7

$$-2 \frac{(\cos(dx+c)+1)^2 (-1+\cos(dx+c))^2}{d(\cos(dx+c))^2 (\sin(dx+c))^5} \left( i\sqrt{(\cos(dx+c)+1)^{-1}} \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}} \text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}, i\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x)`

[Out] 
$$-2/d*(\cos(d*x+c)+1)^2*(-1+\cos(d*x+c))^2*(I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)*\sin(d*x+c)-I*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}+I*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\sin(d*x+c)-I*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}+\cos(d*x+c)-1)/\cos(d*x+c)^2/\sin(d*x+c)^5/(b/\cos(d*x+c))^{3/2}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(b \sec(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate(sec(d*x + c)^3/(b*sec(d*x + c))^(3/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)} \sec(dx+c)}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*sec(d*x + c)/b^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(b \sec(c+dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**3/(b*sec(d*x+c))**(3/2),x)`

[Out] `Integral(sec(c + d*x)**3/(b*sec(c + d*x))**(3/2), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^3}{(b \sec(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="giac")`

[Out] `integrate(sec(d*x + c)^3/(b*sec(d*x + c))^(3/2), x)`



$$3.115 \quad \int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=41

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b \sec(c+dx)}}{b^2 d}$$

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(b^2\*d)

Rubi [A] time = 0.023263, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16, 3771, 2641}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \sec(c+dx)}}{b^2 d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^2/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(b^2\*d)

### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] :> Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c + dx)}{(b \sec(c + dx))^{3/2}} dx &= \frac{\int \sqrt{b \sec(c + dx)} dx}{b^2} \\ &= \frac{(\sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{b^2} \\ &= \frac{2\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{b^2 d} \end{aligned}$$

**Mathematica [A]** time = 0.0216751, size = 41, normalized size = 1.

$$\frac{2\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{b \sec(c + dx)}}{b^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^2/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(b^2\*d)

**Maple [C]** time = 0.135, size = 98, normalized size = 2.4

$$\frac{-2i(-1 + \cos(dx + c))(\cos(dx + c) + 1)^2}{d(\sin(dx + c))^2} \left((\cos(dx + c) + 1)^{-1}\right)^{\frac{5}{2}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \left(\frac{b}{\cos(dx + c)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^2/(b\*sec(d\*x+c))^(3/2), x)

[Out] -2\*I/d\*(1/(cos(d\*x+c)+1))^(5/2)\*(-1+cos(d\*x+c))\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*(cos(d\*x+c)+1)^2/(b/cos(d\*x+c))^(3/2)/(cos(d\*x+c)/(cos(d\*x+c)+1))^(3/2)/sin(d\*x+c)^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b \sec(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^2/(b\*sec(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(sec(d\*x + c)^2/(b\*sec(d\*x + c))^(3/2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)}}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^2/(b\*sec(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b\*sec(d\*x + c))/b^2, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*2/(b\*sec(d\*x+c))\*\*(3/2),x)

[Out] Integral(sec(c + d\*x)\*\*2/(b\*sec(c + d\*x))\*\*(3/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b \sec(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(3/2), x)
```

$$3.116 \quad \int \frac{\sec(c+dx)}{(b \sec(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=41

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

[Out] (2\*EllipticE[(c + d\*x)/2, 2])/(b\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.021754, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {16, 3771, 2639}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (2\*EllipticE[(c + d\*x)/2, 2])/(b\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] :> Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned} \int \frac{\sec(c+dx)}{(b \sec(c+dx))^{3/2}} dx &= \frac{\int \frac{1}{\sqrt{b \sec(c+dx)}} dx}{b} \\ &= \frac{\int \sqrt{\cos(c+dx)} dx}{b \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}} \\ &= \frac{2E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{bd \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0312372, size = 41, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{bd \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (2\*EllipticE[(c + d\*x)/2, 2])/(b\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

**Maple [C]** time = 0.144, size = 311, normalized size = 7.6

$$2 \frac{1}{d (\cos(dx+c))^2 \sin(dx+c)} \left( i \sqrt{(\cos(dx+c)+1)^{-1}} \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}} \text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}, i\right) \cos(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)/(b\*sec(d\*x+c))^(3/2), x)

[Out] 2/d\*(I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*cos(d\*x+c)\*sin(d\*x+c)-I\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*cos(d\*x+c)\*sin(d\*x+c)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)+I\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*sin(d\*x+c)-I\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*sin(d\*x+c)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)-cos(d\*x+c)^2+cos(d\*x+c))/cos(d\*x+c)^2/(b\*cos(d\*x+c))^(3/2)/sin(d\*x+c)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)}{(b \sec(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)/(b\*sec(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(sec(d\*x + c)/(b\*sec(d\*x + c))^(3/2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx + c)}}{b^2 \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)/(b\*sec(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b\*sec(d\*x + c))/(b^2\*sec(d\*x + c)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c + dx)}{(b \sec(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)/(b\*sec(d\*x+c))\*\*(3/2),x)

[Out] Integral(sec(c + d\*x)/(b\*sec(c + d\*x))\*\*(3/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)}{(b \sec(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(3/2), x)
```



$$3.117 \quad \int \frac{1}{(b \sec(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b \sec(c+dx)}}{3b^2d} + \frac{2 \sin(c+dx)}{3bd\sqrt{b \sec(c+dx)}}$$

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*b^2\*d) + (2\*Sin[c + d\*x])/(3\*b\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0329195, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3769, 3771, 2641}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \sec(c+dx)}}{3b^2d} + \frac{2 \sin(c+dx)}{3bd\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(-3/2), x]

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*b^2\*d) + (2\*Sin[c + d\*x])/(3\*b\*d\*Sqrt[b\*Sec[c + d\*x]])

#### Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d^n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]
```

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]
```

#### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{(b \sec(c + dx))^{3/2}} dx &= \frac{2 \sin(c + dx)}{3bd\sqrt{b \sec(c + dx)}} + \frac{\int \sqrt{b \sec(c + dx)} dx}{3b^2} \\ &= \frac{2 \sin(c + dx)}{3bd\sqrt{b \sec(c + dx)}} + \frac{(\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3b^2} \\ &= \frac{2\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)\sqrt{b \sec(c + dx)}}{3b^2d} + \frac{2 \sin(c + dx)}{3bd\sqrt{b \sec(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0492983, size = 59, normalized size = 0.82

$$\frac{\sec^2(c + dx) \left( 2\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \sin(2(c + dx)) \right)}{3d(b \sec(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(-3/2), x]
```

```
[Out] (Sec[c + d*x]^2*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)]))/(3*d*(b*Sec[c + d*x])^(3/2))
```

**Maple [C]** time = 0.155, size = 131, normalized size = 1.8

$$-\frac{2(\cos(dx + c) + 1)^2(-1 + \cos(dx + c))}{3d(\cos(dx + c))^2(\sin(dx + c))^3} \left( i \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(d*x+c))^(3/2), x)
```

```
[Out] -2/3/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
```

$x+c)-\cos(d*x+c)^2+\cos(d*x+c))/\cos(d*x+c)^2/\sin(d*x+c)^3/(b/\cos(d*x+c))^{(3/2)}$   
 $)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*sec(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^(-3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx + c)}}{b^2 \sec(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*sec(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b\*sec(d\*x + c))/(b^2\*sec(d\*x + c)^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*sec(d\*x+c))\*\*(3/2),x)

[Out] Integral((b\*sec(c + d\*x))\*\*(-3/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(-3/2), x)
```

$$3.118 \quad \int \frac{\cos(c+dx)}{(b \sec(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=69

$$\frac{2 \sin(c+dx)}{5d(b \sec(c+dx))^{3/2}} + \frac{6E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

[Out] (6\*EllipticE[(c + d\*x)/2, 2])/(5\*b\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*Sin[c + d\*x])/(5\*d\*(b\*Sec[c + d\*x])^(3/2))

**Rubi [A]** time = 0.0448313, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {16, 3769, 3771, 2639}

$$\frac{2 \sin(c+dx)}{5d(b \sec(c+dx))^{3/2}} + \frac{6E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (6\*EllipticE[(c + d\*x)/2, 2])/(5\*b\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*Sin[c + d\*x])/(5\*d\*(b\*Sec[c + d\*x])^(3/2))

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{(b \sec(c+dx))^{3/2}} dx &= b \int \frac{1}{(b \sec(c+dx))^{5/2}} dx \\
&= \frac{2 \sin(c+dx)}{5d(b \sec(c+dx))^{3/2}} + \frac{3 \int \frac{1}{\sqrt{b \sec(c+dx)}} dx}{5b} \\
&= \frac{2 \sin(c+dx)}{5d(b \sec(c+dx))^{3/2}} + \frac{3 \int \sqrt{\cos(c+dx)} dx}{5b \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}} \\
&= \frac{6E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5bd \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}} + \frac{2 \sin(c+dx)}{5d(b \sec(c+dx))^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0322174, size = 60, normalized size = 0.87

$$\frac{\sqrt{b \sec(c+dx)} \left( \sin(c+dx) + \sin(3(c+dx)) + 12 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| 2\right) \right)}{10b^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d\*x]/(b\*Sec[c + d\*x])^(3/2), x]

```
[Out] (Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Si
n[c + d*x] + Sin[3*(c + d*x)]))/(10*b^2*d)
```

**Maple [C]** time = 0.197, size = 321, normalized size = 4.7

$$-\frac{2}{5d(\cos(dx+c))^2 \sin(dx+c)} \left( 3i \cos(dx+c) \sin(dx+c) \sqrt{(\cos(dx+c)+1)^{-1}} \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}} \text{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x)`

[Out] 
$$-2/5/d*(3*I*\cos(d*x+c)*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-3*I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)*\cos(d*x+c)+3*I*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-3*I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)+\cos(d*x+c)^4+2*\cos(d*x+c)^2-3*\cos(d*x+c))/\cos(d*x+c)^2/(b/\cos(d*x+c))^{3/2}/\sin(d*x+c)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b \sec(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate(cos(d*x + c)/(b*sec(d*x + c))^(3/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)} \cos(dx+c)}{b^2 \sec(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*cos(d*x + c)/(b^2*sec(d*x + c)^2), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{(b \sec(c+dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)/(b\*sec(d\*x+c))\*\*(3/2),x)

[Out] Integral(cos(c + d\*x)/(b\*sec(c + d\*x))\*\*(3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c)}{(b \sec(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)/(b\*sec(d\*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(cos(d\*x + c)/(b\*sec(d\*x + c))^(3/2), x)



$$3.119 \quad \int \frac{\cos^2(c+dx)}{(b \sec(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=98

$$\frac{10\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b \sec(c+dx)}}{21b^2d} + \frac{10 \sin(c+dx)}{21bd\sqrt{b \sec(c+dx)}} + \frac{2b \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}}$$

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*b^2\*d) + (2\*b\*Sin[c + d\*x])/(7\*d\*(b\*Sec[c + d\*x])^(5/2)) + (10\*Sin[c + d\*x])/(21\*b\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0721163, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2641}

$$\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \sec(c+dx)}}{21b^2d} + \frac{10 \sin(c+dx)}{21bd\sqrt{b \sec(c+dx)}} + \frac{2b \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^2/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*b^2\*d) + (2\*b\*Sin[c + d\*x])/(7\*d\*(b\*Sec[c + d\*x])^(5/2)) + (10\*Sin[c + d\*x])/(21\*b\*d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3769

Int[(csc[(c\_)+(d\_)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n+1))/(b\*d\*n), x] + Dist[(n+1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n], x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{(b \sec(c+dx))^{3/2}} dx &= b^2 \int \frac{1}{(b \sec(c+dx))^{7/2}} dx \\
&= \frac{2b \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}} + \frac{5}{7} \int \frac{1}{(b \sec(c+dx))^{3/2}} dx \\
&= \frac{2b \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}} + \frac{10 \sin(c+dx)}{21bd\sqrt{b \sec(c+dx)}} + \frac{5 \int \sqrt{b \sec(c+dx)} dx}{21b^2} \\
&= \frac{2b \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}} + \frac{10 \sin(c+dx)}{21bd\sqrt{b \sec(c+dx)}} + \frac{(5\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{21b^2} \\
&= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx) \middle| 2\right)\sqrt{b \sec(c+dx)}}{21b^2d} + \frac{2b \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}} + \frac{10 \sin(c+dx)}{21bd\sqrt{b \sec(c+dx)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0905225, size = 66, normalized size = 0.67

$$\frac{\sqrt{b \sec(c+dx)} \left( 40\sqrt{\cos(c+dx)} \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + 26 \sin(2(c+dx)) + 3 \sin(4(c+dx)) \right)}{84b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Sec[c + d*x])^(3/2), x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26
*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*b^2*d)
```

**Maple [C]** time = 0.184, size = 153, normalized size = 1.6

$$\frac{2 (\cos(dx + c) + 1)^2 (-1 + \cos(dx + c))}{21 d (\sin(dx + c))^3 (\cos(dx + c))^2} \left( 5 i \operatorname{EllipticF} \left( \frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i \right) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2), x)`

[Out] `-2/21/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(5*I*EllipticF(I*(-1+cos(d*x+c)))/sin(d*x+c), I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*cos(d*x+c)^4+3*cos(d*x+c)^3-5*cos(d*x+c)^2+5*cos(d*x+c))/sin(d*x+c)^3/cos(d*x+c)^2/(b/cos(d*x+c))^(3/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c)^2}{(b \sec(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2), x, algorithm="maxima")`

[Out] `integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(3/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{\sqrt{b \sec(dx + c)} \cos(dx + c)^2}{b^2 \sec(dx + c)^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2), x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^2/(b^2*sec(d*x + c)^2), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c + dx)}{(b \sec(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*\*2/(b\*sec(d\*x+c))\*\*(3/2), x)

[Out] Integral(cos(c + d\*x)\*\*2/(b\*sec(c + d\*x))\*\*(3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c)^2}{(b \sec(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^2/(b\*sec(d\*x+c))^(3/2), x, algorithm="giac")

[Out] integrate(cos(d\*x + c)^2/(b\*sec(d\*x + c))^(3/2), x)

$$3.120 \quad \int \frac{\cos^3(c+dx)}{(b \sec(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=97

$$\frac{2b^2 \sin(c+dx)}{9d(b \sec(c+dx))^{7/2}} + \frac{14 \sin(c+dx)}{45d(b \sec(c+dx))^{3/2}} + \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

[Out] (14\*EllipticE[(c + d\*x)/2, 2])/(15\*b\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b^2\*Sin[c + d\*x])/(9\*d\*(b\*Sec[c + d\*x])^(7/2)) + (14\*Sin[c + d\*x])/(45\*d\*(b\*Sec[c + d\*x])^(3/2))

**Rubi [A]** time = 0.0712229, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2639}

$$\frac{2b^2 \sin(c+dx)}{9d(b \sec(c+dx))^{7/2}} + \frac{14 \sin(c+dx)}{45d(b \sec(c+dx))^{3/2}} + \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^3/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (14\*EllipticE[(c + d\*x)/2, 2])/(15\*b\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b^2\*Sin[c + d\*x])/(9\*d\*(b\*Sec[c + d\*x])^(7/2)) + (14\*Sin[c + d\*x])/(45\*d\*(b\*Sec[c + d\*x])^(3/2))

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n+1))/(b\*d\*n), x] + Dist[(n+1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n], x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{(b \sec(c+dx))^{3/2}} dx &= b^3 \int \frac{1}{(b \sec(c+dx))^{9/2}} dx \\
&= \frac{2b^2 \sin(c+dx)}{9d(b \sec(c+dx))^{7/2}} + \frac{1}{9}(7b) \int \frac{1}{(b \sec(c+dx))^{5/2}} dx \\
&= \frac{2b^2 \sin(c+dx)}{9d(b \sec(c+dx))^{7/2}} + \frac{14 \sin(c+dx)}{45d(b \sec(c+dx))^{3/2}} + \frac{7 \int \frac{1}{\sqrt{b \sec(c+dx)}} dx}{15b} \\
&= \frac{2b^2 \sin(c+dx)}{9d(b \sec(c+dx))^{7/2}} + \frac{14 \sin(c+dx)}{45d(b \sec(c+dx))^{3/2}} + \frac{7 \int \sqrt{\cos(c+dx)} dx}{15b \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}} \\
&= \frac{14E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{15bd \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}} + \frac{2b^2 \sin(c+dx)}{9d(b \sec(c+dx))^{7/2}} + \frac{14 \sin(c+dx)}{45d(b \sec(c+dx))^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0770183, size = 73, normalized size = 0.75

$$\frac{84E\left(\frac{1}{2}(c+dx) \middle| 2\right) + (33 \sin(c+dx) + 5 \sin(3(c+dx))) \cos^{\frac{3}{2}}(c+dx)}{90d \cos^{\frac{3}{2}}(c+dx) (b \sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(b*Sec[c + d*x])^(3/2), x]
```

```
[Out] (84*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]^(3/2)*(33*Sin[c + d*x] + 5*Sin
[3*(c + d*x)]))/(90*d*Cos[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2))
```

**Maple [C]** time = 0.19, size = 333, normalized size = 3.4

$$\frac{2}{45 d \sin(dx+c) (\cos(dx+c))^2} \left( 21 i \sqrt{(\cos(dx+c)+1)^{-1}} \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}} \text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}, i\right) \sin(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x)`

[Out] `2/45/d*(21*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)-21*I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-5*cos(d*x+c)^6+21*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-21*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-2*cos(d*x+c)^4-14*cos(d*x+c)^2+21*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^2/(b/cos(d*x+c))^(3/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{(b \sec(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate(cos(d*x + c)^3/(b*sec(d*x + c))^(3/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)} \cos(dx+c)^3}{b^2 \sec(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^3/(b^2*sec(d*x + c)^2), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**3/(b*sec(d*x+c))**(3/2), x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c)^3}{(b \sec(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2), x, algorithm="giac")`

[Out] `integrate(cos(d*x + c)^3/(b*sec(d*x + c))^(3/2), x)`



$$3.121 \quad \int \frac{\sec^7(c+dx)}{(b \sec(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=100

$$\frac{10\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b \sec(c+dx)}}{21b^3d} + \frac{2 \sin(c+dx)(b \sec(c+dx))^{7/2}}{7b^6d} + \frac{10 \sin(c+dx)(b \sec(c+dx))}{21b^4d}$$

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*b^3\*d) + (10\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(21\*b^4\*d) + (2\*(b\*Sec[c + d\*x])^(7/2)\*Sin[c + d\*x])/(7\*b^6\*d)

**Rubi [A]** time = 0.0571708, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2641}

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^{7/2}}{7b^6d} + \frac{10 \sin(c+dx)(b \sec(c+dx))^{3/2}}{21b^4d} + \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx) \middle| 2\right)\sqrt{b \sec(c+dx)}}{21b^3d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^7/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*b^3\*d) + (10\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(21\*b^4\*d) + (2\*(b\*Sec[c + d\*x])^(7/2)\*Sin[c + d\*x])/(7\*b^6\*d)

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Csc[c + d\*x]\*(b\*Csc[c + d\*x])^(n-1))/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*Csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sec^7(c+dx)}{(b \sec(c+dx))^{5/2}} dx &= \frac{\int (b \sec(c+dx))^{9/2} dx}{b^7} \\
&= \frac{2(b \sec(c+dx))^{7/2} \sin(c+dx)}{7b^6d} + \frac{5 \int (b \sec(c+dx))^{5/2} dx}{7b^5} \\
&= \frac{10(b \sec(c+dx))^{3/2} \sin(c+dx)}{21b^4d} + \frac{2(b \sec(c+dx))^{7/2} \sin(c+dx)}{7b^6d} + \frac{5 \int \sqrt{b \sec(c+dx)} dx}{21b^3} \\
&= \frac{10(b \sec(c+dx))^{3/2} \sin(c+dx)}{21b^4d} + \frac{2(b \sec(c+dx))^{7/2} \sin(c+dx)}{7b^6d} + \frac{(5\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)})}{21b^3} \\
&= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \sec(c+dx)}}{21b^3d} + \frac{10(b \sec(c+dx))^{3/2} \sin(c+dx)}{21b^4d} + \frac{2(b \sec(c+dx))^{7/2} \sin(c+dx)}{7b^6d}
\end{aligned}$$

**Mathematica [A]** time = 0.132118, size = 64, normalized size = 0.64

$$\frac{(b \sec(c+dx))^{5/2} \left( 10 \cos^2(c+dx) \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + 5 \sin(2(c+dx)) + 6 \tan(c+dx) \right)}{21b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(b*Sec[c + d*x])^(5/2), x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*b^5*d)
```

**Maple [C]** time = 0.217, size = 152, normalized size = 1.5

$$-\frac{2(\cos(dx+c)+1)^2(-1+\cos(dx+c))}{21d(\sin(dx+c))^3(\cos(dx+c))^6} \left( 5i(\cos(dx+c))^3 \sin(dx+c) \sqrt{(\cos(dx+c)+1)^{-1}} \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}} \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + 5 \sin(2(c+dx)) + 6 \tan(c+dx) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x)`

[Out] 
$$-2/21/d*(\cos(dx+c)+1)^2*(-1+\cos(dx+c))*(5*I*\cos(dx+c)^3*\sin(dx+c)*(1/(\cos(dx+c)+1))^{1/2}*(\cos(dx+c)/(\cos(dx+c)+1))^{1/2}*EllipticF(I*(-1+\cos(dx+c))/\sin(dx+c),I)-5*\cos(dx+c)^3+5*\cos(dx+c)^2-3*\cos(dx+c)+3)/\sin(dx+c)^3/\cos(dx+c)^6/(b/\cos(dx+c))^{5/2}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^7}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate(sec(d*x + c)^7/(b*sec(d*x + c))^(5/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)} \sec(dx+c)^4}{b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^4/b^3, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^7}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^7/(b*sec(d*x + c))^(5/2), x)
```

$$3.122 \quad \int \frac{\sec^6(c+dx)}{(b \sec(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=100

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^{5/2}}{5b^5d} + \frac{6 \sin(c+dx)\sqrt{b \sec(c+dx)}}{5b^3d} - \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

[Out] (-6\*EllipticE[(c + d\*x)/2, 2])/(5\*b^2\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (6\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(5\*b^3\*d) + (2\*(b\*Sec[c + d\*x])^(5/2)\*Sin[c + d\*x])/(5\*b^5\*d)

**Rubi [A]** time = 0.058089, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2639}

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^{5/2}}{5b^5d} + \frac{6 \sin(c+dx)\sqrt{b \sec(c+dx)}}{5b^3d} - \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^6/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (-6\*EllipticE[(c + d\*x)/2, 2])/(5\*b^2\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (6\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(5\*b^3\*d) + (2\*(b\*Sec[c + d\*x])^(5/2)\*Sin[c + d\*x])/(5\*b^5\*d)

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x])\*(b\*Csc[c + d\*x])^(n-1)/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*Csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sec^6(c+dx)}{(b \sec(c+dx))^{5/2}} dx &= \frac{\int (b \sec(c+dx))^{7/2} dx}{b^6} \\
&= \frac{2(b \sec(c+dx))^{5/2} \sin(c+dx)}{5b^5 d} + \frac{3 \int (b \sec(c+dx))^{3/2} dx}{5b^4} \\
&= \frac{6\sqrt{b \sec(c+dx)} \sin(c+dx)}{5b^3 d} + \frac{2(b \sec(c+dx))^{5/2} \sin(c+dx)}{5b^5 d} - \frac{3 \int \frac{1}{\sqrt{b \sec(c+dx)}} dx}{5b^2} \\
&= \frac{6\sqrt{b \sec(c+dx)} \sin(c+dx)}{5b^3 d} + \frac{2(b \sec(c+dx))^{5/2} \sin(c+dx)}{5b^5 d} - \frac{3 \int \sqrt{\cos(c+dx)} dx}{5b^2 \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}} \\
&= -\frac{6E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5b^2 d \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}} + \frac{6\sqrt{b \sec(c+dx)} \sin(c+dx)}{5b^3 d} + \frac{2(b \sec(c+dx))^{5/2} \sin(c+dx)}{5b^5 d}
\end{aligned}$$

**Mathematica [A]** time = 0.113415, size = 64, normalized size = 0.64

$$\frac{2 \tan(c+dx) (\sec^2(c+dx) + 3) - \frac{6E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{\sqrt{\cos(c+dx)}}}{5b^2 d \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(b*Sec[c + d*x])^(5/2), x]
```

```
[Out] ((-6*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(3 + Sec[c + d*x]^2)
*Tan[c + d*x])/(5*b^2*d*Sqrt[b*Sec[c + d*x]])
```

**Maple [C]** time = 0.222, size = 351, normalized size = 3.5

$$-\frac{2(\cos(dx+c)+1)^2(-1+\cos(dx+c))^2}{5db^5(\sin(dx+c))^5} \left( 3i \operatorname{EllipticF} \left( \frac{i(-1+\cos(dx+c))}{\sin(dx+c)}, i \right) (\cos(dx+c))^3 \sqrt{(\cos(dx+c)+1)^{-1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x)`

[Out] 
$$-2/5/d*(\cos(d*x+c)+1)^2*(-1+\cos(d*x+c))^2*(3*I*\operatorname{EllipticF}(I*(-1+\cos(d*x+c)))/\sin(d*x+c),I)*\cos(d*x+c)^3*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\sin(d*x+c)-3*I*\operatorname{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)^3*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\sin(d*x+c)+3*I*\operatorname{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\cos(d*x+c)^2*\sin(d*x+c)-3*I*\operatorname{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\cos(d*x+c)^2*\sin(d*x+c)+3*\cos(d*x+c)^3-2*\cos(d*x+c)^2-1)*(b/\cos(d*x+c))^{5/2}/b^5/\sin(d*x+c)^5$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^6}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate(sec(d*x + c)^6/(b*sec(d*x + c))^(5/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{\sqrt{b \sec(dx+c)} \sec(dx+c)^3}{b^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^3/b^3, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^6(c + dx)}{(b \sec(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**6/(b*sec(d*x+c))**(5/2), x)`

[Out] `Integral(sec(c + d*x)**6/(b*sec(c + d*x))**(5/2), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^6}{(b \sec(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2), x, algorithm="giac")`

[Out] `integrate(sec(d*x + c)^6/(b*sec(d*x + c))^(5/2), x)`



$$3.123 \quad \int \frac{\sec^5(c+dx)}{(b \sec(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b \sec(c+dx)}}{3b^3d} + \frac{2 \sin(c+dx)(b \sec(c+dx))^{3/2}}{3b^4d}$$

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*b^3\*d) + (2\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(3\*b^4\*d)

**Rubi [A]** time = 0.0383166, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2641}

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^{3/2}}{3b^4d} + \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx) \middle| 2\right)\sqrt{b \sec(c+dx)}}{3b^3d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^5/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*b^3\*d) + (2\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(3\*b^4\*d)

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x])\*(b\*Csc[c + d\*x])^(n-1)/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*Csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\sec^5(c+dx)}{(b \sec(c+dx))^{5/2}} dx &= \frac{\int (b \sec(c+dx))^{5/2} dx}{b^5} \\
 &= \frac{2(b \sec(c+dx))^{3/2} \sin(c+dx)}{3b^4 d} + \frac{\int \sqrt{b \sec(c+dx)} dx}{3b^3} \\
 &= \frac{2(b \sec(c+dx))^{3/2} \sin(c+dx)}{3b^4 d} + \frac{(\sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3b^3} \\
 &= \frac{2\sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \sec(c+dx)}}{3b^3 d} + \frac{2(b \sec(c+dx))^{3/2} \sin(c+dx)}{3b^4 d}
 \end{aligned}$$

**Mathematica [A]** time = 0.065955, size = 51, normalized size = 0.71

$$\frac{2\sqrt{b \sec(c+dx)} \left( \sqrt{\cos(c+dx)} \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + \tan(c+dx) \right)}{3b^3 d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^5/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (2\*Sqrt[b\*Sec[c + d\*x]]\*(Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2] + Tan[c + d\*x]))/(3\*b^3\*d)

**Maple [C]** time = 0.174, size = 130, normalized size = 1.8

$$-\frac{2(\cos(dx+c)+1)^2(-1+\cos(dx+c))}{3d(\cos(dx+c))^4(\sin(dx+c))^3} \left( i\sqrt{(\cos(dx+c)+1)^{-1}} \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}} \text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}, i\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x)`

[Out] 
$$-2/3/d*(\cos(d*x+c)+1)^2*(-1+\cos(d*x+c))*(I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)*\sin(d*x+c)-\cos(d*x+c)+1)/\cos(d*x+c)^4/(b/\cos(d*x+c))^{5/2}/\sin(d*x+c)^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^5}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate(sec(d*x + c)^5/(b*sec(d*x + c))^(5/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)} \sec(dx+c)^2}{b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*sec(d*x + c)^2/b^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5(c+dx)}{(b \sec(c+dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(b*sec(c + d*x))**(5/2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^5}{(b \sec(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(b*sec(d*x + c))^(5/2), x)
```

$$3.124 \quad \int \frac{\sec^4(c+dx)}{(b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=68

$$\frac{2 \sin(c+dx) \sqrt{b \sec(c+dx)}}{b^3 d} - \frac{2E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}}$$

[Out]  $(-2*\text{EllipticE}[(c+d*x)/2, 2])/(b^2*d*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[b*\text{Sec}[c+d*x]]) + (2*\text{Sqrt}[b*\text{Sec}[c+d*x]]*\text{Sin}[c+d*x])/(b^3*d)$

Rubi [A] time = 0.0383202, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3768, 3771, 2639}

$$\frac{2 \sin(c+dx) \sqrt{b \sec(c+dx)}}{b^3 d} - \frac{2E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sec}[c+d*x]^4/(b*\text{Sec}[c+d*x])^{5/2}, x]$

[Out]  $(-2*\text{EllipticE}[(c+d*x)/2, 2])/(b^2*d*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[b*\text{Sec}[c+d*x]]) + (2*\text{Sqrt}[b*\text{Sec}[c+d*x]]*\text{Sin}[c+d*x])/(b^3*d)$

### Rule 16

$\text{Int}[(u_*)*(v_)^{(m_*)}*((b_)*(v_))^{(n_)}, x\_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /;$  FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3768

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_))^{(n_)}, x\_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c+d*x])*(b*\text{Csc}[c+d*x])^{(n-1)}]/(d*(n-1)), x] + \text{Dist}[(b^2*(n-2))/(n-1), \text{Int}[(b*\text{Csc}[c+d*x])^{(n-2)}, x], x] /;$  FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3771

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_*)]*(b_))^{(n_)}, x\_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c+d*x])^{(n-1)}*\text{Sin}[c+d*x]^n, \text{Int}[1/\text{Sin}[c+d*x]^n, x], x] /;$  FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c+dx)}{(b \sec(c+dx))^{5/2}} dx &= \frac{\int (b \sec(c+dx))^{3/2} dx}{b^4} \\
&= \frac{2\sqrt{b \sec(c+dx)} \sin(c+dx)}{b^3 d} - \frac{\int \frac{1}{\sqrt{b \sec(c+dx)}} dx}{b^2} \\
&= \frac{2\sqrt{b \sec(c+dx)} \sin(c+dx)}{b^3 d} - \frac{\int \sqrt{\cos(c+dx)} dx}{b^2 \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}} \\
&= -\frac{2E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}} + \frac{2\sqrt{b \sec(c+dx)} \sin(c+dx)}{b^3 d}
\end{aligned}$$

**Mathematica [A]** time = 0.0525358, size = 51, normalized size = 0.75

$$\frac{2 \tan(c+dx) - \frac{2E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{\sqrt{\cos(c+dx)}}}{b^2 d \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^4/(b\*Sec[c + d\*x])^(5/2), x]

```
[Out] ((-2*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*Tan[c + d*x])/(b^2*d
*Sqrt[b*Sec[c + d*x]])
```

**Maple [C]** time = 0.234, size = 322, normalized size = 4.7

$$-2 \frac{(\cos(dx+c)+1)^2 (-1+\cos(dx+c))^2}{d (\sin(dx+c))^5 (\cos(dx+c))^3} \left( i \sqrt{(\cos(dx+c)+1)^{-1}} \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}} \text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}, i\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x)`

[Out] 
$$-2/d*(\cos(d*x+c)+1)^2*(-1+\cos(d*x+c))^2*(I*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)*\sin(d*x+c)-I*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\cos(d*x+c)*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}+I*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}*\sin(d*x+c)-I*\text{EllipticE}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)*\sin(d*x+c)*(1/(\cos(d*x+c)+1))^{1/2}*(\cos(d*x+c)/(\cos(d*x+c)+1))^{1/2}+\cos(d*x+c)-1)/\sin(d*x+c)^5/\cos(d*x+c)^3/(b/\cos(d*x+c))^{5/2}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate(sec(d*x + c)^4/(b*sec(d*x + c))^(5/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)} \sec(dx+c)}{b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*sec(d*x + c)/b^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{(b \sec(c+dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*4/(b\*sec(d\*x+c))\*\*(5/2),x)

[Out] Integral(sec(c + d\*x)\*\*4/(b\*sec(c + d\*x))\*\*(5/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^4}{(b \sec(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^4/(b\*sec(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(sec(d\*x + c)^4/(b\*sec(d\*x + c))^(5/2), x)



$$3.125 \quad \int \frac{\sec^3(c+dx)}{(b \sec(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=41

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b \sec(c+dx)}}{b^3 d}$$

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(b^3\*d)

**Rubi [A]** time = 0.0218402, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16, 3771, 2641}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \sec(c+dx)}}{b^3 d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^3/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(b^3\*d)

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] :> Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec^3(c+dx)}{(b \sec(c+dx))^{5/2}} dx &= \frac{\int \sqrt{b \sec(c+dx)} dx}{b^3} \\ &= \frac{(\sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{b^3} \\ &= \frac{2\sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \sec(c+dx)}}{b^3 d} \end{aligned}$$

**Mathematica [A]** time = 0.0156618, size = 41, normalized size = 1.

$$\frac{2\sqrt{\cos(c+dx)} \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{b \sec(c+dx)}}{b^3 d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^3/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(b^3\*d)

**Maple [C]** time = 0.14, size = 98, normalized size = 2.4

$$\frac{-2i(-1 + \cos(dx+c))(\cos(dx+c)+1)^2}{d(\sin(dx+c))^2} \left((\cos(dx+c)+1)^{-1}\right)^{\frac{7}{2}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx+c))}{\sin(dx+c)}, i\right) \left(\frac{b}{\cos(dx+c)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^3/(b\*sec(d\*x+c))^(5/2), x)

[Out] -2\*I/d\*(1/(cos(d\*x+c)+1))^(7/2)\*(-1+cos(d\*x+c))\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c), I)\*(cos(d\*x+c)+1)^2/(b/cos(d\*x+c))^(5/2)/(cos(d\*x+c)/(cos(d\*x+c)+1))^(5/2)/sin(d\*x+c)^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^3/(b\*sec(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(sec(d\*x + c)^3/(b\*sec(d\*x + c))^(5/2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)}}{b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^3/(b\*sec(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b\*sec(d\*x + c))/b^3, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(b \sec(c+dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*3/(b\*sec(d\*x+c))\*\*(5/2),x)

[Out] Integral(sec(c + d\*x)\*\*3/(b\*sec(c + d\*x))\*\*(5/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c))^(5/2), x)
```

$$3.126 \quad \int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=41

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}}$$

[Out] (2\*EllipticE[(c + d\*x)/2, 2])/(b^2\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0226226, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16, 3771, 2639}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^2/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (2\*EllipticE[(c + d\*x)/2, 2])/(b^2\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{5/2}} dx &= \frac{\int \frac{1}{\sqrt{b \sec(c+dx)}} dx}{b^2} \\ &= \frac{\int \sqrt{\cos(c+dx)} dx}{b^2 \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}} \\ &= \frac{2E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0364556, size = 38, normalized size = 0.93

$$\frac{2E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{d \cos^{\frac{5}{2}}(c+dx) (b \sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^2/(b\*Sec[c + d\*x])^(5/2),x]

[Out] (2\*EllipticE[(c + d\*x)/2, 2])/(d\*Cos[c + d\*x]^(5/2)\*(b\*Sec[c + d\*x])^(5/2))

**Maple [C]** time = 0.143, size = 311, normalized size = 7.6

$$2 \frac{1}{d (\cos(dx+c))^3 \sin(dx+c)} \left( i \sqrt{(\cos(dx+c)+1)^{-1}} \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}} \text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}, i\right) \cos(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^2/(b\*sec(d\*x+c))^(5/2),x)

[Out] 2/d\*(I\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*cos(d\*x+c)\*sin(d\*x+c)-I\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*cos(d\*x+c)\*sin(d\*x+c)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)+I\*EllipticF(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*(1/(cos(d\*x+c)+1))^(1/2)\*(cos(d\*x+c)/(cos(d\*x+c)+1))^(1/2)\*sin(d\*x+c)-I\*EllipticE(I\*(-1+cos(d\*x+c))/sin(d\*x+c),I)\*sin(d\*x+c)\*(1/(cos(d\*x+c)+1))^(1/2)\*

$\cos(dx+c)/(\cos(dx+c)+1))^{1/2}-\cos(dx+c)^2+\cos(dx+c))/\cos(dx+c)^3/(b/c$   
 $\cos(dx+c))^{5/2}/\sin(dx+c)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^2/(b\*sec(dx+c))^(5/2),x, algorithm="maxima")

[Out] integrate(sec(dx + c)^2/(b\*sec(dx + c))^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)}}{b^3 \sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^2/(b\*sec(dx+c))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b\*sec(dx + c))/(b^3\*sec(dx + c)), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)\*\*2/(b\*sec(dx+c))\*\*(5/2),x)

[Out] Integral(sec(c + dx)\*\*2/(b\*sec(c + dx))\*\*(5/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(5/2), x)
```



$$3.127 \quad \int \frac{\sec(c+dx)}{(b \sec(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b \sec(c+dx)}}{3b^3d} + \frac{2 \sin(c+dx)}{3b^2d\sqrt{b \sec(c+dx)}}$$

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*b^3\*d) + (2\*Sin[c + d\*x])/(3\*b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0400432, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {16, 3769, 3771, 2641}

$$\frac{2 \sin(c+dx)}{3b^2d\sqrt{b \sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx) \middle| 2\right)\sqrt{b \sec(c+dx)}}{3b^3d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (2\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(3\*b^3\*d) + (2\*Sin[c + d\*x])/(3\*b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n+1))/(b\*d\*n), x] + Dist[(n+1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{(b \sec(c+dx))^{5/2}} dx &= \frac{\int \frac{1}{(b \sec(c+dx))^{3/2}} dx}{b} \\
&= \frac{2 \sin(c+dx)}{3b^2 d \sqrt{b \sec(c+dx)}} + \frac{\int \sqrt{b \sec(c+dx)} dx}{3b^3} \\
&= \frac{2 \sin(c+dx)}{3b^2 d \sqrt{b \sec(c+dx)}} + \frac{(\sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3b^3} \\
&= \frac{2 \sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \sec(c+dx)}}{3b^3 d} + \frac{2 \sin(c+dx)}{3b^2 d \sqrt{b \sec(c+dx)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0306355, size = 62, normalized size = 0.86

$$\frac{\sec^2(c+dx) \left( 2 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + \sin(2(c+dx)) \right)}{3bd(b \sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]/(b\*Sec[c + d\*x])^(5/2), x]

```
[Out] (Sec[c + d*x]^2*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c
+ d*x)]))/(3*b*d*(b*Sec[c + d*x])^(3/2))
```

**Maple [C]** time = 0.147, size = 131, normalized size = 1.8

$$-\frac{(-2 + 2 \cos(dx+c)) (\cos(dx+c) + 1)^2}{3d (\sin(dx+c))^3 (\cos(dx+c))^3} \left( i \operatorname{EllipticF}\left(\frac{i(-1 + \cos(dx+c))}{\sin(dx+c)}, i\right) \sqrt{(\cos(dx+c) + 1)^{-1}} \sqrt{\frac{\cos(dx+c)}{\cos(dx+c) + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x)`

[Out] `-2/3/d*(-1+cos(d*x+c))*(I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)^2+cos(d*x+c))*(cos(d*x+c)+1)^2/sin(d*x+c)^3/cos(d*x+c)^3/(b/cos(d*x+c))^(5/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate(sec(d*x + c)/(b*sec(d*x + c))^(5/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)}}{b^3 \sec(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))/(b^3*sec(d*x + c)^2), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(b \sec(c+dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)/(b*sec(c + d*x))**(5/2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)}{(b \sec(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(5/2), x)
```

$$3.128 \quad \int \frac{1}{(b \sec(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=72

$$\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}$$

[Out] (6\*EllipticE[(c + d\*x)/2, 2])/(5\*b^2\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*Sin[c + d\*x])/(5\*b\*d\*(b\*Sec[c + d\*x])^(3/2))

**Rubi [A]** time = 0.0337027, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3769, 3771, 2639}

$$\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(-5/2), x]

[Out] (6\*EllipticE[(c + d\*x)/2, 2])/(5\*b^2\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*Sin[c + d\*x])/(5\*b\*d\*(b\*Sec[c + d\*x])^(3/2))

#### Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]
```

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]
```

#### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{(b \sec(c + dx))^{5/2}} dx &= \frac{2 \sin(c + dx)}{5bd(b \sec(c + dx))^{3/2}} + \frac{3 \int \frac{1}{\sqrt{b \sec(c + dx)}} dx}{5b^2} \\ &= \frac{2 \sin(c + dx)}{5bd(b \sec(c + dx))^{3/2}} + \frac{3 \int \sqrt{\cos(c + dx)} dx}{5b^2 \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} \\ &= \frac{6E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^2 d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2 \sin(c + dx)}{5bd(b \sec(c + dx))^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.0180601, size = 60, normalized size = 0.83

$$\frac{\sqrt{b \sec(c + dx)} \left( \sin(c + dx) + \sin(3(c + dx)) + 12 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{10b^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(-5/2), x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Si
n[c + d*x] + Sin[3*(c + d*x)]))/(10*b^3*d)
```

**Maple [C]** time = 0.199, size = 321, normalized size = 4.5

$$-\frac{2}{5d(\cos(dx+c))^3 \sin(dx+c)} \left( 3i \cos(dx+c) \sin(dx+c) \sqrt{(\cos(dx+c)+1)^{-1}} \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}} \text{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}, I\right) - 3i \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(d*x+c))^(5/2), x)
```

```
[Out] -2/5/d*(3*I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c), I)-3*I*(1/(cos(d*x
```

$(+c)+1))^{1/2} * (\cos(dx+c) / (\cos(dx+c)+1))^{1/2} * \text{EllipticF}(I * (-1+\cos(dx+c)) / \sin(dx+c), I) * \sin(dx+c) * \cos(dx+c) + 3 * I * \sin(dx+c) * (1 / (\cos(dx+c)+1))^{1/2} * (\cos(dx+c) / (\cos(dx+c)+1))^{1/2} * \text{EllipticE}(I * (-1+\cos(dx+c)) / \sin(dx+c), I) - 3 * I * (1 / (\cos(dx+c)+1))^{1/2} * (\cos(dx+c) / (\cos(dx+c)+1))^{1/2} * \text{EllipticF}(I * (-1+\cos(dx+c)) / \sin(dx+c), I) * \sin(dx+c) + \cos(dx+c)^4 + 2 * \cos(dx+c)^2 - 3 * \cos(dx+c)) / \cos(dx+c)^3 / \sin(dx+c) / (b / \cos(dx+c))^{5/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(dx+c))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*sec(dx+c))^(5/2),x, algorithm="maxima")

[Out] integrate((b\*sec(dx + c))^(-5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)}}{b^3 \sec(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*sec(dx+c))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b\*sec(dx + c))/(b^3\*sec(dx + c)^3), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(c+dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*sec(dx+c))\*\*(5/2),x)

```
[Out] Integral((b*sec(c + d*x))**(-5/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2), x)
```



$$3.129 \quad \int \frac{\cos(c+dx)}{(b \sec(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=97

$$\frac{10\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b \sec(c+dx)}}{21b^3d} + \frac{10 \sin(c+dx)}{21b^2d\sqrt{b \sec(c+dx)}} + \frac{2 \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}}$$

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*b^3\*d) + (2\*Sin[c + d\*x])/(7\*d\*(b\*Sec[c + d\*x])^(5/2)) + (10\*Sin[c + d\*x])/(21\*b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0634044, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {16, 3769, 3771, 2641}

$$\frac{10 \sin(c+dx)}{21b^2d\sqrt{b \sec(c+dx)}} + \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx) \middle| 2\right)\sqrt{b \sec(c+dx)}}{21b^3d} + \frac{2 \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*b^3\*d) + (2\*Sin[c + d\*x])/(7\*d\*(b\*Sec[c + d\*x])^(5/2)) + (10\*Sin[c + d\*x])/(21\*b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n+1))/(b\*d\*n), x] + Dist[(n+1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n], x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{(b \sec(c+dx))^{5/2}} dx &= b \int \frac{1}{(b \sec(c+dx))^{7/2}} dx \\
&= \frac{2 \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}} + \frac{5 \int \frac{1}{(b \sec(c+dx))^{3/2}} dx}{7b} \\
&= \frac{2 \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}} + \frac{10 \sin(c+dx)}{21b^2 d \sqrt{b \sec(c+dx)}} + \frac{5 \int \sqrt{b \sec(c+dx)} dx}{21b^3} \\
&= \frac{2 \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}} + \frac{10 \sin(c+dx)}{21b^2 d \sqrt{b \sec(c+dx)}} + \frac{(5 \sqrt{\cos(c+dx)} \sqrt{b \sec(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{21b^3} \\
&= \frac{10 \sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \sec(c+dx)}}{21b^3 d} + \frac{2 \sin(c+dx)}{7d(b \sec(c+dx))^{5/2}} + \frac{10 \sin(c+dx)}{21b^2 d \sqrt{b \sec(c+dx)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0689545, size = 66, normalized size = 0.68

$$\frac{\sqrt{b \sec(c+dx)} \left( 40 \sqrt{\cos(c+dx)} \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + 26 \sin(2(c+dx)) + 3 \sin(4(c+dx)) \right)}{84b^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Sec[c + d*x])^(5/2), x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26
*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*b^3*d)
```

**Maple [C]** time = 0.178, size = 153, normalized size = 1.6

$$\frac{2 (\cos(dx + c) + 1)^2 (-1 + \cos(dx + c))}{21 d (\sin(dx + c))^3 (\cos(dx + c))^3} \left( 5 i \operatorname{EllipticF} \left( \frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i \right) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)/(b*sec(d*x+c))^(5/2), x)`

[Out] `-2/21/d*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))*(5*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*cos(d*x+c)^4+3*cos(d*x+c)^3-5*cos(d*x+c)^2+5*cos(d*x+c))/sin(d*x+c)^3/cos(d*x+c)^3/(b/cos(d*x+c))^(5/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c)}{(b \sec(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)/(b*sec(d*x+c))^(5/2), x, algorithm="maxima")`

[Out] `integrate(cos(d*x + c)/(b*sec(d*x + c))^(5/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{\sqrt{b \sec(dx + c)} \cos(dx + c)}{b^3 \sec(dx + c)^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)/(b*sec(d*x+c))^(5/2), x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*cos(d*x + c)/(b^3*sec(d*x + c)^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)/(b\*sec(d\*x+c))\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c)}{(b \sec(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)/(b\*sec(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(cos(d\*x + c)/(b\*sec(d\*x + c))^(5/2), x)

$$3.130 \quad \int \frac{\cos^2(c+dx)}{(b \sec(c+dx))^{5/2}} dx$$

Optimal. Leaf size=98

$$\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{14\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}} + \frac{2b\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}$$

[Out] (14\*EllipticE[(c + d\*x)/2, 2])/(15\*b^2\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b\*Sin[c + d\*x])/(9\*d\*(b\*Sec[c + d\*x])^(7/2)) + (14\*Sin[c + d\*x])/(45\*b\*d\*(b\*Sec[c + d\*x])^(3/2))

**Rubi [A]** time = 0.0723242, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {16, 3769, 3771, 2639}

$$\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{14\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}} + \frac{2b\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^2/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (14\*EllipticE[(c + d\*x)/2, 2])/(15\*b^2\*d\*Sqrt[Cos[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]) + (2\*b\*Sin[c + d\*x])/(9\*d\*(b\*Sec[c + d\*x])^(7/2)) + (14\*Sin[c + d\*x])/(45\*b\*d\*(b\*Sec[c + d\*x])^(3/2))

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c + dx)}{(b \sec(c + dx))^{5/2}} dx &= b^2 \int \frac{1}{(b \sec(c + dx))^{9/2}} dx \\
&= \frac{2b \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{7}{9} \int \frac{1}{(b \sec(c + dx))^{5/2}} dx \\
&= \frac{2b \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14 \sin(c + dx)}{45bd(b \sec(c + dx))^{3/2}} + \frac{7 \int \frac{1}{\sqrt{b \sec(c + dx)}} dx}{15b^2} \\
&= \frac{2b \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14 \sin(c + dx)}{45bd(b \sec(c + dx))^{3/2}} + \frac{7 \int \sqrt{\cos(c + dx)} dx}{15b^2 \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} \\
&= \frac{14E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15b^2 d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2b \sin(c + dx)}{9d(b \sec(c + dx))^{7/2}} + \frac{14 \sin(c + dx)}{45bd(b \sec(c + dx))^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.120966, size = 73, normalized size = 0.74

$$\frac{4(33 \sin(c + dx) + 5 \sin(3(c + dx))) \cos(c + dx) + \frac{336E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{\sqrt{\cos(c + dx)}}}{360b^2 d \sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Sec[c + d*x])^(5/2), x]
```

```
[Out] ((336*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 4*Cos[c + d*x]*(33*Si
n[c + d*x] + 5*Sin[3*(c + d*x)]))/(360*b^2*d*Sqrt[b*Sec[c + d*x]])
```

**Maple [C]** time = 0.183, size = 333, normalized size = 3.4

$$-\frac{2}{45 d \sin(dx+c) (\cos(dx+c))^3} \left( 21 i \cos(dx+c) \sin(dx+c) \sqrt{(\cos(dx+c)+1)^{-1}} \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}} \operatorname{EllipticE} \left( \frac{i(-1}{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x)`

[Out] `-2/45/d*(21*I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-21*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)+5*cos(d*x+c)^6+21*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-21*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^4+14*cos(d*x+c)^2-21*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^3/(b/cos(d*x+c))^(5/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(5/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{\sqrt{b \sec(dx+c)} \cos(dx+c)^2}{b^3 \sec(dx+c)^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))*cos(d*x + c)^2/(b^3*sec(d*x + c)^3), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(5/2), x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c)^2}{(b \sec(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2), x, algorithm="giac")`

[Out] `integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(5/2), x)`



$$3.131 \quad \int \frac{1}{(b \sec(c+dx))^{7/2}} dx$$

**Optimal.** Leaf size=100

$$\frac{10\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{b \sec(c+dx)}}{21b^4d} + \frac{10 \sin(c+dx)}{21b^3d\sqrt{b \sec(c+dx)}} + \frac{2 \sin(c+dx)}{7bd(b \sec(c+dx))^{5/2}}$$

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*b^4\*d) + (2\*Sin[c + d\*x])/(7\*b\*d\*(b\*Sec[c + d\*x])^(5/2)) + (10\*Sin[c + d\*x])/(21\*b^3\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0512744, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3769, 3771, 2641}

$$\frac{10 \sin(c+dx)}{21b^3d\sqrt{b \sec(c+dx)}} + \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx) \middle| 2\right)\sqrt{b \sec(c+dx)}}{21b^4d} + \frac{2 \sin(c+dx)}{7bd(b \sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(-7/2), x]

[Out] (10\*Sqrt[Cos[c + d\*x]]\*EllipticF[(c + d\*x)/2, 2]\*Sqrt[b\*Sec[c + d\*x]])/(21\*b^4\*d) + (2\*Sin[c + d\*x])/(7\*b\*d\*(b\*Sec[c + d\*x])^(5/2)) + (10\*Sin[c + d\*x])/(21\*b^3\*d\*Sqrt[b\*Sec[c + d\*x]])

#### Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

#### Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
  Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(b \sec(c + dx))^{7/2}} dx &= \frac{2 \sin(c + dx)}{7bd(b \sec(c + dx))^{5/2}} + \frac{5 \int \frac{1}{(b \sec(c + dx))^{3/2}} dx}{7b^2} \\
 &= \frac{2 \sin(c + dx)}{7bd(b \sec(c + dx))^{5/2}} + \frac{10 \sin(c + dx)}{21b^3 d \sqrt{b \sec(c + dx)}} + \frac{5 \int \sqrt{b \sec(c + dx)} dx}{21b^4} \\
 &= \frac{2 \sin(c + dx)}{7bd(b \sec(c + dx))^{5/2}} + \frac{10 \sin(c + dx)}{21b^3 d \sqrt{b \sec(c + dx)}} + \frac{(5 \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{21b^4} \\
 &= \frac{10 \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{21b^4 d} + \frac{2 \sin(c + dx)}{7bd(b \sec(c + dx))^{5/2}} + \frac{10 \sin(c + dx)}{21b^3 d \sqrt{b \sec(c + dx)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0204968, size = 66, normalized size = 0.66

$$\frac{\sqrt{b \sec(c + dx)} \left( 40 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 26 \sin(2(c + dx)) + 3 \sin(4(c + dx)) \right)}{84b^4 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(-7/2), x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26
*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*b^4*d)
```

**Maple [C]** time = 0.187, size = 153, normalized size = 1.5

$$-\frac{2(\cos(dx + c) + 1)^2(-1 + \cos(dx + c))}{21d(\cos(dx + c))^4(\sin(dx + c))^3} \left( 5i \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}, i\right) \sqrt{(\cos(dx + c) + 1)^{-1}} \sqrt{\frac{\cos(dx + c)}{\cos(dx + c) + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*sec(d*x+c))^(7/2),x)`

[Out] 
$$-2/21/d*(\cos(dx+c)+1)^2*(-1+\cos(dx+c))*(5*I*EllipticF(I*(-1+\cos(dx+c))/\sin(dx+c),I)*(1/(\cos(dx+c)+1))^{1/2}*(\cos(dx+c)/(\cos(dx+c)+1))^{1/2}*\sin(dx+c)-3*\cos(dx+c)^4+3*\cos(dx+c)^3-5*\cos(dx+c)^2+5*\cos(dx+c))/(b/\cos(dx+c))^{7/2}/\cos(dx+c)^4/\sin(dx+c)^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(dx + c))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*sec(d*x+c))^(7/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^(7/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec(dx + c)}}{b^4 \sec(dx + c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*sec(d*x+c))^(7/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sec(d*x + c))/(b^4*sec(d*x + c)^4), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*sec(d*x+c))^(7/2),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(dx + c))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*sec(d\*x+c))^(7/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(7/2), x)

### 3.132 $\int \sec^{\frac{9}{2}}(c + dx) \sqrt{b \sec(c + dx)} dx$

**Optimal.** Leaf size=107

$$\frac{\sin(c + dx) \sec^{\frac{7}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{4d} + \frac{3 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{8d} + \frac{3 \sqrt{b \sec(c + dx)} \tanh^{-1}(\sin(c + dx))}{8d \sqrt{\sec(c + dx)}}$$

[Out] (3\*ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(8\*d\*Sqrt[Sec[c + d\*x]]) + (3\*Sec[c + d\*x]^(3/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(8\*d) + (Sec[c + d\*x]^(7/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(4\*d)

**Rubi [A]** time = 0.0332845, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 3768, 3770}

$$\frac{\sin(c + dx) \sec^{\frac{7}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{4d} + \frac{3 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{8d} + \frac{3 \sqrt{b \sec(c + dx)} \tanh^{-1}(\sin(c + dx))}{8d \sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(9/2)\*Sqrt[b\*Sec[c + d\*x]], x]

[Out] (3\*ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(8\*d\*Sqrt[Sec[c + d\*x]]) + (3\*Sec[c + d\*x]^(3/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(8\*d) + (Sec[c + d\*x]^(7/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(4\*d)

#### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x])\*(b\*Csc[c + d\*x])^(n - 1)/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
 \int \sec^{\frac{9}{2}}(c + dx) \sqrt{b \sec(c + dx)} dx &= \frac{\sqrt{b \sec(c + dx)} \int \sec^5(c + dx) dx}{\sqrt{\sec(c + dx)}} \\
 &= \frac{\sec^{\frac{7}{2}}(c + dx) \sqrt{b \sec(c + dx)} \sin(c + dx)}{4d} + \frac{(3\sqrt{b \sec(c + dx)}) \int \sec^3(c + dx) dx}{4\sqrt{\sec(c + dx)}} \\
 &= \frac{3 \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)} \sin(c + dx)}{8d} + \frac{\sec^{\frac{7}{2}}(c + dx) \sqrt{b \sec(c + dx)} \sin(c + dx)}{4d} \\
 &= \frac{3 \tanh^{-1}(\sin(c + dx)) \sqrt{b \sec(c + dx)}}{8d\sqrt{\sec(c + dx)}} + \frac{3 \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)} \sin(c + dx)}{8d} + \dots
 \end{aligned}$$

**Mathematica [A]** time = 0.141481, size = 64, normalized size = 0.6

$$\frac{\sqrt{b \sec(c + dx)} \left( 3 \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) \sec(c + dx) (2 \sec^2(c + dx) + 3) \right)}{8d\sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)*Sqrt[b*Sec[c + d*x]], x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(3*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(3 + 2*Sec[c + d*x]^2)*Tan[c + d*x]))/(8*d*Sqrt[Sec[c + d*x]])
```

**Maple [A]** time = 0.178, size = 131, normalized size = 1.2

$$\frac{\cos(dx + c)}{8d} \left( 3 (\cos(dx + c))^4 \ln \left( -\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) - 3 (\cos(dx + c))^4 \ln \left( -\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)*(b*sec(d*x+c))^(1/2), x)
```

```
[Out] 1/8/d*(3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*cos(d*x+c)^2*sin(d*x+c)+2*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(9/2)*(b/cos(d*x+c))^(1/2)
```

**Maxima [B]** time = 2.48916, size = 2236, normalized size = 20.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
```

c), cos(2\*d\*x + 2\*c))) - 44\*(cos(8\*d\*x + 8\*c) + 4\*cos(6\*d\*x + 6\*c) + 6\*cos(4\*d\*x + 4\*c) + 4\*cos(2\*d\*x + 2\*c) + 1)\*sin(5/2\*arctan2(sin(2\*d\*x + 2\*c), cos(2\*d\*x + 2\*c))) + 44\*(cos(8\*d\*x + 8\*c) + 4\*cos(6\*d\*x + 6\*c) + 6\*cos(4\*d\*x + 4\*c) + 4\*cos(2\*d\*x + 2\*c) + 1)\*sin(3/2\*arctan2(sin(2\*d\*x + 2\*c), cos(2\*d\*x + 2\*c))) + 12\*(cos(8\*d\*x + 8\*c) + 4\*cos(6\*d\*x + 6\*c) + 6\*cos(4\*d\*x + 4\*c) + 4\*cos(2\*d\*x + 2\*c) + 1)\*sin(1/2\*arctan2(sin(2\*d\*x + 2\*c), cos(2\*d\*x + 2\*c))))\*sqrt(b)/((2\*(4\*cos(6\*d\*x + 6\*c) + 6\*cos(4\*d\*x + 4\*c) + 4\*cos(2\*d\*x + 2\*c) + 1)\*cos(8\*d\*x + 8\*c) + cos(8\*d\*x + 8\*c)^2 + 8\*(6\*cos(4\*d\*x + 4\*c) + 4\*cos(2\*d\*x + 2\*c) + 1)\*cos(6\*d\*x + 6\*c) + 16\*cos(6\*d\*x + 6\*c)^2 + 12\*(4\*cos(2\*d\*x + 2\*c) + 1)\*cos(4\*d\*x + 4\*c) + 36\*cos(4\*d\*x + 4\*c)^2 + 16\*cos(2\*d\*x + 2\*c)^2 + 4\*(2\*sin(6\*d\*x + 6\*c) + 3\*sin(4\*d\*x + 4\*c) + 2\*sin(2\*d\*x + 2\*c))\*sin(8\*d\*x + 8\*c) + sin(8\*d\*x + 8\*c)^2 + 16\*(3\*sin(4\*d\*x + 4\*c) + 2\*sin(2\*d\*x + 2\*c))\*sin(6\*d\*x + 6\*c) + 16\*sin(6\*d\*x + 6\*c)^2 + 36\*sin(4\*d\*x + 4\*c)^2 + 48\*sin(4\*d\*x + 4\*c)\*sin(2\*d\*x + 2\*c) + 16\*sin(2\*d\*x + 2\*c)^2 + 8\*cos(2\*d\*x + 2\*c) + 1)\*d)

**Fricas [A]** time = 1.60803, size = 608, normalized size = 5.68

$$\left[ \frac{3 \sqrt{b} \cos(dx+c)^3 \log\left( \frac{b \cos(dx+c)^2 - 2 \sqrt{b} \sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b}{\cos(dx+c)^2} \right) + \frac{2(3 \cos(dx+c)^2 + 2) \sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{\cos(dx+c)}}}{16 d \cos(dx+c)^3}, - \frac{3 \sqrt{-b} \arctan\left( \frac{\sqrt{\cos(dx+c)} \sin(dx+c)}{\cos(dx+c)} \right)}{\cos(dx+c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(9/2)\*(b\*sec(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/16\*(3\*sqrt(b)\*cos(d\*x + c)^3\*log(-(b\*cos(d\*x + c)^2 - 2\*sqrt(b)\*sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c) - 2\*b)/cos(d\*x + c)^2) + 2\*(3\*cos(d\*x + c)^2 + 2)\*sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/sqrt(cos(d\*x + c)))/(d\*cos(d\*x + c)^3), -1/8\*(3\*sqrt(-b)\*arctan(sqrt(-b)\*sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c)/b)\*cos(d\*x + c)^3 - (3\*cos(d\*x + c)^2 + 2)\*sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/sqrt(cos(d\*x + c)))/(d\*cos(d\*x + c)^3)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(9/2)*(b*sec(d*x+c))**(1/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \sec(dx + c)^{\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(9/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^(9/2), x)`

### 3.133 $\int \sec^{\frac{7}{2}}(c + dx) \sqrt{b \sec(c + dx)} dx$

**Optimal.** Leaf size=70

$$\frac{\sin^3(c + dx) \sec^{\frac{5}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{3d} + \frac{\sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{b \sec(c + dx)}}{d}$$

[Out] (Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/d + (Sec[c + d\*x]^(5/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*d)

**Rubi [A]** time = 0.0165725, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 3767}

$$\frac{\sin^3(c + dx) \sec^{\frac{5}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{3d} + \frac{\sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{b \sec(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(7/2)\*Sqrt[b\*Sec[c + d\*x]],x]

[Out] (Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/d + (Sec[c + d\*x]^(5/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*d)

#### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

#### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

#### Rubi steps

$$\begin{aligned}
\int \sec^{\frac{7}{2}}(c+dx) \sqrt{b \sec(c+dx)} dx &= \frac{\sqrt{b \sec(c+dx)} \int \sec^4(c+dx) dx}{\sqrt{\sec(c+dx)}} \\
&= -\frac{\sqrt{b \sec(c+dx)} \operatorname{Subst}\left(\int (1+x^2) dx, x, -\tan(c+dx)\right)}{d \sqrt{\sec(c+dx)}} \\
&= \frac{\sqrt{\sec(c+dx)} \sqrt{b \sec(c+dx)} \sin(c+dx)}{d} + \frac{\sec^{\frac{5}{2}}(c+dx) \sqrt{b \sec(c+dx)} \sin^3(c+dx)}{3d}
\end{aligned}$$

**Mathematica [A]** time = 0.0867656, size = 45, normalized size = 0.64

$$\frac{\left(\frac{1}{3} \tan^3(c+dx) + \tan(c+dx)\right) \sqrt{b \sec(c+dx)}}{d \sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^(7/2)\*Sqrt[b\*Sec[c + d\*x]],x]

[Out] (Sqrt[b\*Sec[c + d\*x]]\*(Tan[c + d\*x] + Tan[c + d\*x]^3/3))/(d\*Sqrt[Sec[c + d\*x]])

**Maple [A]** time = 0.129, size = 52, normalized size = 0.7

$$\frac{(2(\cos(dx+c))^2+1)\cos(dx+c)\sin(dx+c)}{3d} \left((\cos(dx+c))^{-1}\right)^{\frac{7}{2}} \sqrt{\frac{b}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(7/2)\*(b\*sec(d\*x+c))^(1/2),x)

[Out] 1/3/d\*(2\*cos(d\*x+c)^2+1)\*(1/cos(d\*x+c))^(7/2)\*(b/cos(d\*x+c))^(1/2)\*cos(d\*x+c)\*sin(d\*x+c)

**Maxima [B]** time = 2.07997, size = 397, normalized size = 5.67

$$\frac{4((3 \cos(2dx+2c)+1) \sin(2dx+2c) + 3(2(3 \cos(4dx+4c)+3 \cos(2dx+2c)+1) \cos(6dx+6c) + \cos(6dx+6c))^2 + 6(3 \cos(2dx+2c)+1) \cos(4dx+4c) \sin(4dx+4c))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)
*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)
)*sin(2*d*x + 2*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) +
1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4
*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x +
4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*
x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 +
6*cos(2*d*x + 2*c) + 1)*d)
```

**Fricas [A]** time = 1.41191, size = 115, normalized size = 1.64

$$\frac{(2 \cos(dx + c)^2 + 1) \sqrt{\frac{b}{\cos(dx+c)} \sin(dx + c)}}{3 d \cos(dx + c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(d*x + c)^2 + 1)*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)
)^(5/2))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \sec(dx + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^(7/2), x)
```

### 3.134 $\int \sec^{\frac{5}{2}}(c + dx) \sqrt{b \sec(c + dx)} dx$

**Optimal.** Leaf size=72

$$\frac{\sin(c + dx) \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{2d} + \frac{\sqrt{b \sec(c + dx)} \tanh^{-1}(\sin(c + dx))}{2d \sqrt{\sec(c + dx)}}$$

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(2\*d\*Sqrt[Sec[c + d\*x]]) + (Sec[c + d\*x]^(3/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(2\*d)

**Rubi [A]** time = 0.0192353, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 3768, 3770}

$$\frac{\sin(c + dx) \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{2d} + \frac{\sqrt{b \sec(c + dx)} \tanh^{-1}(\sin(c + dx))}{2d \sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(5/2)\*Sqrt[b\*Sec[c + d\*x]],x]

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(2\*d\*Sqrt[Sec[c + d\*x]]) + (Sec[c + d\*x]^(3/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(2\*d)

#### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \sec^{\frac{5}{2}}(c + dx) \sqrt{b \sec(c + dx)} dx &= \frac{\sqrt{b \sec(c + dx)} \int \sec^3(c + dx) dx}{\sqrt{\sec(c + dx)}} \\ &= \frac{\sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)} \sin(c + dx)}{2d} + \frac{\sqrt{b \sec(c + dx)} \int \sec(c + dx) dx}{2\sqrt{\sec(c + dx)}} \\ &= \frac{\tanh^{-1}(\sin(c + dx)) \sqrt{b \sec(c + dx)}}{2d\sqrt{\sec(c + dx)}} + \frac{\sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)} \sin(c + dx)}{2d} \end{aligned}$$

**Mathematica [A]** time = 0.0517154, size = 50, normalized size = 0.69

$$\frac{\sqrt{b \sec(c + dx)} (\tanh^{-1}(\sin(c + dx)) + \tan(c + dx) \sec(c + dx))}{2d\sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]], x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(2*d*Sqrt[Sec[c + d*x]])
```

**Maple [A]** time = 0.135, size = 112, normalized size = 1.6

$$\frac{\cos(dx + c)}{2d} \left( \ln \left( \frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^2 - \ln \left( \frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2), x)
```

```
[Out] 1/2/d*(ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+sin(d*x+c))*cos(d*x+c)*(1/cos(d
```

$$*x+c))^{(5/2)}*(b/\cos(d*x+c))^{(1/2)}$$

**Maxima [B]** time = 2.11265, size = 892, normalized size = 12.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(5/2)\*(b\*sec(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] 
$$-1/4*(4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) + (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) - 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*\sqrt{b}/((2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*d)$$

**Fricas [A]** time = 1.5485, size = 528, normalized size = 7.33

$$\left[ \frac{\sqrt{b} \cos(dx + c) \log \left( -\frac{b \cos(dx+c)^2 - 2\sqrt{b} \sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b}{\cos(dx+c)^2} \right) + \frac{2\sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4d \cos(dx + c)}, \sqrt{-b} \arctan \left( \frac{\sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)}}{b} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)), -1/2*(sqrt(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b*cos(d*x + c) - sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \sec(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^(5/2), x)
```

$$3.135 \quad \int \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)} dx$$

**Optimal.** Leaf size=32

$$\frac{\sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{b \sec(c + dx)}}{d}$$

[Out] (Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/d

**Rubi [A]** time = 0.0117126, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 3767, 8}

$$\frac{\sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{b \sec(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(3/2)\*Sqrt[b\*Sec[c + d\*x]],x]

[Out] (Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/d

#### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

#### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned}
\int \sec^{\frac{3}{2}}(c+dx) \sqrt{b \sec(c+dx)} dx &= \frac{\sqrt{b \sec(c+dx)} \int \sec^2(c+dx) dx}{\sqrt{\sec(c+dx)}} \\
&= -\frac{\sqrt{b \sec(c+dx)} \operatorname{Subst}\left(\int 1 dx, x, -\tan(c+dx)\right)}{d \sqrt{\sec(c+dx)}} \\
&= \frac{\sqrt{\sec(c+dx)} \sqrt{b \sec(c+dx)} \sin(c+dx)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.0176707, size = 32, normalized size = 1.

$$\frac{\sin(c+dx) \sqrt{\sec(c+dx)} \sqrt{b \sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^(3/2)\*Sqrt[b\*Sec[c + d\*x]],x]

[Out] (Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/d

**Maple [A]** time = 0.13, size = 39, normalized size = 1.2

$$\frac{\cos(dx+c) \sin(dx+c)}{d} \left( (\cos(dx+c))^{-1} \right)^{\frac{3}{2}} \sqrt{\frac{b}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(3/2)\*(b\*sec(d\*x+c))^(1/2),x)

[Out] 1/d\*cos(d\*x+c)\*sin(d\*x+c)\*(1/cos(d\*x+c))^(3/2)\*(b/cos(d\*x+c))^(1/2)

**Maxima [A]** time = 2.14705, size = 73, normalized size = 2.28

$$\frac{2 \sqrt{b} \sin(2 dx + 2 c)}{(\cos(2 dx + 2 c)^2 + \sin(2 dx + 2 c)^2 + 2 \cos(2 dx + 2 c) + 1) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(3/2)\*(b\*sec(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] 2\*sqrt(b)\*sin(2\*d\*x + 2\*c)/((cos(2\*d\*x + 2\*c)^2 + sin(2\*d\*x + 2\*c)^2 + 2\*cos(2\*d\*x + 2\*c) + 1)\*d)

**Fricas [A]** time = 1.36651, size = 78, normalized size = 2.44

$$\frac{\sqrt{\frac{b}{\cos(dx+c)} \sin(dx+c)}}{d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(3/2)\*(b\*sec(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/(d\*sqrt(cos(d\*x + c)))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*(3/2)\*(b\*sec(d\*x+c))\*\*(1/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx+c)} \sec(dx+c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(3/2)\*(b\*sec(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b\*sec(d\*x + c))\*sec(d\*x + c)^(3/2), x)

### 3.136 $\int \sqrt{\sec(c + dx)} \sqrt{b \sec(c + dx)} dx$

**Optimal.** Leaf size=33

$$\frac{\sqrt{b \sec(c + dx)} \tanh^{-1}(\sin(c + dx))}{d \sqrt{\sec(c + dx)}}$$

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(d\*Sqrt[Sec[c + d\*x]])

**Rubi [A]** time = 0.006833, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 3770}

$$\frac{\sqrt{b \sec(c + dx)} \tanh^{-1}(\sin(c + dx))}{d \sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]],x]

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(d\*Sqrt[Sec[c + d\*x]])

#### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

#### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \sqrt{\sec(c + dx)} \sqrt{b \sec(c + dx)} dx &= \frac{\sqrt{b \sec(c + dx)} \int \sec(c + dx) dx}{\sqrt{\sec(c + dx)}} \\ &= \frac{\tanh^{-1}(\sin(c + dx)) \sqrt{b \sec(c + dx)}}{d \sqrt{\sec(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0113776, size = 33, normalized size = 1.

$$\frac{\sqrt{b \sec(c + dx)} \tanh^{-1}(\sin(c + dx))}{d \sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]],x]

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(d\*Sqrt[Sec[c + d\*x]])

**Maple [A]** time = 0.116, size = 52, normalized size = 1.6

$$-2 \frac{\cos(dx + c) \sqrt{(\cos(dx + c))^{-1}}}{d} \operatorname{Artanh}\left(\frac{-1 + \cos(dx + c)}{\sin(dx + c)}\right) \sqrt{\frac{b}{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(1/2)\*(b\*sec(d\*x+c))^(1/2),x)

[Out] -2/d\*arctanh((-1+cos(d\*x+c))/sin(d\*x+c))\*cos(d\*x+c)\*(1/cos(d\*x+c))^(1/2)\*(b/cos(d\*x+c))^(1/2)

**Maxima [B]** time = 2.06356, size = 88, normalized size = 2.67

$$\frac{\sqrt{b}(\log(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \sin(dx + c) + 1) - \log(\cos(dx + c)^2 + \sin(dx + c)^2 - 2 \sin(dx + c) + 1))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(1/2)\*(b\*sec(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/2\*sqrt(b)\*(log(cos(d\*x + c)^2 + sin(d\*x + c)^2 + 2\*sin(d\*x + c) + 1) - log(cos(d\*x + c)^2 + sin(d\*x + c)^2 - 2\*sin(d\*x + c) + 1))/d

**Fricas [A]** time = 1.49578, size = 290, normalized size = 8.79

$$\left[ \frac{\sqrt{b} \log \left( \frac{b \cos(dx+c)^2 - 2\sqrt{b} \sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b}{\cos(dx+c)^2} \right)}{2d}, -\frac{\sqrt{-b} \arctan \left( \frac{\sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c)}{b} \right)}{d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(1/2)\*(b\*sec(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/2\*sqrt(b)\*log(-(b\*cos(d\*x + c))^2 - 2\*sqrt(b)\*sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c) - 2\*b)/cos(d\*x + c)^2)/d, -sqrt(-b)\*arctan(sqrt(-b)\*sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c)/b)/d]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(c + dx)} \sqrt{\sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*(1/2)\*(b\*sec(d\*x+c))\*\*(1/2),x)

[Out] Integral(sqrt(b\*sec(c + d\*x))\*sqrt(sec(c + d\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c)} \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(1/2)\*(b\*sec(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b\*sec(d\*x + c))\*sqrt(sec(d\*x + c)), x)

$$3.137 \quad \int \frac{\sqrt{b \sec(c+dx)}}{\sqrt{\sec(c+dx)}} dx$$

**Optimal.** Leaf size=24

$$\frac{x\sqrt{b \sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

[Out] (x\*Sqrt[b\*Sec[c + d\*x]])/Sqrt[Sec[c + d\*x]]

**Rubi [A]** time = 0.0022498, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 8}

$$\frac{x\sqrt{b \sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b\*Sec[c + d\*x]]/Sqrt[Sec[c + d\*x]],x]

[Out] (x\*Sqrt[b\*Sec[c + d\*x]])/Sqrt[Sec[c + d\*x]]

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b \sec(c+dx)}}{\sqrt{\sec(c+dx)}} dx &= \frac{\sqrt{b \sec(c+dx)}}{\sqrt{\sec(c+dx)}} \int 1 dx \\ &= \frac{x\sqrt{b \sec(c+dx)}}{\sqrt{\sec(c+dx)}} \end{aligned}$$



**Mathematica [A]** time = 0.0144343, size = 24, normalized size = 1.

$$\frac{x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b\*Sec[c + d\*x]]/Sqrt[Sec[c + d\*x]],x]

[Out] (x\*Sqrt[b\*Sec[c + d\*x]])/Sqrt[Sec[c + d\*x]]

**Maple [A]** time = 0.108, size = 32, normalized size = 1.3

$$\frac{dx+c}{d} \sqrt{\frac{b}{\cos(dx+c)}} \frac{1}{\sqrt{(\cos(dx+c))^{-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(1/2)/sec(d\*x+c)^(1/2),x)

[Out] 1/d\*(d\*x+c)\*(b/cos(d\*x+c))^(1/2)/(1/cos(d\*x+c))^(1/2)

**Maxima [A]** time = 1.7862, size = 35, normalized size = 1.46

$$\frac{2\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(1/2)/sec(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] 2\*sqrt(b)\*arctan(sin(d\*x + c)/(cos(d\*x + c) + 1))/d

**Fricas [A]** time = 1.63846, size = 266, normalized size = 11.08

$$\left[ \frac{\sqrt{-b} \log \left( -2 \sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 2b \cos(dx+c)^2 - b \right)}{2d}, \frac{\sqrt{b} \arctan \left( \frac{\sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{b} \sqrt{\cos(dx+c)}} \right)}{d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(1/2)/sec(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/2\*sqrt(-b)\*log(-2\*sqrt(-b)\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + 2\*b\*cos(d\*x + c)^2 - b)/d, sqrt(b)\*arctan(sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/(sqrt(b)\*sqrt(cos(d\*x + c))))/d]

**Sympy [A]** time = 1.55017, size = 5, normalized size = 0.21

$$\sqrt{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(1/2)/sec(d\*x+c)\*\*(1/2),x)

[Out] sqrt(b)\*x

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \sec(dx+c)}}{\sqrt{\sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(1/2)/sec(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b\*sec(d\*x + c))/sqrt(sec(d\*x + c)), x)

$$3.138 \quad \int \frac{\sqrt{b \sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=32

$$\frac{\sin(c+dx)\sqrt{b \sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

[Out] (Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[Sec[c + d\*x]])

**Rubi [A]** time = 0.0066801, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 2637}

$$\frac{\sin(c+dx)\sqrt{b \sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b\*Sec[c + d\*x]]/Sec[c + d\*x]^(3/2), x]

[Out] (Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[Sec[c + d\*x]])

#### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

#### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\int \frac{\sqrt{b \sec(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx = \frac{\sqrt{b \sec(c + dx)} \int \cos(c + dx) dx}{\sqrt{\sec(c + dx)}} \\ = \frac{\sqrt{b \sec(c + dx)} \sin(c + dx)}{d \sqrt{\sec(c + dx)}}$$

**Mathematica [A]** time = 0.0355135, size = 32, normalized size = 1.

$$\frac{\sin(c + dx) \sqrt{b \sec(c + dx)}}{d \sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b\*Sec[c + d\*x]]/Sec[c + d\*x]^(3/2), x]

[Out] (Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[Sec[c + d\*x]])

**Maple [A]** time = 0.127, size = 41, normalized size = 1.3

$$\frac{\sin(dx + c)}{d \cos(dx + c)} \sqrt{\frac{b}{\cos(dx + c)}} \left( (\cos(dx + c))^{-1} \right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(1/2)/sec(d\*x+c)^(3/2), x)

[Out] 1/d\*sin(d\*x+c)\*(b/cos(d\*x+c))^(1/2)/(1/cos(d\*x+c))^(3/2)/cos(d\*x+c)

**Maxima [A]** time = 2.05672, size = 18, normalized size = 0.56

$$\frac{\sqrt{b} \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(1/2)/sec(d\*x+c)^(3/2), x, algorithm="maxima")

[Out]  $\sqrt{b} \sin(dx + c)/d$

**Fricas [A]** time = 1.40192, size = 76, normalized size = 2.38

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")`

[Out]  $\sqrt{b/\cos(dx+c)} \sqrt{\cos(dx+c)} \sin(dx+c)/d$

**Sympy [A]** time = 20.9001, size = 36, normalized size = 1.12

$$\begin{cases} \frac{\sqrt{b} \tan(c+dx)}{d \sec(c+dx)} & \text{for } d \neq 0 \\ \frac{x \sqrt{b} \sec(c)}{\sec^2(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)`

[Out] `Piecewise((sqrt(b)*tan(c + d*x)/(d*sec(c + d*x)), Ne(d, 0)), (x*sqrt(b*sec(c))/sec(c)**(3/2), True))`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \sec(dx+c)}}{\sec(dx+c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")`

[Out] `integrate(sqrt(b*sec(d*x + c))/sec(d*x + c)^(3/2), x)`

$$3.139 \quad \int \frac{\sqrt{b \sec(c+dx)}}{\sec^2(c+dx)} dx$$

**Optimal.** Leaf size=63

$$\frac{x\sqrt{b \sec(c+dx)}}{2\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)\sqrt{b \sec(c+dx)}}{2d \sec^{\frac{3}{2}}(c+dx)}$$

[Out] (x\*Sqrt[b\*Sec[c + d\*x]])/(2\*Sqrt[Sec[c + d\*x]]) + (Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(2\*d\*Sec[c + d\*x]^(3/2))

**Rubi [A]** time = 0.0137245, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 2635, 8}

$$\frac{x\sqrt{b \sec(c+dx)}}{2\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)\sqrt{b \sec(c+dx)}}{2d \sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b\*Sec[c + d\*x]]/Sec[c + d\*x]^(5/2), x]

[Out] (x\*Sqrt[b\*Sec[c + d\*x]])/(2\*Sqrt[Sec[c + d\*x]]) + (Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(2\*d\*Sec[c + d\*x]^(3/2))

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x])\*(b\*Sin[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b \sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)} dx &= \frac{\sqrt{b \sec(c+dx)} \int \cos^2(c+dx) dx}{\sqrt{\sec(c+dx)}} \\ &= \frac{\sqrt{b \sec(c+dx)} \sin(c+dx)}{2d \sec^{\frac{3}{2}}(c+dx)} + \frac{\sqrt{b \sec(c+dx)} \int 1 dx}{2\sqrt{\sec(c+dx)}} \\ &= \frac{x\sqrt{b \sec(c+dx)}}{2\sqrt{\sec(c+dx)}} + \frac{\sqrt{b \sec(c+dx)} \sin(c+dx)}{2d \sec^{\frac{3}{2}}(c+dx)} \end{aligned}$$

**Mathematica [A]** time = 0.0639679, size = 45, normalized size = 0.71

$$\frac{(2(c+dx) + \sin(2(c+dx)))\sqrt{b \sec(c+dx)}}{4d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b\*Sec[c + d\*x]]/Sec[c + d\*x]^(5/2), x]

[Out] (Sqrt[b\*Sec[c + d\*x]]\*(2\*(c + d\*x) + Sin[2\*(c + d\*x)]))/(4\*d\*Sqrt[Sec[c + d\*x]])

**Maple [A]** time = 0.14, size = 54, normalized size = 0.9

$$\frac{\cos(dx+c)\sin(dx+c)+dx+c}{2d(\cos(dx+c))^2} \sqrt{\frac{b}{\cos(dx+c)}} \left((\cos(dx+c))^{-1}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(1/2)/sec(d\*x+c)^(5/2), x)

[Out] 1/2/d\*(cos(d\*x+c)\*sin(d\*x+c)+d\*x+c)\*(b/cos(d\*x+c))^(1/2)/cos(d\*x+c)^2/(1/cos(d\*x+c))^(5/2)

**Maxima [A]** time = 2.01943, size = 34, normalized size = 0.54

$$\frac{(2dx + 2c + \sin(2dx + 2c))\sqrt{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(1/2)/sec(d\*x+c)^(5/2),x, algorithm="maxima")

[Out] 1/4\*(2\*d\*x + 2\*c + sin(2\*d\*x + 2\*c))\*sqrt(b)/d

**Fricas [A]** time = 1.6486, size = 428, normalized size = 6.79

$$\left[ \frac{2 \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + \sqrt{-b} \log\left(-2 \sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 2b \cos(dx+c)^2 - b\right)}{4d}, \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(1/2)/sec(d\*x+c)^(5/2),x, algorithm="fricas")

[Out] [1/4\*(2\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + sqrt(-b)\*log(-2\*sqrt(-b)\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + 2\*b\*cos(d\*x + c)^2 - b))/d, 1/2\*(sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + sqrt(b)\*arctan(sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/(sqrt(b)\*sqrt(cos(d\*x + c)))))/d]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(1/2)/sec(d\*x+c)\*\*(5/2),x)



[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \sec(dx + c)}}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(1/2)/sec(d\*x+c)^(5/2),x, algorithm="giac")

[Out] integrate(sqrt(b\*sec(d\*x + c))/sec(d\*x + c)^(5/2), x)

$$3.140 \quad \int \frac{\sqrt{b \sec(c+dx)}}{7 \sec^2(c+dx)} dx$$

**Optimal.** Leaf size=70

$$\frac{\sin(c+dx)\sqrt{b \sec(c+dx)}}{d\sqrt{\sec(c+dx)}} - \frac{\sin^3(c+dx)\sqrt{b \sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

[Out] (Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[Sec[c + d\*x]]) - (Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*d\*Sqrt[Sec[c + d\*x]])

**Rubi [A]** time = 0.0163579, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 2633}

$$\frac{\sin(c+dx)\sqrt{b \sec(c+dx)}}{d\sqrt{\sec(c+dx)}} - \frac{\sin^3(c+dx)\sqrt{b \sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b\*Sec[c + d\*x]]/Sec[c + d\*x]^(7/2), x]

[Out] (Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[Sec[c + d\*x]]) - (Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*d\*Sqrt[Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 2633

Int[sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

### Rubi steps

$$\int \frac{\sqrt{b \sec(c+dx)}^7}{\sec^2(c+dx)} dx = \frac{\sqrt{b \sec(c+dx)} \int \cos^3(c+dx) dx}{\sqrt{\sec(c+dx)}}$$

$$= -\frac{\sqrt{b \sec(c+dx)} \text{Subst}\left(\int (1-x^2) dx, x, -\sin(c+dx)\right)}{d\sqrt{\sec(c+dx)}}$$

$$= \frac{\sqrt{b \sec(c+dx)} \sin(c+dx)}{d\sqrt{\sec(c+dx)}} - \frac{\sqrt{b \sec(c+dx)} \sin^3(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

**Mathematica [A]** time = 0.108713, size = 45, normalized size = 0.64

$$\frac{\sin(c+dx)(\cos(2(c+dx))+5)\sqrt{b \sec(c+dx)}}{6d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b\*Sec[c + d\*x]]/Sec[c + d\*x]^(7/2), x]

[Out] ((5 + Cos[2\*(c + d\*x)])\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(6\*d\*Sqrt[Sec[c + d\*x]])

**Maple [A]** time = 0.135, size = 52, normalized size = 0.7

$$\frac{((\cos(dx+c))^2+2)\sin(dx+c)}{3d(\cos(dx+c))^3} \sqrt{\frac{b}{\cos(dx+c)}} ((\cos(dx+c))^{-1})^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(1/2)/sec(d\*x+c)^(7/2), x)

[Out] 1/3/d\*(cos(d\*x+c)^2+2)\*(b/cos(d\*x+c))^(1/2)\*sin(d\*x+c)/(1/cos(d\*x+c))^(7/2)/cos(d\*x+c)^3

**Maxima [A]** time = 2.07164, size = 57, normalized size = 0.81

$$\frac{\sqrt{b}\left(\sin(3dx+3c)+9\sin\left(\frac{1}{3}\arctan\left(\frac{\sin(3dx+3c)}{\cos(3dx+3c)}\right)\right)\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(1/2)/sec(d\*x+c)^(7/2),x, algorithm="maxima")

[Out] 1/12\*sqrt(b)\*(sin(3\*d\*x + 3\*c) + 9\*sin(1/3\*arctan2(sin(3\*d\*x + 3\*c), cos(3\*d\*x + 3\*c))))/d

**Fricas [A]** time = 1.38295, size = 130, normalized size = 1.86

$$\frac{(\cos(dx+c)^3 + 2\cos(dx+c))\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{3d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(1/2)/sec(d\*x+c)^(7/2),x, algorithm="fricas")

[Out] 1/3\*(cos(d\*x + c)^3 + 2\*cos(d\*x + c))\*sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/(d\*sqrt(cos(d\*x + c)))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(1/2)/sec(d\*x+c)\*\*(7/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \sec(dx+c)}}{\sec(dx+c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))/sec(d*x + c)^(7/2), x)
```

$$3.141 \quad \int \frac{\sqrt{b \sec(c+dx)}}{\sec^2(c+dx)} dx$$

**Optimal.** Leaf size=98

$$\frac{3x\sqrt{b \sec(c+dx)}}{8\sqrt{\sec(c+dx)}} + \frac{3 \sin(c+dx)\sqrt{b \sec(c+dx)}}{8d \sec^{\frac{3}{2}}(c+dx)} + \frac{\sin(c+dx)\sqrt{b \sec(c+dx)}}{4d \sec^{\frac{7}{2}}(c+dx)}$$

[Out] (3\*x\*Sqrt[b\*Sec[c + d\*x]])/(8\*Sqrt[Sec[c + d\*x]]) + (Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(4\*d\*Sec[c + d\*x]^(7/2)) + (3\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(8\*d\*Sec[c + d\*x]^(3/2))

**Rubi [A]** time = 0.0251071, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 2635, 8}

$$\frac{3x\sqrt{b \sec(c+dx)}}{8\sqrt{\sec(c+dx)}} + \frac{3 \sin(c+dx)\sqrt{b \sec(c+dx)}}{8d \sec^{\frac{3}{2}}(c+dx)} + \frac{\sin(c+dx)\sqrt{b \sec(c+dx)}}{4d \sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b\*Sec[c + d\*x]]/Sec[c + d\*x]^(9/2), x]

[Out] (3\*x\*Sqrt[b\*Sec[c + d\*x]])/(8\*Sqrt[Sec[c + d\*x]]) + (Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(4\*d\*Sec[c + d\*x]^(7/2)) + (3\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(8\*d\*Sec[c + d\*x]^(3/2))

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_.)\*((b\_.)\*(v\_))^(n\_.), x\_Symbol] := Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_.), x\_Symbol] := -Simp[(b\*Cos[c + d\*x])\*(b\*SIN[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*SIN[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

**Rule 8**

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

**Rubi steps**

$$\begin{aligned}
 \int \frac{\sqrt{b \sec(c+dx)}}{\sec^{\frac{9}{2}}(c+dx)} dx &= \frac{\sqrt{b \sec(c+dx)} \int \cos^4(c+dx) dx}{\sqrt{\sec(c+dx)}} \\
 &= \frac{\sqrt{b \sec(c+dx)} \sin(c+dx)}{4d \sec^{\frac{7}{2}}(c+dx)} + \frac{(3\sqrt{b \sec(c+dx)}) \int \cos^2(c+dx) dx}{4\sqrt{\sec(c+dx)}} \\
 &= \frac{\sqrt{b \sec(c+dx)} \sin(c+dx)}{4d \sec^{\frac{7}{2}}(c+dx)} + \frac{3\sqrt{b \sec(c+dx)} \sin(c+dx)}{8d \sec^{\frac{3}{2}}(c+dx)} + \frac{(3\sqrt{b \sec(c+dx)}) \int 1 dx}{8\sqrt{\sec(c+dx)}} \\
 &= \frac{3x\sqrt{b \sec(c+dx)}}{8\sqrt{\sec(c+dx)}} + \frac{\sqrt{b \sec(c+dx)} \sin(c+dx)}{4d \sec^{\frac{7}{2}}(c+dx)} + \frac{3\sqrt{b \sec(c+dx)} \sin(c+dx)}{8d \sec^{\frac{3}{2}}(c+dx)}
 \end{aligned}$$

**Mathematica [A]** time = 0.116334, size = 55, normalized size = 0.56

$$\frac{(12(c+dx) + 8 \sin(2(c+dx)) + \sin(4(c+dx)))\sqrt{b \sec(c+dx)}}{32d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b\*Sec[c + d\*x]]/Sec[c + d\*x]^(9/2), x]

[Out] (Sqrt[b\*Sec[c + d\*x]]\*(12\*(c + d\*x) + 8\*Sin[2\*(c + d\*x)] + Sin[4\*(c + d\*x)])) / (32\*d\*Sqrt[Sec[c + d\*x]])

**Maple [A]** time = 0.157, size = 74, normalized size = 0.8

$$\frac{2 (\cos(dx+c))^3 \sin(dx+c) + 3 \cos(dx+c) \sin(dx+c) + 3 dx + 3c}{8d (\cos(dx+c))^4} \sqrt{\frac{b}{\cos(dx+c)}} \left( (\cos(dx+c))^{-1} \right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(1/2)/sec(d\*x+c)^(9/2), x)

[Out]  $1/8/d*(2*\cos(d*x+c)^3*\sin(d*x+c)+3*\cos(d*x+c)*\sin(d*x+c)+3*d*x+3*c)*(b/\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^4/(1/\cos(d*x+c))^{(9/2)}$

**Maxima [A]** time = 2.13244, size = 66, normalized size = 0.67

$$\frac{\left(12dx + 12c + \sin(4dx + 4c) + 8 \sin\left(\frac{1}{2} \arctan(\sin(4dx + 4c), \cos(4dx + 4c))\right)\right)\sqrt{b}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")`

[Out]  $1/32*(12*d*x + 12*c + \sin(4*d*x + 4*c) + 8*\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))))*sqrt(b)/d$

**Fricas [A]** time = 1.78951, size = 537, normalized size = 5.48

$$\left[ \frac{2(2 \cos(dx+c)^4 + 3 \cos(dx+c)^2) \sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{\cos(dx+c)}} + 3 \sqrt{-b} \log\left(-2 \sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 2b \cos(dx+c)^2 - \dots\right) \right] / 16d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")`

[Out]  $[1/16*(2*(2*\cos(d*x + c)^4 + 3*\cos(d*x + c)^2)*sqrt(b/\cos(d*x + c))*\sin(d*x + c)/sqrt(\cos(d*x + c)) + 3*sqrt(-b)*log(-2*sqrt(-b)*sqrt(b/\cos(d*x + c))*\cos(d*x + c)^{(3/2)}*\sin(d*x + c) + 2*b*\cos(d*x + c)^2 - b))/d, 1/8*((2*\cos(d*x + c)^4 + 3*\cos(d*x + c)^2)*sqrt(b/\cos(d*x + c))*\sin(d*x + c)/sqrt(\cos(d*x + c)) + 3*sqrt(b)*arctan(sqrt(b/\cos(d*x + c))*\sin(d*x + c)/(sqrt(b)*sqrt(\cos(d*x + c)))))/d]$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(1/2)/sec(d\*x+c)\*\*(9/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \sec(dx + c)}}{\sec(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(1/2)/sec(d\*x+c)^(9/2),x, algorithm="giac")

[Out] integrate(sqrt(b\*sec(d\*x + c))/sec(d\*x + c)^(9/2), x)

### 3.142 $\int \sec^{\frac{7}{2}}(c + dx)(b \sec(c + dx))^{3/2} dx$

**Optimal.** Leaf size=110

$$\frac{b \sin(c + dx) \sec^{\frac{7}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{4d} + \frac{3b \sin(c + dx) \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{8d} + \frac{3b \sqrt{b \sec(c + dx)} \tanh^{-1}(\sin(c + dx))}{8d \sqrt{\sec(c + dx)}}$$

[Out] (3\*b\*ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(8\*d\*Sqrt[Sec[c + d\*x]]) + (3\*b\*Sec[c + d\*x]^(3/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(8\*d) + (b\*Sec[c + d\*x]^(7/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(4\*d)

**Rubi [A]** time = 0.0348635, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 3768, 3770}

$$\frac{b \sin(c + dx) \sec^{\frac{7}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{4d} + \frac{3b \sin(c + dx) \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{8d} + \frac{3b \sqrt{b \sec(c + dx)} \tanh^{-1}(\sin(c + dx))}{8d \sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(7/2)\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] (3\*b\*ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(8\*d\*Sqrt[Sec[c + d\*x]]) + (3\*b\*Sec[c + d\*x]^(3/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(8\*d) + (b\*Sec[c + d\*x]^(7/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(4\*d)

#### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> -Simp[(b\*Csc[c + d\*x]\*(b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
 \int \sec^{\frac{7}{2}}(c + dx)(b \sec(c + dx))^{3/2} dx &= \frac{(b\sqrt{b \sec(c + dx)}) \int \sec^5(c + dx) dx}{\sqrt{\sec(c + dx)}} \\
 &= \frac{b \sec^{\frac{7}{2}}(c + dx) \sqrt{b \sec(c + dx)} \sin(c + dx)}{4d} + \frac{(3b\sqrt{b \sec(c + dx)}) \int \sec^3(c + dx) dx}{4\sqrt{\sec(c + dx)}} \\
 &= \frac{3b \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)} \sin(c + dx)}{8d} + \frac{b \sec^{\frac{7}{2}}(c + dx) \sqrt{b \sec(c + dx)} \sin(c + dx)}{4d} \\
 &= \frac{3b \tanh^{-1}(\sin(c + dx)) \sqrt{b \sec(c + dx)}}{8d\sqrt{\sec(c + dx)}} + \frac{3b \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)} \sin(c + dx)}{8d}
 \end{aligned}$$

**Mathematica [A]** time = 0.11093, size = 64, normalized size = 0.58

$$\frac{(b \sec(c + dx))^{3/2} \left( 3 \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) \sec(c + dx) (2 \sec^2(c + dx) + 3) \right)}{8d \sec^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^(3/2), x]
```

```
[Out] ((b*Sec[c + d*x])^(3/2)*(3*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(3 + 2*Sec[c + d*x]^2)*Tan[c + d*x]))/(8*d*Sec[c + d*x]^(3/2))
```

**Maple [A]** time = 0.139, size = 131, normalized size = 1.2

$$-\frac{\cos(dx + c)}{8d} \left( 3 (\cos(dx + c))^4 \ln\left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)}\right) - 3 (\cos(dx + c))^4 \ln\left(-\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(3/2), x)
```

```
[Out] -1/8/d*(3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-3*cos(d*x+c)^2*sin(d*x+c)-2*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(7/2)*(b/cos(d*x+c))^(3/2)
```

**Maxima [B]** time = 2.63137, size = 2352, normalized size = 21.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) + 4*b*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) + 4*b*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) + 4*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) + 4*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*
```

```
(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4
4*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
44*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 12*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*sqrt(b)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*d)
```

**Fricas [A]** time = 1.58469, size = 621, normalized size = 5.65

$$\frac{3b^2 \cos(dx+c)^3 \log\left(\frac{b \cos(dx+c)^2 - 2\sqrt{b} \sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b}{\cos(dx+c)^2}\right) + \frac{2(3b \cos(dx+c)^2 + 2b) \sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{\cos(dx+c)}}}{16d \cos(dx+c)^3}, - \frac{3\sqrt{-bb} a}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(7/2)\*(b\*sec(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/16\*(3\*b^(3/2)\*cos(d\*x + c)^3\*log(-(b\*cos(d\*x + c)^2 - 2\*sqrt(b)\*sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c) - 2\*b)/cos(d\*x + c)^2) + 2\*(3\*b\*cos(d\*x + c)^2 + 2\*b)\*sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/sqrt(cos(d\*x + c)))/(d\*cos(d\*x + c)^3), -1/8\*(3\*sqrt(-b)\*b\*arctan(sqrt(-b)\*sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c)/b)\*cos(d\*x + c)^3 - (3\*b\*cos(d\*x + c)^2 + 2\*b)\*sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/sqrt(cos(d\*x + c)))/(d\*cos(d\*x + c)^3)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*(7/2)\*(b\*sec(d\*x+c))\*\*(3/2), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \sec(dx + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(7/2)\*(b\*sec(d\*x+c))^(3/2), x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(3/2)\*sec(d\*x + c)^(7/2), x)

### 3.143 $\int \sec^{\frac{5}{2}}(c + dx)(b \sec(c + dx))^{3/2} dx$

**Optimal.** Leaf size=72

$$\frac{b \sin^3(c + dx) \sec^{\frac{5}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{3d} + \frac{b \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{b \sec(c + dx)}}{d}$$

[Out] (b\*Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/d + (b\*Sec[c + d\*x]^(5/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*d)

**Rubi [A]** time = 0.0169584, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 3767}

$$\frac{b \sin^3(c + dx) \sec^{\frac{5}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{3d} + \frac{b \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{b \sec(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(5/2)\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] (b\*Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/d + (b\*Sec[c + d\*x]^(5/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*d)

#### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b}, m, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

#### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

#### Rubi steps

$$\begin{aligned}
\int \sec^{\frac{5}{2}}(c+dx)(b \sec(c+dx))^{3/2} dx &= \frac{(b\sqrt{b \sec(c+dx)}) \int \sec^4(c+dx) dx}{\sqrt{\sec(c+dx)}} \\
&= -\frac{(b\sqrt{b \sec(c+dx)}) \operatorname{Subst}\left(\int (1+x^2) dx, x, -\tan(c+dx)\right)}{d\sqrt{\sec(c+dx)}} \\
&= \frac{b\sqrt{\sec(c+dx)}\sqrt{b \sec(c+dx)} \sin(c+dx)}{d} + \frac{b \sec^{\frac{5}{2}}(c+dx)\sqrt{b \sec(c+dx)} \sin^3(c+dx)}{3d}
\end{aligned}$$

**Mathematica [A]** time = 0.088353, size = 45, normalized size = 0.62

$$\frac{\left(\frac{1}{3} \tan^3(c+dx) + \tan(c+dx)\right) (b \sec(c+dx))^{3/2}}{d \sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^(5/2)\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] ((b\*Sec[c + d\*x])^(3/2)\*(Tan[c + d\*x] + Tan[c + d\*x]^3/3))/(d\*Sec[c + d\*x]^(3/2))

**Maple [A]** time = 0.118, size = 52, normalized size = 0.7

$$\frac{(2 \cos(dx+c)^2 + 1) \cos(dx+c) \sin(dx+c)}{3d} \left( (\cos(dx+c))^{-1} \right)^{\frac{5}{2}} \left( \frac{b}{\cos(dx+c)} \right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(5/2)\*(b\*sec(d\*x+c))^(3/2), x)

[Out] 1/3/d\*(2\*cos(d\*x+c)^2+1)\*cos(d\*x+c)\*sin(d\*x+c)\*(1/cos(d\*x+c))^(5/2)\*(b/cos(d\*x+c))^(3/2)

**Maxima [B]** time = 2.10283, size = 404, normalized size = 5.61

$$\frac{4(3b \cos(6dx+6c) \sin(2d)}{3(2(3 \cos(4dx+4c) + 3 \cos(2dx+2c) + 1) \cos(6dx+6c) + \cos(6dx+6c)^2 + 6(3 \cos(2dx+2c) + 1) \cos(4dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] 
$$-4/3*(3*b*\cos(6*d*x + 6*c)*\sin(2*d*x + 2*c) + 9*b*\cos(4*d*x + 4*c)*\sin(2*d*x + 2*c) - (3*b*\cos(2*d*x + 2*c) + b)*\sin(6*d*x + 6*c) - 3*(3*b*\cos(2*d*x + 2*c) + b)*\sin(4*d*x + 4*c))*\sqrt{b}/((2*(3*\cos(4*d*x + 4*c) + 3*\cos(2*d*x + 2*c) + 1)*\cos(6*d*x + 6*c) + \cos(6*d*x + 6*c)^2 + 6*(3*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + 9*\cos(4*d*x + 4*c)^2 + 9*\cos(2*d*x + 2*c)^2 + 6*(\sin(4*d*x + 4*c) + \sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + \sin(6*d*x + 6*c)^2 + 9*\sin(4*d*x + 4*c)^2 + 18*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*\sin(2*d*x + 2*c)^2 + 6*\cos(2*d*x + 2*c) + 1)*d)$$

**Fricas [A]** time = 1.35406, size = 117, normalized size = 1.62

$$\frac{(2b \cos(dx + c)^2 + b) \sqrt{\frac{b}{\cos(dx+c)} \sin(dx + c)}}{3d \cos(dx + c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] 
$$1/3*(2*b*\cos(d*x + c)^2 + b)*\sqrt{b/\cos(d*x + c)}*\sin(d*x + c)/(d*\cos(d*x + c)^(5/2))$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(5/2)*(b*sec(d*x+c))**(3/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \sec(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^(5/2), x)
```

$$3.144 \quad \int \sec^{\frac{3}{2}}(c + dx)(b \sec(c + dx))^{3/2} dx$$

**Optimal.** Leaf size=74

$$\frac{b \sin(c + dx) \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{2d} + \frac{b \sqrt{b \sec(c + dx)} \tanh^{-1}(\sin(c + dx))}{2d \sqrt{\sec(c + dx)}}$$

[Out] (b\*ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(2\*d\*Sqrt[Sec[c + d\*x]]) + (b\*Sec[c + d\*x]^(3/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(2\*d)

**Rubi [A]** time = 0.0207255, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 3768, 3770}

$$\frac{b \sin(c + dx) \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{2d} + \frac{b \sqrt{b \sec(c + dx)} \tanh^{-1}(\sin(c + dx))}{2d \sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(3/2)\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] (b\*ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(2\*d\*Sqrt[Sec[c + d\*x]]) + (b\*Sec[c + d\*x]^(3/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(2\*d)

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \sec^{\frac{3}{2}}(c + dx)(b \sec(c + dx))^{3/2} dx &= \frac{(b\sqrt{b \sec(c + dx)}) \int \sec^3(c + dx) dx}{\sqrt{\sec(c + dx)}} \\ &= \frac{b \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)} \sin(c + dx)}{2d} + \frac{(b\sqrt{b \sec(c + dx)}) \int \sec(c + dx) dx}{2\sqrt{\sec(c + dx)}} \\ &= \frac{b \tanh^{-1}(\sin(c + dx)) \sqrt{b \sec(c + dx)}}{2d\sqrt{\sec(c + dx)}} + \frac{b \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)} \sin(c + dx)}{2d} \end{aligned}$$

**Mathematica [A]** time = 0.0584711, size = 50, normalized size = 0.68

$$\frac{(b \sec(c + dx))^{3/2} (\tanh^{-1}(\sin(c + dx)) + \tan(c + dx) \sec(c + dx))}{2d \sec^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2), x]
```

```
[Out] ((b*Sec[c + d*x])^(3/2)*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x])
)/(2*d*Sec[c + d*x]^(3/2))
```

**Maple [A]** time = 0.117, size = 112, normalized size = 1.5

$$\frac{\cos(dx + c)}{2d} \left( \ln \left( -\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^2 - \ln \left( -\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(3/2), x)
```

```
[Out] 1/2/d*(ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-ln(-(-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+sin(d*x+c))*cos(d*x+c)*(1/cos(d
```

$$*x+c))^{(3/2)}*(b/\cos(d*x+c))^{(3/2)}$$

**Maxima [B]** time = 2.19128, size = 933, normalized size = 12.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(3/2)\*(b\*sec(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] 
$$-1/4*(4*(b*\sin(4*d*x + 4*c) + 2*b*\sin(2*d*x + 2*c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4*(b*\sin(4*d*x + 4*c) + 2*b*\sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - (b*\cos(4*d*x + 4*c))^2 + 4*b*\cos(2*d*x + 2*c)^2 + b*\sin(4*d*x + 4*c)^2 + 4*b*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*b*\sin(2*d*x + 2*c)^2 + 2*(2*b*\cos(2*d*x + 2*c) + b)*\cos(4*d*x + 4*c) + 4*b*\cos(2*d*x + 2*c) + b)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 1) + (b*\cos(4*d*x + 4*c)^2 + 4*b*\cos(2*d*x + 2*c)^2 + b*\sin(4*d*x + 4*c)^2 + 4*b*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*b*\sin(2*d*x + 2*c)^2 + 2*(2*b*\cos(2*d*x + 2*c) + b)*\cos(4*d*x + 4*c) + 4*b*\cos(2*d*x + 2*c) + b)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 1) - 4*(b*\cos(4*d*x + 4*c) + 2*b*\cos(2*d*x + 2*c) + b)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(b*\cos(4*d*x + 4*c) + 2*b*\cos(2*d*x + 2*c) + b)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*sqrt(b)/((2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*d)$$

**Fricas [A]** time = 1.64844, size = 536, normalized size = 7.24

$$\left[ \frac{b^3 \cos(dx+c) \log\left( -\frac{b \cos(dx+c)^2 - 2\sqrt{b} \sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b}{\cos(dx+c)^2} \right) + \frac{2b \sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4d \cos(dx+c)}, -\frac{\sqrt{-b} \arctan\left( \frac{\sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}}}{\cos(dx+c)} \right)}{\cos(dx+c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b^(3/2)*cos(d*x + c)*log(-(b*cos(d*x + c))^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c)))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2 + 2*b*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)), -1/2*(sqrt(-b)*b*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)*cos(d*x + c) - b*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{3}{2}} \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^(3/2), x)
```

$$3.145 \quad \int \sqrt{\sec(c + dx)}(b \sec(c + dx))^{3/2} dx$$

Optimal. Leaf size=33

$$\frac{b \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{b \sec(c + dx)}}{d}$$

[Out] (b\*Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/d

**Rubi [A]** time = 0.0113825, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 3767, 8}

$$\frac{b \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{b \sec(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d\*x]]\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] (b\*Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/d

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned} \int \sqrt{\sec(c+dx)}(b \sec(c+dx))^{3/2} dx &= \frac{(b\sqrt{b \sec(c+dx)}) \int \sec^2(c+dx) dx}{\sqrt{\sec(c+dx)}} \\ &= -\frac{(b\sqrt{b \sec(c+dx)}) \operatorname{Subst}\left(\int 1 dx, x, -\tan(c+dx)\right)}{d\sqrt{\sec(c+dx)}} \\ &= \frac{b\sqrt{\sec(c+dx)}\sqrt{b \sec(c+dx)} \sin(c+dx)}{d} \end{aligned}$$

**Mathematica [A]** time = 0.0203905, size = 32, normalized size = 0.97

$$\frac{\sin(c+dx)(b \sec(c+dx))^{3/2}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[c + d\*x]]\*(b\*Sec[c + d\*x])^(3/2), x]

[Out] ((b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(d\*Sqrt[Sec[c + d\*x]])

**Maple [A]** time = 0.113, size = 39, normalized size = 1.2

$$\frac{\cos(dx+c)\sin(dx+c)}{d} \sqrt{(\cos(dx+c))^{-1} \left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(1/2)\*(b\*sec(d\*x+c))^(3/2), x)

[Out] 1/d\*(1/cos(d\*x+c))^(1/2)\*(b/cos(d\*x+c))^(3/2)\*cos(d\*x+c)\*sin(d\*x+c)

**Maxima [A]** time = 2.03621, size = 73, normalized size = 2.21

$$\frac{2b^{\frac{3}{2}} \sin(2dx+2c)}{(\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2\cos(2dx+2c) + 1)d}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out]  $2*b^{(3/2)}*\sin(2*d*x + 2*c)/((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)*d)$

**Fricas [A]** time = 1.41589, size = 81, normalized size = 2.45

$$\frac{b\sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `b*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(1/2)*(b*sec(d*x+c))**(3/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx+c))^{\frac{3}{2}} \sqrt{\sec(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")`

[Out] `integrate((b*sec(d*x + c))^(3/2)*sqrt(sec(d*x + c)), x)`

$$3.146 \quad \int \frac{(b \sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}} dx$$

**Optimal.** Leaf size=34

$$\frac{b\sqrt{b \sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{\sec(c+dx)}}$$

[Out] (b\*ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(d\*Sqrt[Sec[c + d\*x]])

**Rubi [A]** time = 0.0074038, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 3770}

$$\frac{b\sqrt{b \sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(3/2)/Sqrt[Sec[c + d\*x]],x]

[Out] (b\*ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(d\*Sqrt[Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned} \int \frac{(b \sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}} dx &= \frac{(b\sqrt{b \sec(c+dx)}) \int \sec(c+dx) dx}{\sqrt{\sec(c+dx)}} \\ &= \frac{b \tanh^{-1}(\sin(c+dx))\sqrt{b \sec(c+dx)}}{d\sqrt{\sec(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0210147, size = 33, normalized size = 0.97

$$\frac{(b \sec(c + dx))^{3/2} \tanh^{-1}(\sin(c + dx))}{d \sec^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Sec[c + d\*x])^(3/2)/Sqrt[Sec[c + d\*x]],x]

[Out] (ArcTanh[Sin[c + d\*x]]\*(b\*Sec[c + d\*x])^(3/2))/(d\*Sec[c + d\*x]^(3/2))

**Maple [A]** time = 0.117, size = 52, normalized size = 1.5

$$-2 \frac{\cos(dx + c)}{d \sqrt{(\cos(dx + c))^{-1}}} \operatorname{Arctanh}\left(\frac{-1 + \cos(dx + c)}{\sin(dx + c)}\right) \left(\frac{b}{\cos(dx + c)}\right)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(1/2),x)

[Out] -2/d\*arctanh((-1+cos(d\*x+c))/sin(d\*x+c))\*cos(d\*x+c)\*(b/cos(d\*x+c))^(3/2)/(1/cos(d\*x+c))^(1/2)

**Maxima [B]** time = 2.06124, size = 92, normalized size = 2.71

$$\frac{(b \log(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \sin(dx + c) + 1) - b \log(\cos(dx + c)^2 + \sin(dx + c)^2 - 2 \sin(dx + c) + 1)) \sqrt{b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/2\*(b\*log(cos(d\*x + c)^2 + sin(d\*x + c)^2 + 2\*sin(d\*x + c) + 1) - b\*log(cos(d\*x + c)^2 + sin(d\*x + c)^2 - 2\*sin(d\*x + c) + 1))\*sqrt(b)/d

**Fricas [A]** time = 1.78313, size = 293, normalized size = 8.62

$$\left[ \frac{b^{\frac{3}{2}} \log\left(\frac{b \cos(dx+c)^2 - 2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) - 2b}{\cos(dx+c)^2}\right)}{2d}, \frac{\sqrt{-b} \arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b}\right)}{d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/2\*b^(3/2)\*log(-(b\*cos(d\*x + c))^2 - 2\*sqrt(b)\*sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c) - 2\*b)/cos(d\*x + c)^2)/d, -sqrt(-b)\*b\*arctan(sqrt(-b)\*sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c)/b)/d]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(c + dx))^{\frac{3}{2}}}{\sqrt{\sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(3/2)/sec(d\*x+c)\*\*(1/2),x)

[Out] Integral((b\*sec(c + d\*x))\*\*(3/2)/sqrt(sec(c + d\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx + c))^{\frac{3}{2}}}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(3/2)/sqrt(sec(d\*x + c)), x)

$$3.147 \quad \int \frac{(b \sec(c+dx))^{3/2}}{\sec^2(c+dx)} dx$$

**Optimal.** Leaf size=25

$$\frac{bx\sqrt{b \sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

[Out] (b\*x\*Sqrt[b\*Sec[c + d\*x]])/Sqrt[Sec[c + d\*x]]

**Rubi [A]** time = 0.002594, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 8}

$$\frac{bx\sqrt{b \sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(3/2)/Sec[c + d\*x]^(3/2),x]

[Out] (b\*x\*Sqrt[b\*Sec[c + d\*x]])/Sqrt[Sec[c + d\*x]]

### Rule 17

Int[(u\_)\*((a\_)\*(v\_))^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned} \int \frac{(b \sec(c+dx))^{3/2}}{\sec^2(c+dx)} dx &= \frac{(b\sqrt{b \sec(c+dx)}) \int 1 dx}{\sqrt{\sec(c+dx)}} \\ &= \frac{bx\sqrt{b \sec(c+dx)}}{\sqrt{\sec(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0245737, size = 24, normalized size = 0.96

$$\frac{x(b \sec(c + dx))^{3/2}}{\sec^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Sec[c + d\*x])^(3/2)/Sec[c + d\*x]^(3/2),x]

[Out] (x\*(b\*Sec[c + d\*x])^(3/2))/Sec[c + d\*x]^(3/2)

**Maple [A]** time = 0.089, size = 32, normalized size = 1.3

$$\frac{dx + c}{d} \left( \frac{b}{\cos(dx + c)} \right)^{\frac{3}{2}} \left( (\cos(dx + c))^{-1} \right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(3/2),x)

[Out] 1/d\*(d\*x+c)\*(b/cos(d\*x+c))^(3/2)/(1/cos(d\*x+c))^(3/2)

**Maxima [A]** time = 1.78698, size = 35, normalized size = 1.4

$$\frac{2b^{\frac{3}{2}} \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(3/2),x, algorithm="maxima")

[Out] 2\*b^(3/2)\*arctan(sin(d\*x + c)/(cos(d\*x + c) + 1))/d

**Fricas [A]** time = 1.85659, size = 269, normalized size = 10.76

$$\left[ \frac{\sqrt{-bb} \log \left( -2 \sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 2b \cos(dx+c)^2 - b \right)}{2d}, \frac{b^{\frac{3}{2}} \arctan \left( \frac{\sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{b} \sqrt{\cos(dx+c)}} \right)}{d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] [1/2\*sqrt(-b)\*b\*log(-2\*sqrt(-b)\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + 2\*b\*cos(d\*x + c)^2 - b)/d, b^(3/2)\*arctan(sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/(sqrt(b)\*sqrt(cos(d\*x + c))))/d]

**Sympy [A]** time = 38.5588, size = 5, normalized size = 0.2

$$b^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(3/2)/sec(d\*x+c)\*\*(3/2),x)

[Out] b\*\*(3/2)\*x

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(3/2)/sec(d\*x + c)^(3/2), x)

$$3.148 \quad \int \frac{(b \sec(c+dx))^{3/2}}{\sec^2(c+dx)} dx$$

**Optimal.** Leaf size=33

$$\frac{b \sin(c+dx) \sqrt{b \sec(c+dx)}}{d \sqrt{\sec(c+dx)}}$$

[Out] (b\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[Sec[c + d\*x]])

**Rubi [A]** time = 0.0074591, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 2637}

$$\frac{b \sin(c+dx) \sqrt{b \sec(c+dx)}}{d \sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(3/2)/Sec[c + d\*x]^(5/2),x]

[Out] (b\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_.))^(m\_.)\*((b\_.)\*(v\_.))^(n\_.), x\_Symbol] := Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps



$$\int \frac{(b \sec(c + dx))^{3/2}}{\sec^2(c + dx)} dx = \frac{(b\sqrt{b \sec(c + dx)}) \int \cos(c + dx) dx}{\sqrt{\sec(c + dx)}}$$

$$= \frac{b\sqrt{b \sec(c + dx)} \sin(c + dx)}{d\sqrt{\sec(c + dx)}}$$

**Mathematica [A]** time = 0.0440589, size = 32, normalized size = 0.97

$$\frac{\sin(c + dx)(b \sec(c + dx))^{3/2}}{d \sec^2(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Sec[c + d\*x])^(3/2)/Sec[c + d\*x]^(5/2), x]

[Out] ((b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(d\*Sec[c + d\*x]^(3/2))

**Maple [A]** time = 0.114, size = 41, normalized size = 1.2

$$\frac{\sin(dx + c)}{d \cos(dx + c)} \left( \frac{b}{\cos(dx + c)} \right)^{\frac{3}{2}} \left( (\cos(dx + c))^{-1} \right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(5/2), x)

[Out] 1/d\*sin(d\*x+c)\*(b/cos(d\*x+c))^(3/2)/(1/cos(d\*x+c))^(5/2)/cos(d\*x+c)

**Maxima [A]** time = 2.00161, size = 18, normalized size = 0.55

$$\frac{b^{\frac{3}{2}} \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(5/2),x, algorithm="maxima")

[Out] b^(3/2)\*sin(d\*x + c)/d

**Fricas [A]** time = 1.57804, size = 78, normalized size = 2.36

$$\frac{b \sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(5/2),x, algorithm="fricas")

[Out] b\*sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c)/d

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(3/2)/sec(d\*x+c)\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(3/2)/sec(d\*x + c)^(5/2), x)

$$3.149 \quad \int \frac{(b \sec(c+dx))^{3/2}}{\sec^2(c+dx)} dx$$

**Optimal.** Leaf size=65

$$\frac{bx\sqrt{b \sec(c+dx)}}{2\sqrt{\sec(c+dx)}} + \frac{b \sin(c+dx)\sqrt{b \sec(c+dx)}}{2d \sec^{\frac{3}{2}}(c+dx)}$$

[Out] (b\*x\*Sqrt[b\*Sec[c + d\*x]])/(2\*Sqrt[Sec[c + d\*x]]) + (b\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(2\*d\*Sec[c + d\*x]^(3/2))

**Rubi [A]** time = 0.0146445, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 2635, 8}

$$\frac{bx\sqrt{b \sec(c+dx)}}{2\sqrt{\sec(c+dx)}} + \frac{b \sin(c+dx)\sqrt{b \sec(c+dx)}}{2d \sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(3/2)/Sec[c + d\*x]^(7/2), x]

[Out] (b\*x\*Sqrt[b\*Sec[c + d\*x]])/(2\*Sqrt[Sec[c + d\*x]]) + (b\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(2\*d\*Sec[c + d\*x]^(3/2))

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x])\*(b\*Ssin[c + d\*x])^(n - 1)/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Ssin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned} \int \frac{(b \sec(c + dx))^{3/2}}{\sec^2(c + dx)} dx &= \frac{(b\sqrt{b \sec(c + dx)}) \int \cos^2(c + dx) dx}{\sqrt{\sec(c + dx)}} \\ &= \frac{b\sqrt{b \sec(c + dx)} \sin(c + dx)}{2d \sec^{\frac{3}{2}}(c + dx)} + \frac{(b\sqrt{b \sec(c + dx)}) \int 1 dx}{2\sqrt{\sec(c + dx)}} \\ &= \frac{bx\sqrt{b \sec(c + dx)}}{2\sqrt{\sec(c + dx)}} + \frac{b\sqrt{b \sec(c + dx)} \sin(c + dx)}{2d \sec^{\frac{3}{2}}(c + dx)} \end{aligned}$$

**Mathematica [A]** time = 0.0792508, size = 45, normalized size = 0.69

$$\frac{(2(c + dx) + \sin(2(c + dx)))(b \sec(c + dx))^{3/2}}{4d \sec^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Sec[c + d\*x])^(3/2)/Sec[c + d\*x]^(7/2), x]

[Out] ((b\*Sec[c + d\*x])^(3/2)\*(2\*(c + d\*x) + Sin[2\*(c + d\*x)]))/(4\*d\*Sec[c + d\*x]^(3/2))

**Maple [A]** time = 0.124, size = 54, normalized size = 0.8

$$\frac{\cos(dx + c) \sin(dx + c) + dx + c}{2d (\cos(dx + c))^2} \left( \frac{b}{\cos(dx + c)} \right)^{\frac{3}{2}} ((\cos(dx + c))^{-1})^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(7/2), x)

[Out] 1/2/d\*(cos(d\*x+c)\*sin(d\*x+c)+d\*x+c)\*(b/cos(d\*x+c))^(3/2)/cos(d\*x+c)^2/(1/cos(d\*x+c))^(7/2)

---

**Maxima [A]** time = 2.03789, size = 38, normalized size = 0.58

$$\frac{(2(dx+c)b + b \sin(2dx+2c))\sqrt{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(7/2),x, algorithm="maxima")

[Out] 1/4\*(2\*(d\*x + c)\*b + b\*sin(2\*d\*x + 2\*c))\*sqrt(b)/d

---

**Fricas [A]** time = 1.96696, size = 436, normalized size = 6.71

$$\left[ \frac{2b \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + \sqrt{-b} \log\left(-2\sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 2b \cos(dx+c)^2 - \right)}{4d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(7/2),x, algorithm="fricas")

[Out] [1/4\*(2\*b\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + sqrt(-b)\*b\*log(-2\*sqrt(-b)\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + 2\*b\*cos(d\*x + c)^2 - b))/d, 1/2\*(b\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + b^(3/2)\*arctan(sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/(sqrt(b)\*sqrt(cos(d\*x + c)))))/d]

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(3/2)/sec(d\*x+c)\*\*(7/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx + c))^{\frac{3}{2}}}{\sec(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(3/2)/sec(d\*x + c)^(7/2), x)

$$3.150 \quad \int \frac{(b \sec(c+dx))^{3/2}}{\sec^2(c+dx)} dx$$

**Optimal.** Leaf size=72

$$\frac{b \sin(c+dx) \sqrt{b \sec(c+dx)}}{d \sqrt{\sec(c+dx)}} - \frac{b \sin^3(c+dx) \sqrt{b \sec(c+dx)}}{3d \sqrt{\sec(c+dx)}}$$

[Out] (b\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[Sec[c + d\*x]]) - (b\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*d\*Sqrt[Sec[c + d\*x]])

**Rubi [A]** time = 0.0170607, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 2633}

$$\frac{b \sin(c+dx) \sqrt{b \sec(c+dx)}}{d \sqrt{\sec(c+dx)}} - \frac{b \sin^3(c+dx) \sqrt{b \sec(c+dx)}}{3d \sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(3/2)/Sec[c + d\*x]^(9/2),x]

[Out] (b\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[Sec[c + d\*x]]) - (b\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*d\*Sqrt[Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 2633

Int[sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

### Rubi steps

$$\begin{aligned} \int \frac{(b \sec(c + dx))^{3/2}}{\sec^2(c + dx)} dx &= \frac{(b\sqrt{b \sec(c + dx)}) \int \cos^3(c + dx) dx}{\sqrt{\sec(c + dx)}} \\ &= -\frac{(b\sqrt{b \sec(c + dx)}) \text{Subst}\left(\int (1 - x^2) dx, x, -\sin(c + dx)\right)}{d\sqrt{\sec(c + dx)}} \\ &= \frac{b\sqrt{b \sec(c + dx)} \sin(c + dx)}{d\sqrt{\sec(c + dx)}} - \frac{b\sqrt{b \sec(c + dx)} \sin^3(c + dx)}{3d\sqrt{\sec(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.138329, size = 45, normalized size = 0.62

$$\frac{\sin(c + dx)(\cos(2(c + dx)) + 5)(b \sec(c + dx))^{3/2}}{6d \sec^2(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Sec[c + d\*x])^(3/2)/Sec[c + d\*x]^(9/2), x]

[Out] ((5 + Cos[2\*(c + d\*x)])\*(b\*Sec[c + d\*x])^(3/2)\*Sin[c + d\*x])/(6\*d\*Sec[c + d\*x]^(3/2))

**Maple [A]** time = 0.109, size = 52, normalized size = 0.7

$$\frac{((\cos(dx + c))^2 + 2) \sin(dx + c)}{3d (\cos(dx + c))^3} \left(\frac{b}{\cos(dx + c)}\right)^{\frac{3}{2}} ((\cos(dx + c))^{-1})^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(9/2), x)

[Out] 1/3/d\*(cos(d\*x+c)^2+2)\*sin(d\*x+c)\*(b/cos(d\*x+c))^(3/2)/cos(d\*x+c)^3/(1/cos(d\*x+c))^(9/2)

**Maxima [A]** time = 2.08281, size = 61, normalized size = 0.85

$$\frac{\left(b \sin(3dx + 3c) + 9b \sin\left(\frac{1}{3} \arctan(\sin(3dx + 3c), \cos(3dx + 3c))\right)\right) \sqrt{b}}{12d}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")`

[Out]  $1/12*(b*\sin(3*d*x + 3*c) + 9*b*\sin(1/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))))*\sqrt{b}/d$

**Fricas [A]** time = 1.66819, size = 135, normalized size = 1.88

$$\frac{(b \cos(dx + c)^3 + 2b \cos(dx + c)) \sqrt{\frac{b}{\cos(dx+c)}} \sin(dx + c)}{3d\sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")`

[Out]  $1/3*(b*\cos(d*x + c)^3 + 2*b*\cos(d*x + c))*\sqrt{b/\cos(d*x + c)}*\sin(d*x + c)/(d*\sqrt{\cos(d*x + c)})$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(9/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx + c))^{\frac{3}{2}}}{\sec(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)/sec(d*x + c)^(9/2), x)
```

$$3.151 \quad \int \frac{(b \sec(c+dx))^{3/2}}{\sec^2(c+dx)} dx$$

**Optimal.** Leaf size=101

$$\frac{3bx\sqrt{b \sec(c+dx)}}{8\sqrt{\sec(c+dx)}} + \frac{3b \sin(c+dx)\sqrt{b \sec(c+dx)}}{8d \sec^{\frac{3}{2}}(c+dx)} + \frac{b \sin(c+dx)\sqrt{b \sec(c+dx)}}{4d \sec^{\frac{7}{2}}(c+dx)}$$

[Out] (3\*b\*x\*Sqrt[b\*Sec[c + d\*x]])/(8\*Sqrt[Sec[c + d\*x]]) + (b\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(4\*d\*Sec[c + d\*x]^(7/2)) + (3\*b\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(8\*d\*Sec[c + d\*x]^(3/2))

**Rubi [A]** time = 0.0275267, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 2635, 8}

$$\frac{3bx\sqrt{b \sec(c+dx)}}{8\sqrt{\sec(c+dx)}} + \frac{3b \sin(c+dx)\sqrt{b \sec(c+dx)}}{8d \sec^{\frac{3}{2}}(c+dx)} + \frac{b \sin(c+dx)\sqrt{b \sec(c+dx)}}{4d \sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(3/2)/Sec[c + d\*x]^(11/2), x]

[Out] (3\*b\*x\*Sqrt[b\*Sec[c + d\*x]])/(8\*Sqrt[Sec[c + d\*x]]) + (b\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(4\*d\*Sec[c + d\*x]^(7/2)) + (3\*b\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(8\*d\*Sec[c + d\*x]^(3/2))

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x])\*(b\*SIN[c + d\*x])^(n - 1)/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*SIN[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

**Rule 8**

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

**Rubi steps**

$$\begin{aligned}
 \int \frac{(b \sec(c + dx))^{3/2}}{\sec^{11/2}(c + dx)} dx &= \frac{(b\sqrt{b \sec(c + dx)}) \int \cos^4(c + dx) dx}{\sqrt{\sec(c + dx)}} \\
 &= \frac{b\sqrt{b \sec(c + dx)} \sin(c + dx)}{4d \sec^{7/2}(c + dx)} + \frac{(3b\sqrt{b \sec(c + dx)}) \int \cos^2(c + dx) dx}{4\sqrt{\sec(c + dx)}} \\
 &= \frac{b\sqrt{b \sec(c + dx)} \sin(c + dx)}{4d \sec^{7/2}(c + dx)} + \frac{3b\sqrt{b \sec(c + dx)} \sin(c + dx)}{8d \sec^{3/2}(c + dx)} + \frac{(3b\sqrt{b \sec(c + dx)}) \int 1 dx}{8\sqrt{\sec(c + dx)}} \\
 &= \frac{3bx\sqrt{b \sec(c + dx)}}{8\sqrt{\sec(c + dx)}} + \frac{b\sqrt{b \sec(c + dx)} \sin(c + dx)}{4d \sec^{7/2}(c + dx)} + \frac{3b\sqrt{b \sec(c + dx)} \sin(c + dx)}{8d \sec^{3/2}(c + dx)}
 \end{aligned}$$

**Mathematica [A]** time = 0.154344, size = 55, normalized size = 0.54

$$\frac{(12(c + dx) + 8 \sin(2(c + dx)) + \sin(4(c + dx)))(b \sec(c + dx))^{3/2}}{32d \sec^{3/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Sec[c + d\*x])^(3/2)/Sec[c + d\*x]^(11/2), x]

[Out] ((b\*Sec[c + d\*x])^(3/2)\*(12\*(c + d\*x) + 8\*Sin[2\*(c + d\*x)] + Sin[4\*(c + d\*x)]))/(32\*d\*Sec[c + d\*x]^(3/2))

**Maple [A]** time = 0.123, size = 74, normalized size = 0.7

$$\frac{2 (\cos(dx + c))^3 \sin(dx + c) + 3 \cos(dx + c) \sin(dx + c) + 3 dx + 3c \left(\frac{b}{\cos(dx + c)}\right)^{3/2} ((\cos(dx + c))^{-1})^{-11/2}}{8d (\cos(dx + c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(11/2), x)

[Out]  $1/8/d*(2*\cos(d*x+c)^3*\sin(d*x+c)+3*\cos(d*x+c)*\sin(d*x+c)+3*d*x+3*c)*(b/\cos(d*x+c))^{(3/2)}/(1/\cos(d*x+c))^{(11/2)}/\cos(d*x+c)^4$

**Maxima [A]** time = 2.153, size = 72, normalized size = 0.71

$$\frac{\left(12(dx+c)b + b \sin(4dx+4c) + 8b \sin\left(\frac{1}{2} \arctan(\sin(4dx+4c), \cos(4dx+4c))\right)\right) \sqrt{b}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(11/2),x, algorithm="maxima")`

[Out]  $1/32*(12*(d*x + c)*b + b*\sin(4*d*x + 4*c) + 8*b*\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))))*\sqrt{b}/d$

**Fricas [A]** time = 2.0165, size = 551, normalized size = 5.46

$$\left[ \frac{3\sqrt{-b}b \log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 2b \cos(dx+c)^2 - b\right) + \frac{2(2b \cos(dx+c)^4 + 3b \cos(dx+c)^2) \sqrt{\frac{b}{\cos(dx+c)}}}{\sqrt{\cos(dx+c)}}}{16d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(11/2),x, algorithm="fricas")`

[Out]  $[1/16*(3*\sqrt{-b}*b*\log(-2*\sqrt{-b}*\sqrt{b/\cos(d*x+c)}*\cos(d*x+c)^{(3/2)}*\sin(d*x+c) + 2*b*\cos(d*x+c)^2 - b) + 2*(2*b*\cos(d*x+c)^4 + 3*b*\cos(d*x+c)^2)*\sqrt{b/\cos(d*x+c)}*\sin(d*x+c)/\sqrt{\cos(d*x+c)})/d, 1/8*(3*b^{(3/2)}*\arctan(\sqrt{b/\cos(d*x+c)}*\sin(d*x+c)/(\sqrt{b}*\sqrt{\cos(d*x+c)}))) + (2*b*\cos(d*x+c)^4 + 3*b*\cos(d*x+c)^2)*\sqrt{b/\cos(d*x+c)}*\sin(d*x+c)/\sqrt{\cos(d*x+c)})/d]$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(3/2)/sec(d\*x+c)\*\*(11/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx + c))^{\frac{3}{2}}}{\sec(dx + c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(3/2)/sec(d\*x+c)^(11/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(3/2)/sec(d\*x + c)^(11/2), x)

### 3.152 $\int \sec^{\frac{7}{2}}(c + dx)(b \sec(c + dx))^{5/2} dx$

**Optimal.** Leaf size=116

$$\frac{b^2 \sin^5(c + dx) \sec^{\frac{9}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{5d} + \frac{2b^2 \sin^3(c + dx) \sec^{\frac{5}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{3d} + \frac{b^2 \sin(c + dx) \sqrt{\sec(c + dx)}}{d}$$

[Out] (b^2\*Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/d + (2\*b^2\*Sec[c + d\*x]^(5/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*d) + (b^2\*Sec[c + d\*x]^(9/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x]^5)/(5\*d)

**Rubi [A]** time = 0.0239371, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 3767}

$$\frac{b^2 \sin^5(c + dx) \sec^{\frac{9}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{5d} + \frac{2b^2 \sin^3(c + dx) \sec^{\frac{5}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{3d} + \frac{b^2 \sin(c + dx) \sqrt{\sec(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(7/2)\*(b\*Sec[c + d\*x])^(5/2), x]

[Out] (b^2\*Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/d + (2\*b^2\*Sec[c + d\*x]^(5/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*d) + (b^2\*Sec[c + d\*x]^(9/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x]^5)/(5\*d)

#### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

#### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

#### Rubi steps

$$\begin{aligned}
\int \sec^{\frac{7}{2}}(c+dx)(b \sec(c+dx))^{5/2} dx &= \frac{(b^2 \sqrt{b \sec(c+dx)}) \int \sec^6(c+dx) dx}{\sqrt{\sec(c+dx)}} \\
&= -\frac{(b^2 \sqrt{b \sec(c+dx)}) \text{Subst}\left(\int (1+2x^2+x^4) dx, x, -\tan(c+dx)\right)}{d \sqrt{\sec(c+dx)}} \\
&= \frac{b^2 \sqrt{\sec(c+dx)} \sqrt{b \sec(c+dx)} \sin(c+dx)}{d} + \frac{2b^2 \sec^{\frac{5}{2}}(c+dx) \sqrt{b \sec(c+dx)} \sin^3(c+dx)}{3d}
\end{aligned}$$

**Mathematica [A]** time = 0.216542, size = 57, normalized size = 0.49

$$\frac{\left(\frac{1}{5} \tan^5(c+dx) + \frac{2}{3} \tan^3(c+dx) + \tan(c+dx)\right) (b \sec(c+dx))^{5/2}}{d \sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^(7/2)\*(b\*Sec[c + d\*x])^(5/2), x]

[Out] ((b\*Sec[c + d\*x])^(5/2)\*(Tan[c + d\*x] + (2\*Tan[c + d\*x]^3)/3 + Tan[c + d\*x]^5/5))/(d\*Sec[c + d\*x]^(5/2))

**Maple [A]** time = 0.119, size = 62, normalized size = 0.5

$$\frac{(8 (\cos(dx+c))^4 + 4 (\cos(dx+c))^2 + 3) \cos(dx+c) \sin(dx+c)}{15d} \left((\cos(dx+c))^{-1}\right)^{\frac{7}{2}} \left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(7/2)\*(b\*sec(d\*x+c))^(5/2), x)

[Out] 1/15/d\*(8\*cos(d\*x+c)^4+4\*cos(d\*x+c)^2+3)\*(1/cos(d\*x+c))^(7/2)\*(b/cos(d\*x+c))^(5/2)\*cos(d\*x+c)\*sin(d\*x+c)

**Maxima [B]** time = 2.53733, size = 952, normalized size = 8.21

result too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(7/2)\*(b\*sec(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -16/15*(5*(2*b^2*\sin(4*d*x + 4*c) + b^2*\sin(2*d*x + 2*c))*\cos(10*d*x + 10*c) \\ & + 25*(2*b^2*\sin(4*d*x + 4*c) + b^2*\sin(2*d*x + 2*c))*\cos(8*d*x + 8*c) + 5 \\ & 0*(2*b^2*\sin(4*d*x + 4*c) + b^2*\sin(2*d*x + 2*c))*\cos(6*d*x + 6*c) - (10*b^2 \\ & *2*\cos(4*d*x + 4*c) + 5*b^2*\cos(2*d*x + 2*c) + b^2)*\sin(10*d*x + 10*c) - 5*( \\ & 10*b^2*\cos(4*d*x + 4*c) + 5*b^2*\cos(2*d*x + 2*c) + b^2)*\sin(8*d*x + 8*c) - \\ & 10*(10*b^2*\cos(4*d*x + 4*c) + 5*b^2*\cos(2*d*x + 2*c) + b^2)*\sin(6*d*x + 6*c) \\ & ))*\sqrt{b}/((2*(5*\cos(8*d*x + 8*c) + 10*\cos(6*d*x + 6*c) + 10*\cos(4*d*x + 4 \\ & *c) + 5*\cos(2*d*x + 2*c) + 1)*\cos(10*d*x + 10*c) + \cos(10*d*x + 10*c)^2 + 1 \\ & 0*(10*\cos(6*d*x + 6*c) + 10*\cos(4*d*x + 4*c) + 5*\cos(2*d*x + 2*c) + 1)*\cos( \\ & 8*d*x + 8*c) + 25*\cos(8*d*x + 8*c)^2 + 20*(10*\cos(4*d*x + 4*c) + 5*\cos(2*d* \\ & x + 2*c) + 1)*\cos(6*d*x + 6*c) + 100*\cos(6*d*x + 6*c)^2 + 20*(5*\cos(2*d*x + \\ & 2*c) + 1)*\cos(4*d*x + 4*c) + 100*\cos(4*d*x + 4*c)^2 + 25*\cos(2*d*x + 2*c)^ \\ & 2 + 10*(\sin(8*d*x + 8*c) + 2*\sin(6*d*x + 6*c) + 2*\sin(4*d*x + 4*c) + \sin(2* \\ & d*x + 2*c))*\sin(10*d*x + 10*c) + \sin(10*d*x + 10*c)^2 + 50*(2*\sin(6*d*x + 6 \\ & *c) + 2*\sin(4*d*x + 4*c) + \sin(2*d*x + 2*c))*\sin(8*d*x + 8*c) + 25*\sin(8*d* \\ & x + 8*c)^2 + 100*(2*\sin(4*d*x + 4*c) + \sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + \\ & 100*\sin(6*d*x + 6*c)^2 + 100*\sin(4*d*x + 4*c)^2 + 100*\sin(4*d*x + 4*c)*\sin \\ & (2*d*x + 2*c) + 25*\sin(2*d*x + 2*c)^2 + 10*\cos(2*d*x + 2*c) + 1)*d) \end{aligned}$$

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**Fricas [A]** time = 1.72835, size = 158, normalized size = 1.36

$$\frac{(8b^2 \cos(dx+c)^4 + 4b^2 \cos(dx+c)^2 + 3b^2) \sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{15d \cos(dx+c)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(7/2)\*(b\*sec(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] 
$$\frac{1}{15}*(8*b^2*\cos(d*x + c)^4 + 4*b^2*\cos(d*x + c)^2 + 3*b^2)*\sqrt{b/\cos(d*x + c)}*\sin(d*x + c)/(d*\cos(d*x + c)^{(9/2)})$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(7/2)*(b*sec(d*x+c))**(5/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \sec(dx + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")`

[Out] `integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c)^(7/2), x)`

### 3.153 $\int \sec^{\frac{3}{2}}(c + dx)(b \sec(c + dx))^{5/2} dx$

**Optimal.** Leaf size=76

$$\frac{b^2 \sin^3(c + dx) \sec^{\frac{5}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{3d} + \frac{b^2 \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{b \sec(c + dx)}}{d}$$

[Out]  $(b^2 \sqrt{\sec[c + d*x]} \sqrt{b \sec[c + d*x]} \sin[c + d*x])/d + (b^2 \sec[c + d*x]^{(5/2)} \sqrt{b \sec[c + d*x]} \sin[c + d*x]^3)/(3*d)$

**Rubi [A]** time = 0.0174445, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 3767}

$$\frac{b^2 \sin^3(c + dx) \sec^{\frac{5}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{3d} + \frac{b^2 \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{b \sec(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\sec[c + d*x]^{(3/2)} * (b * \sec[c + d*x])^{(5/2)}, x]$

[Out]  $(b^2 \sqrt{\sec[c + d*x]} \sqrt{b \sec[c + d*x]} \sin[c + d*x])/d + (b^2 \sec[c + d*x]^{(5/2)} \sqrt{b \sec[c + d*x]} \sin[c + d*x]^3)/(3*d)$

#### Rule 17

$\text{Int}[(u_.) * ((a_.) * (v_))^{(m_)} * ((b_.) * (v_))^{(n_)}, x\_Symbol] \rightarrow \text{Dist}[(a^{(m + 1/2)} * b^{(n - 1/2)} * \sqrt{b*v}) / \sqrt{a*v}, \text{Int}[u*v^{(m + n)}, x], x] /;$  FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

#### Rule 3767

$\text{Int}[\csc[(c_.) + (d_.) * (x_)]^{(n_)}, x\_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], \text{Cot}[c + d*x]], x] /;$  FreeQ[{c, d}, x] && IGtQ[n/2, 0]

#### Rubi steps

$$\begin{aligned} \int \sec^{\frac{3}{2}}(c+dx)(b \sec(c+dx))^{5/2} dx &= \frac{(b^2 \sqrt{b \sec(c+dx)}) \int \sec^4(c+dx) dx}{\sqrt{\sec(c+dx)}} \\ &= -\frac{(b^2 \sqrt{b \sec(c+dx)}) \operatorname{Subst}\left(\int (1+x^2) dx, x, -\tan(c+dx)\right)}{d \sqrt{\sec(c+dx)}} \\ &= \frac{b^2 \sqrt{\sec(c+dx)} \sqrt{b \sec(c+dx)} \sin(c+dx)}{d} + \frac{b^2 \sec^{\frac{5}{2}}(c+dx) \sqrt{b \sec(c+dx)} \sin^3(c+dx)}{3d} \end{aligned}$$

**Mathematica [A]** time = 0.0789691, size = 45, normalized size = 0.59

$$\frac{\left(\frac{1}{3} \tan^3(c+dx) + \tan(c+dx)\right) (b \sec(c+dx))^{5/2}}{d \sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^(3/2)\*(b\*Sec[c + d\*x])^(5/2), x]

[Out] ((b\*Sec[c + d\*x])^(5/2)\*(Tan[c + d\*x] + Tan[c + d\*x]^3/3))/(d\*Sec[c + d\*x]^(5/2))

**Maple [A]** time = 0.106, size = 52, normalized size = 0.7

$$\frac{(2(\cos(dx+c))^2+1)\cos(dx+c)\sin(dx+c)}{3d} \left((\cos(dx+c))^{-1}\right)^{\frac{3}{2}} \left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(3/2)\*(b\*sec(d\*x+c))^(5/2), x)

[Out] 1/3/d\*(2\*cos(d\*x+c)^2+1)\*(1/cos(d\*x+c))^(3/2)\*(b/cos(d\*x+c))^(5/2)\*cos(d\*x+c)\*sin(d\*x+c)

**Maxima [B]** time = 1.9785, size = 420, normalized size = 5.53

$$\frac{4(3b^2 \cos(6dx+6c) \sin(2dx) + 3(2(3 \cos(4dx+4c) + 3 \cos(2dx+2c) + 1) \cos(6dx+6c) + \cos(6dx+6c)^2 + 6(3 \cos(2dx+2c) + 1) \cos(4dx+4c))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] 
$$-4/3*(3*b^2*\cos(6*d*x + 6*c)*\sin(2*d*x + 2*c) + 9*b^2*\cos(4*d*x + 4*c)*\sin(2*d*x + 2*c) - (3*b^2*\cos(2*d*x + 2*c) + b^2)*\sin(6*d*x + 6*c) - 3*(3*b^2*\cos(2*d*x + 2*c) + b^2)*\sin(4*d*x + 4*c))*\sqrt{b}/((2*(3*\cos(4*d*x + 4*c) + 3*\cos(2*d*x + 2*c) + 1)*\cos(6*d*x + 6*c) + \cos(6*d*x + 6*c)^2 + 6*(3*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + 9*\cos(4*d*x + 4*c)^2 + 9*\cos(2*d*x + 2*c)^2 + 6*(\sin(4*d*x + 4*c) + \sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + \sin(6*d*x + 6*c)^2 + 9*\sin(4*d*x + 4*c)^2 + 18*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*\sin(2*d*x + 2*c)^2 + 6*\cos(2*d*x + 2*c) + 1)*d)$$

**Fricas [A]** time = 1.6932, size = 123, normalized size = 1.62

$$\frac{(2b^2 \cos(dx + c)^2 + b^2) \sqrt{\frac{b}{\cos(dx+c)} \sin(dx + c)}}{3d \cos(dx + c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] 
$$1/3*(2*b^2*\cos(d*x + c)^2 + b^2)*\sqrt{b/\cos(d*x + c)}*\sin(d*x + c)/(d*\cos(d*x + c)^(5/2))$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**(5/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c)^(3/2), x)
```

### 3.154 $\int \sqrt{\sec(c + dx)}(b \sec(c + dx))^{5/2} dx$

**Optimal.** Leaf size=78

$$\frac{b^2 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{2d} + \frac{b^2 \sqrt{b \sec(c + dx)} \tanh^{-1}(\sin(c + dx))}{2d \sqrt{\sec(c + dx)}}$$

[Out] (b^2\*ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(2\*d\*Sqrt[Sec[c + d\*x]]) + (b^2\*Sec[c + d\*x]^(3/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(2\*d)

**Rubi [A]** time = 0.020254, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 3768, 3770}

$$\frac{b^2 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx) \sqrt{b \sec(c + dx)}}{2d} + \frac{b^2 \sqrt{b \sec(c + dx)} \tanh^{-1}(\sin(c + dx))}{2d \sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d\*x]]\*(b\*Sec[c + d\*x])^(5/2), x]

[Out] (b^2\*ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(2\*d\*Sqrt[Sec[c + d\*x]]) + (b^2\*Sec[c + d\*x]^(3/2)\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(2\*d)

#### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x])\*(b\*Csc[c + d\*x])^(n - 1)/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \sqrt{\sec(c+dx)}(b \sec(c+dx))^{5/2} dx &= \frac{(b^2 \sqrt{b \sec(c+dx)}) \int \sec^3(c+dx) dx}{\sqrt{\sec(c+dx)}} \\ &= \frac{b^2 \sec^{\frac{3}{2}}(c+dx) \sqrt{b \sec(c+dx)} \sin(c+dx)}{2d} + \frac{(b^2 \sqrt{b \sec(c+dx)}) \int \sec(c+dx) dx}{2\sqrt{\sec(c+dx)}} \\ &= \frac{b^2 \tanh^{-1}(\sin(c+dx)) \sqrt{b \sec(c+dx)}}{2d\sqrt{\sec(c+dx)}} + \frac{b^2 \sec^{\frac{3}{2}}(c+dx) \sqrt{b \sec(c+dx)} \sin(c+dx)}{2d} \end{aligned}$$

**Mathematica [A]** time = 0.0767913, size = 50, normalized size = 0.64

$$\frac{(b \sec(c+dx))^{5/2} (\tanh^{-1}(\sin(c+dx)) + \tan(c+dx) \sec(c+dx))}{2d \sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[c + d\*x]]\*(b\*Sec[c + d\*x])^(5/2), x]

[Out] ((b\*Sec[c + d\*x])^(5/2)\*(ArcTanh[Sin[c + d\*x]] + Sec[c + d\*x]\*Tan[c + d\*x]))/(2\*d\*Sec[c + d\*x]^(5/2))

**Maple [A]** time = 0.132, size = 112, normalized size = 1.4

$$\frac{\cos(dx+c)}{2d} \left( \ln \left( -\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)} \right) (\cos(dx+c))^2 - \ln \left( -\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)} \right) (\cos(dx+c)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(1/2)\*(b\*sec(d\*x+c))^(5/2), x)

[Out] 1/2/d\*(ln(-(-1+cos(d\*x+c)-sin(d\*x+c))/sin(d\*x+c))\*cos(d\*x+c)^2-ln(-(-1+cos(d\*x+c)+sin(d\*x+c))/sin(d\*x+c))\*cos(d\*x+c)^2+sin(d\*x+c))\*cos(d\*x+c)\*(1/cos(d\*x+c))^(1/2)\*(b/cos(d\*x+c))^(5/2)



**Maxima [B]** time = 2.19142, size = 1008, normalized size = 12.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(1/2)\*(b\*sec(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] 
$$-1/4*(4*(b^2*\sin(4*d*x + 4*c) + 2*b^2*\sin(2*d*x + 2*c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4*(b^2*\sin(4*d*x + 4*c) + 2*b^2*\sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - (b^2*\cos(4*d*x + 4*c)^2 + 4*b^2*\cos(2*d*x + 2*c)^2 + b^2*\sin(4*d*x + 4*c)^2 + 4*b^2*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*b^2*\sin(2*d*x + 2*c)^2 + 4*b^2*\cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*\cos(2*d*x + 2*c) + b^2)*\cos(4*d*x + 4*c))*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) + (b^2*\cos(4*d*x + 4*c)^2 + 4*b^2*\cos(2*d*x + 2*c)^2 + b^2*\sin(4*d*x + 4*c)^2 + 4*b^2*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*b^2*\sin(2*d*x + 2*c)^2 + 4*b^2*\cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*\cos(2*d*x + 2*c) + b^2)*\cos(4*d*x + 4*c))*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) - 4*(b^2*\cos(4*d*x + 4*c) + 2*b^2*\cos(2*d*x + 2*c) + b^2)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(b^2*\cos(4*d*x + 4*c) + 2*b^2*\cos(2*d*x + 2*c) + b^2)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*\sqrt{b}/((2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*d)$$

**Fricas [A]** time = 1.97983, size = 544, normalized size = 6.97

$$\frac{\left[ \frac{5}{b^2} \cos(dx+c) \log \left( -\frac{b \cos(dx+c)^2 - 2\sqrt{b} \sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b}{\cos(dx+c)^2} \right) + \frac{2b^2 \sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{\cos(dx+c)}} \sqrt{-bb^2} \arctan \left( \frac{\sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}}}{\cos(dx+c)} \right) \right]}{4d \cos(dx+c)},$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(1/2)\*(b\*sec(d\*x+c))^(5/2),x, algorithm="fricas")

```
[Out] [1/4*(b^(5/2)*cos(d*x + c)*log(-(b*cos(d*x + c))^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*b^2*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)), -1/2*(sqrt(-b)*b^2*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)*cos(d*x + c) - b^2*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(b*sec(d*x+c))**(5/2), x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{5}{2}} \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(5/2), x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*sqrt(sec(d*x + c)), x)
```

$$3.155 \quad \int \frac{(b \sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}} dx$$

Optimal. Leaf size=35

$$\frac{b^2 \sin(c+dx) \sqrt{\sec(c+dx)} \sqrt{b \sec(c+dx)}}{d}$$

[Out] (b^2\*Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/d

**Rubi [A]** time = 0.011894, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 3767, 8}

$$\frac{b^2 \sin(c+dx) \sqrt{\sec(c+dx)} \sqrt{b \sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(5/2)/Sqrt[Sec[c + d\*x]],x]

[Out] (b^2\*Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/d

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned} \int \frac{(b \sec(c + dx))^{5/2}}{\sqrt{\sec(c + dx)}} dx &= \frac{(b^2 \sqrt{b \sec(c + dx)}) \int \sec^2(c + dx) dx}{\sqrt{\sec(c + dx)}} \\ &= -\frac{(b^2 \sqrt{b \sec(c + dx)}) \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{d \sqrt{\sec(c + dx)}} \\ &= \frac{b^2 \sqrt{\sec(c + dx)} \sqrt{b \sec(c + dx)} \sin(c + dx)}{d} \end{aligned}$$

**Mathematica [A]** time = 0.0309176, size = 32, normalized size = 0.91

$$\frac{\sin(c + dx)(b \sec(c + dx))^{5/2}}{d \sec^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Sec[c + d\*x])^(5/2)/Sqrt[Sec[c + d\*x]], x]

[Out] ((b\*Sec[c + d\*x])^(5/2)\*Sin[c + d\*x])/(d\*Sec[c + d\*x]^(3/2))

**Maple [A]** time = 0.127, size = 39, normalized size = 1.1

$$\frac{\cos(dx + c) \sin(dx + c)}{d} \left( \frac{b}{\cos(dx + c)} \right)^{\frac{5}{2}} \frac{1}{\sqrt{(\cos(dx + c))^{-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(5/2)/sec(d\*x+c)^(1/2), x)

[Out] 1/d\*(b/cos(d\*x+c))^(5/2)\*cos(d\*x+c)\*sin(d\*x+c)/(1/cos(d\*x+c))^(1/2)

**Maxima [A]** time = 2.06034, size = 73, normalized size = 2.09

$$\frac{2 b^2 \sin(2 dx + 2 c)}{(\cos(2 dx + 2 c)^2 + \sin(2 dx + 2 c)^2 + 2 \cos(2 dx + 2 c) + 1) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")`

[Out]  $2*b^{5/2}*\sin(2*d*x + 2*c)/((\cos(2*d*x + 2*c))^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)*d$

**Fricas [A]** time = 1.63597, size = 84, normalized size = 2.4

$$\frac{b^2 \sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")`

[Out]  $b^2*\sqrt{b/\cos(d*x + c)}*\sin(d*x + c)/(d*\sqrt{\cos(d*x + c)})$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx+c))^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)/sqrt(sec(d*x + c)), x)
```

$$3.156 \quad \int \frac{(b \sec(c+dx))^{5/2}}{\sec^2(c+dx)} dx$$

**Optimal.** Leaf size=36

$$\frac{b^2 \sqrt{b \sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{d \sqrt{\sec(c+dx)}}$$

[Out] (b^2\*ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(d\*Sqrt[Sec[c + d\*x]])

**Rubi [A]** time = 0.00772, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 3770}

$$\frac{b^2 \sqrt{b \sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{d \sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(5/2)/Sec[c + d\*x]^(3/2), x]

[Out] (b^2\*ArcTanh[Sin[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])/(d\*Sqrt[Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\int \frac{(b \sec(c + dx))^{5/2}}{\sec^{\frac{3}{2}}(c + dx)} dx = \frac{(b^2 \sqrt{b \sec(c + dx)}) \int \sec(c + dx) dx}{\sqrt{\sec(c + dx)}} \\ = \frac{b^2 \tanh^{-1}(\sin(c + dx)) \sqrt{b \sec(c + dx)}}{d \sqrt{\sec(c + dx)}}$$

**Mathematica [A]** time = 0.0342288, size = 33, normalized size = 0.92

$$\frac{(b \sec(c + dx))^{5/2} \tanh^{-1}(\sin(c + dx))}{d \sec^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Sec[c + d\*x])^(5/2)/Sec[c + d\*x]^(3/2), x]

[Out] (ArcTanh[Sin[c + d\*x]]\*(b\*Sec[c + d\*x])^(5/2))/(d\*Sec[c + d\*x]^(5/2))

**Maple [A]** time = 0.097, size = 52, normalized size = 1.4

$$-2 \frac{\cos(dx + c)}{d ((\cos(dx + c))^{-1})^{3/2}} \operatorname{Arctanh}\left(\frac{-1 + \cos(dx + c)}{\sin(dx + c)}\right) \left(\frac{b}{\cos(dx + c)}\right)^{5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(5/2)/sec(d\*x+c)^(3/2), x)

[Out] -2/d\*arctanh((-1+cos(d\*x+c))/sin(d\*x+c))\*cos(d\*x+c)\*(b/cos(d\*x+c))^(5/2)/(1/cos(d\*x+c))^(3/2)

**Maxima [B]** time = 1.91586, size = 97, normalized size = 2.69

$$\frac{(b^2 \log(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \sin(dx + c) + 1) - b^2 \log(\cos(dx + c)^2 + \sin(dx + c)^2 - 2 \sin(dx + c) + 1)) \sqrt{b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((b\*sec(d\*x+c))^(5/2)/sec(d\*x+c)^(3/2),x, algorithm="maxima")

[Out]  $\frac{1}{2} * (b^2 * \log(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 * \sin(dx + c) + 1) - b^2 * \log(\cos(dx + c)^2 + \sin(dx + c)^2 - 2 * \sin(dx + c) + 1)) * \sqrt{b} / d$

**Fricas [A]** time = 1.84508, size = 296, normalized size = 8.22

$$\left[ \frac{b^{\frac{5}{2}} \log\left(\frac{b \cos(dx+c)^2 - 2\sqrt{b} \sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b}{\cos(dx+c)^2}\right)}{2d}, \frac{\sqrt{-b} b^2 \arctan\left(\frac{\sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c)}{b}\right)}{d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(5/2)/sec(d\*x+c)^(3/2),x, algorithm="fricas")

[Out]  $\frac{1}{2} * b^{5/2} * \log(-(b * \cos(dx + c))^2 - 2 * \sqrt{b} * \sqrt{b / \cos(dx + c)} * \sqrt{c \cos(dx + c)} * \sin(dx + c) - 2 * b) / \cos(dx + c)^2 / d, -\sqrt{-b} * b^2 * \arctan(\sqrt{-b} * \sqrt{b / \cos(dx + c)} * \sqrt{c \cos(dx + c)} * \sin(dx + c) / b) / d]$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(5/2)/sec(d\*x+c)\*\*(3/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx + c))^{\frac{5}{2}}}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)/sec(d*x + c)^(3/2), x)
```

$$3.157 \quad \int \frac{(b \sec(c+dx))^{5/2}}{\sec^2(c+dx)} dx$$

**Optimal.** Leaf size=27

$$\frac{b^2 x \sqrt{b \sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

[Out] (b^2\*x\*Sqrt[b\*Sec[c + d\*x]])/Sqrt[Sec[c + d\*x]]

**Rubi [A]** time = 0.0026389, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 8}

$$\frac{b^2 x \sqrt{b \sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(5/2)/Sec[c + d\*x]^(5/2),x]

[Out] (b^2\*x\*Sqrt[b\*Sec[c + d\*x]])/Sqrt[Sec[c + d\*x]]

### Rule 17

Int[(u\_)\*((a\_)\*(v\_))^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned} \int \frac{(b \sec(c+dx))^{5/2}}{\sec^2(c+dx)} dx &= \frac{(b^2 \sqrt{b \sec(c+dx)}) \int 1 dx}{\sqrt{\sec(c+dx)}} \\ &= \frac{b^2 x \sqrt{b \sec(c+dx)}}{\sqrt{\sec(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0194576, size = 24, normalized size = 0.89

$$\frac{x(b \sec(c + dx))^{5/2}}{\sec^2(c + dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Sec[c + d\*x])^(5/2)/Sec[c + d\*x]^(5/2),x]

[Out] (x\*(b\*Sec[c + d\*x])^(5/2))/Sec[c + d\*x]^(5/2)

**Maple [A]** time = 0.091, size = 32, normalized size = 1.2

$$\frac{dx + c}{d} \left( \frac{b}{\cos(dx + c)} \right)^{5/2} \left( (\cos(dx + c))^{-1} \right)^{-5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(5/2)/sec(d\*x+c)^(5/2),x)

[Out] 1/d\*(d\*x+c)\*(b/cos(d\*x+c))^(5/2)/(1/cos(d\*x+c))^(5/2)

**Maxima [A]** time = 1.82461, size = 35, normalized size = 1.3

$$\frac{2b^{5/2} \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(5/2)/sec(d\*x+c)^(5/2),x, algorithm="maxima")

[Out] 2\*b^(5/2)\*arctan(sin(d\*x + c)/(cos(d\*x + c) + 1))/d

**Fricas [A]** time = 1.97332, size = 271, normalized size = 10.04

$$\left[ \frac{\sqrt{-b} b^2 \log\left(-2 \sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 2b \cos(dx+c)^2 - b\right)}{2d}, \frac{b^{\frac{5}{2}} \arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{b} \sqrt{\cos(dx+c)}}\right)}{d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(5/2)/sec(d\*x+c)^(5/2),x, algorithm="fricas")

[Out] [1/2\*sqrt(-b)\*b^2\*log(-2\*sqrt(-b)\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + 2\*b\*cos(d\*x + c)^2 - b)/d, b^(5/2)\*arctan(sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/(sqrt(b)\*sqrt(cos(d\*x + c))))/d]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(5/2)/sec(d\*x+c)\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx + c))^{\frac{5}{2}}}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(5/2)/sec(d\*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(5/2)/sec(d\*x + c)^(5/2), x)

$$3.158 \quad \int \frac{(b \sec(c+dx))^{5/2}}{\sec^2(c+dx)} dx$$

**Optimal.** Leaf size=35

$$\frac{b^2 \sin(c+dx) \sqrt{b \sec(c+dx)}}{d \sqrt{\sec(c+dx)}}$$

[Out] (b^2\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[Sec[c + d\*x]])

**Rubi [A]** time = 0.0070795, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 2637}

$$\frac{b^2 \sin(c+dx) \sqrt{b \sec(c+dx)}}{d \sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(5/2)/Sec[c + d\*x]^(7/2),x]

[Out] (b^2\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\int \frac{(b \sec(c + dx))^{5/2}}{\sec^2(c + dx)} dx = \frac{(b^2 \sqrt{b \sec(c + dx)}) \int \cos(c + dx) dx}{\sqrt{\sec(c + dx)}}$$

$$= \frac{b^2 \sqrt{b \sec(c + dx)} \sin(c + dx)}{d \sqrt{\sec(c + dx)}}$$

**Mathematica [A]** time = 0.0652117, size = 32, normalized size = 0.91

$$\frac{\sin(c + dx)(b \sec(c + dx))^{5/2}}{d \sec^2(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Sec[c + d\*x])^(5/2)/Sec[c + d\*x]^(7/2), x]

[Out] ((b\*Sec[c + d\*x])^(5/2)\*Sin[c + d\*x])/(d\*Sec[c + d\*x]^(5/2))

**Maple [A]** time = 0.105, size = 41, normalized size = 1.2

$$\frac{\sin(dx + c)}{d \cos(dx + c)} \left( \frac{b}{\cos(dx + c)} \right)^{5/2} \left( (\cos(dx + c))^{-1} \right)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(5/2)/sec(d\*x+c)^(7/2), x)

[Out] 1/d\*sin(d\*x+c)\*(b/cos(d\*x+c))^(5/2)/(1/cos(d\*x+c))^(7/2)/cos(d\*x+c)

**Maxima [A]** time = 1.87907, size = 18, normalized size = 0.51

$$\frac{b^{5/2} \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(5/2)/sec(d\*x+c)^(7/2),x, algorithm="maxima")

[Out] b^(5/2)\*sin(d\*x + c)/d

**Fricas [A]** time = 1.66211, size = 81, normalized size = 2.31

$$\frac{b^2 \sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(5/2)/sec(d\*x+c)^(7/2),x, algorithm="fricas")

[Out] b^2\*sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c)/d

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(5/2)/sec(d\*x+c)\*\*(7/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(5/2)/sec(d\*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(5/2)/sec(d\*x + c)^(7/2), x)



$$3.159 \quad \int \frac{(b \sec(c+dx))^{5/2}}{\sec^2(c+dx)} dx$$

**Optimal.** Leaf size=69

$$\frac{b^2 x \sqrt{b \sec(c+dx)}}{2 \sqrt{\sec(c+dx)}} + \frac{b^2 \sin(c+dx) \sqrt{b \sec(c+dx)}}{2d \sec^{\frac{3}{2}}(c+dx)}$$

[Out] (b^2\*x\*Sqrt[b\*Sec[c + d\*x]])/(2\*Sqrt[Sec[c + d\*x]]) + (b^2\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(2\*d\*Sec[c + d\*x]^(3/2))

**Rubi [A]** time = 0.0154454, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 2635, 8}

$$\frac{b^2 x \sqrt{b \sec(c+dx)}}{2 \sqrt{\sec(c+dx)}} + \frac{b^2 \sin(c+dx) \sqrt{b \sec(c+dx)}}{2d \sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(5/2)/Sec[c + d\*x]^(9/2), x]

[Out] (b^2\*x\*Sqrt[b\*Sec[c + d\*x]])/(2\*Sqrt[Sec[c + d\*x]]) + (b^2\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(2\*d\*Sec[c + d\*x]^(3/2))

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x])\*(b\*SIN[c + d\*x])^(n - 1)/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*SIN[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

### Rubi steps

$$\begin{aligned} \int \frac{(b \sec(c + dx))^{5/2}}{\sec^2(c + dx)} dx &= \frac{(b^2 \sqrt{b \sec(c + dx)}) \int \cos^2(c + dx) dx}{\sqrt{\sec(c + dx)}} \\ &= \frac{b^2 \sqrt{b \sec(c + dx)} \sin(c + dx)}{2d \sec^{\frac{3}{2}}(c + dx)} + \frac{(b^2 \sqrt{b \sec(c + dx)}) \int 1 dx}{2\sqrt{\sec(c + dx)}} \\ &= \frac{b^2 x \sqrt{b \sec(c + dx)}}{2\sqrt{\sec(c + dx)}} + \frac{b^2 \sqrt{b \sec(c + dx)} \sin(c + dx)}{2d \sec^{\frac{3}{2}}(c + dx)} \end{aligned}$$

**Mathematica [A]** time = 0.1127, size = 45, normalized size = 0.65

$$\frac{(2(c + dx) + \sin(2(c + dx)))(b \sec(c + dx))^{5/2}}{4d \sec^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] `Integrate[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2), x]`

[Out] `((b*Sec[c + d*x])^(5/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sec[c + d*x]^(5/2))`

**Maple [A]** time = 0.105, size = 54, normalized size = 0.8

$$\frac{\cos(dx + c) \sin(dx + c) + dx + c}{2d (\cos(dx + c))^2} \left( \frac{b}{\cos(dx + c)} \right)^{\frac{5}{2}} \left( (\cos(dx + c))^{-1} \right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2), x)`

[Out] `1/2/d*(cos(d*x+c)*sin(d*x+c)+d*x+c)*(b/cos(d*x+c))^(5/2)/cos(d*x+c)^2/(1/cos(d*x+c))^(9/2)`

---

**Maxima [A]** time = 2.12771, size = 43, normalized size = 0.62

$$\frac{(2(dx+c)b^2 + b^2 \sin(2dx+2c))\sqrt{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(5/2)/sec(d\*x+c)^(9/2),x, algorithm="maxima")

[Out] 1/4\*(2\*(d\*x + c)\*b^2 + b^2\*sin(2\*d\*x + 2\*c))\*sqrt(b)/d

---

**Fricas [A]** time = 2.04022, size = 444, normalized size = 6.43

$$\left[ \frac{2b^2 \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + \sqrt{-b} b^2 \log\left(-2\sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 2b \cos(dx+c)^2\right)}{4d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(5/2)/sec(d\*x+c)^(9/2),x, algorithm="fricas")

[Out] [1/4\*(2\*b^2\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + sqrt(-b)\*b^2\*log(-2\*sqrt(-b)\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + 2\*b\*cos(d\*x + c)^2 - b))/d, 1/2\*(b^2\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + b^(5/2)\*arctan(sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/(sqrt(b)\*sqrt(cos(d\*x + c)))))/d]

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(5/2)/sec(d\*x+c)\*\*(9/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx + c))^{\frac{5}{2}}}{\sec(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(5/2)/sec(d\*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(5/2)/sec(d\*x + c)^(9/2), x)

$$3.160 \quad \int \frac{(b \sec(c+dx))^{5/2}}{\sec^2(c+dx)} dx$$

**Optimal.** Leaf size=76

$$\frac{b^2 \sin(c+dx) \sqrt{b \sec(c+dx)}}{d \sqrt{\sec(c+dx)}} - \frac{b^2 \sin^3(c+dx) \sqrt{b \sec(c+dx)}}{3d \sqrt{\sec(c+dx)}}$$

[Out] (b^2\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[Sec[c + d\*x]]) - (b^2\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*d\*Sqrt[Sec[c + d\*x]])

**Rubi [A]** time = 0.0168081, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 2633}

$$\frac{b^2 \sin(c+dx) \sqrt{b \sec(c+dx)}}{d \sqrt{\sec(c+dx)}} - \frac{b^2 \sin^3(c+dx) \sqrt{b \sec(c+dx)}}{3d \sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(5/2)/Sec[c + d\*x]^(11/2), x]

[Out] (b^2\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[Sec[c + d\*x]]) - (b^2\*Sqrt[b\*Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*d\*Sqrt[Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 2633

Int[sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

### Rubi steps

$$\int \frac{(b \sec(c + dx))^{5/2}}{\sec^{11/2}(c + dx)} dx = \frac{(b^2 \sqrt{b \sec(c + dx)}) \int \cos^3(c + dx) dx}{\sqrt{\sec(c + dx)}}$$

$$= -\frac{(b^2 \sqrt{b \sec(c + dx)}) \text{Subst} \left( \int (1 - x^2) dx, x, -\sin(c + dx) \right)}{d \sqrt{\sec(c + dx)}}$$

$$= \frac{b^2 \sqrt{b \sec(c + dx)} \sin(c + dx)}{d \sqrt{\sec(c + dx)}} - \frac{b^2 \sqrt{b \sec(c + dx)} \sin^3(c + dx)}{3d \sqrt{\sec(c + dx)}}$$

**Mathematica [A]** time = 0.162517, size = 45, normalized size = 0.59

$$\frac{\sin(c + dx)(\cos(2(c + dx)) + 5)(b \sec(c + dx))^{5/2}}{6d \sec^2(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Sec[c + d\*x])^(5/2)/Sec[c + d\*x]^(11/2), x]

[Out] ((5 + Cos[2\*(c + d\*x)])\*(b\*Sec[c + d\*x])^(5/2)\*Sin[c + d\*x])/(6\*d\*Sec[c + d\*x]^(5/2))

**Maple [A]** time = 0.114, size = 52, normalized size = 0.7

$$\frac{((\cos(dx + c))^2 + 2) \sin(dx + c)}{3d (\cos(dx + c))^3} \left( \frac{b}{\cos(dx + c)} \right)^{5/2} ((\cos(dx + c))^{-1})^{-11/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(5/2)/sec(d\*x+c)^(11/2), x)

[Out] 1/3/d\*(cos(d\*x+c)^2+2)\*sin(d\*x+c)\*(b/cos(d\*x+c))^(5/2)/cos(d\*x+c)^3/(1/cos(d\*x+c))^(11/2)

**Maxima [A]** time = 2., size = 66, normalized size = 0.87

$$\frac{\left( b^2 \sin(3dx + 3c) + 9b^2 \sin\left(\frac{1}{3} \arctan(\sin(3dx + 3c), \cos(3dx + 3c))\right) \right) \sqrt{b}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="maxima")`

[Out]  $\frac{1}{12}*(b^2*\sin(3*d*x + 3*c) + 9*b^2*\sin(\frac{1}{3}*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))))*\sqrt{b}/d$

**Fricas [A]** time = 1.64336, size = 140, normalized size = 1.84

$$\frac{(b^2 \cos(dx + c)^3 + 2b^2 \cos(dx + c)) \sqrt{\frac{b}{\cos(dx+c)}} \sin(dx + c)}{3d\sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="fricas")`

[Out]  $\frac{1}{3}*(b^2*\cos(d*x + c)^3 + 2*b^2*\cos(d*x + c))*\sqrt{b/\cos(d*x + c)}*\sin(d*x + c)/(d*\sqrt{\cos(d*x + c)})$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(11/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx + c))^{\frac{5}{2}}}{\sec(dx + c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)/sec(d*x + c)^(11/2), x)
```



$$3.161 \quad \int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=72

$$\frac{\sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{b \sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{b \sec(c+dx)}}$$

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[Sec[c + d\*x]])/(2\*d\*Sqrt[b\*Sec[c + d\*x]]) + (Sec[c + d\*x]^(5/2)\*Sin[c + d\*x])/(2\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0192948, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 3768, 3770}

$$\frac{\sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{b \sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(7/2)/Sqrt[b\*Sec[c + d\*x]], x]

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[Sec[c + d\*x]])/(2\*d\*Sqrt[b\*Sec[c + d\*x]]) + (Sec[c + d\*x]^(5/2)\*Sin[c + d\*x])/(2\*d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x])\*(b\*Csc[c + d\*x])^(n - 1)/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{b \sec(c+dx)}} dx &= \frac{\sqrt{\sec(c+dx)} \int \sec^3(c+dx) dx}{\sqrt{b \sec(c+dx)}} \\ &= \frac{\sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2d\sqrt{b \sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)} \int \sec(c+dx) dx}{2\sqrt{b \sec(c+dx)}} \\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{2d\sqrt{b \sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2d\sqrt{b \sec(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0574469, size = 50, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)} \left( \tanh^{-1}(\sin(c+dx)) + \tan(c+dx) \sec(c+dx) \right)}{2d\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/Sqrt[b*Sec[c + d*x]], x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(2
*d*Sqrt[b*Sec[c + d*x]])
```

**Maple [A]** time = 0.137, size = 112, normalized size = 1.6

$$\frac{\cos(dx+c)}{2d} \left( \ln \left( -\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)} \right) (\cos(dx+c))^2 - \ln \left( -\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)} \right) (\cos(dx+c)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2), x)
```

```
[Out] 1/2/d*(ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-ln(-(-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+sin(d*x+c))*(1/cos(d*x+c))^(7/2)
```

)\*cos(d\*x+c)/(b/cos(d\*x+c))^(1/2)

**Maxima [B]** time = 2.02468, size = 892, normalized size = 12.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(7/2)/(b\*sec(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] 
$$-1/4*(4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) + (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) - 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))/((2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\sqrt{b}*d)$$

**Fricas [A]** time = 1.96414, size = 533, normalized size = 7.4

$$\left[ \frac{\sqrt{b} \cos(dx+c) \log\left( \frac{b \cos(dx+c)^2 - 2\sqrt{b} \sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b}{\cos(dx+c)^2} \right) + \frac{2\sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4bd \cos(dx+c)}, \sqrt{-b} \arctan\left( \frac{\sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}}}{\sqrt{\cos(dx+c)}} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c))^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b*d*cos(d*x + c)), -1/2*(sqrt(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b*cos(d*x + c) - sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b*d*cos(d*x + c))]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{\sqrt{b} \sec(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/sqrt(b*sec(d*x + c)), x)
```

$$3.162 \quad \int \frac{\sec^5(c+dx)}{\sqrt{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=32

$$\frac{\sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{d \sqrt{b \sec(c+dx)}}$$

[Out] (Sec[c + d\*x]^(3/2)\*Sin[c + d\*x])/(d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0114377, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 3767, 8}

$$\frac{\sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{d \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(5/2)/Sqrt[b\*Sec[c + d\*x]],x]

[Out] (Sec[c + d\*x]^(3/2)\*Sin[c + d\*x])/(d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{b \sec(c+dx)}} dx &= \frac{\sqrt{\sec(c+dx)} \int \sec^2(c+dx) dx}{\sqrt{b \sec(c+dx)}} \\
&= -\frac{\sqrt{\sec(c+dx)} \text{Subst}\left(\int 1 dx, x, -\tan(c+dx)\right)}{d\sqrt{b \sec(c+dx)}} \\
&= \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{d\sqrt{b \sec(c+dx)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0256689, size = 32, normalized size = 1.

$$\frac{\sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{d\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^(5/2)/Sqrt[b\*Sec[c + d\*x]], x]

[Out] (Sec[c + d\*x]^(3/2)\*Sin[c + d\*x])/(d\*Sqrt[b\*Sec[c + d\*x]])

**Maple [A]** time = 0.119, size = 39, normalized size = 1.2

$$\frac{\cos(dx+c) \sin(dx+c)}{d} \left( (\cos(dx+c))^{-1} \right)^{\frac{5}{2}} \frac{1}{\sqrt{\frac{b}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(5/2)/(b\*sec(d\*x+c))^(1/2), x)

[Out] 1/d\*(1/cos(d\*x+c))^(5/2)\*cos(d\*x+c)\*sin(d\*x+c)/(b/cos(d\*x+c))^(1/2)

**Maxima [B]** time = 2.03861, size = 80, normalized size = 2.5

$$\frac{2\sqrt{b} \sin(2dx+2c)}{(b \cos(2dx+2c)^2 + b \sin(2dx+2c)^2 + 2b \cos(2dx+2c) + b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out]  $2*\sqrt{b}*\sin(2*d*x + 2*c)/((b*\cos(2*d*x + 2*c)^2 + b*\sin(2*d*x + 2*c)^2 + 2*b*\cos(2*d*x + 2*c) + b)*d)$

**Fricas [A]** time = 1.64232, size = 81, normalized size = 2.53

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{bd\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `sqrt(b/cos(d*x + c))*sin(d*x + c)/(b*d*sqrt(cos(d*x + c)))`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(1/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b \sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c)), x)
```



$$3.163 \quad \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=33

$$\frac{\sqrt{\sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{b \sec(c+dx)}}$$

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[Sec[c + d\*x]])/(d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0074269, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 3770}

$$\frac{\sqrt{\sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(3/2)/Sqrt[b\*Sec[c + d\*x]],x]

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[Sec[c + d\*x]])/(d\*Sqrt[b\*Sec[c + d\*x]])

#### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

#### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b \sec(c+dx)}} dx = \frac{\sqrt{\sec(c+dx)} \int \sec(c+dx) dx}{\sqrt{b \sec(c+dx)}} \\ = \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{d\sqrt{b \sec(c+dx)}}$$

**Mathematica [A]** time = 0.0164863, size = 33, normalized size = 1.

$$\frac{\sqrt{\sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^(3/2)/Sqrt[b\*Sec[c + d\*x]], x]

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[Sec[c + d\*x]])/(d\*Sqrt[b\*Sec[c + d\*x]])

**Maple [A]** time = 0.12, size = 52, normalized size = 1.6

$$-2 \frac{((\cos(dx+c))^{-1})^{3/2} \cos(dx+c)}{d} \operatorname{Arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right) \frac{1}{\sqrt{\frac{b}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(3/2)/(b\*sec(d\*x+c))^(1/2), x)

[Out] -2/d\*arctanh((-1+cos(d\*x+c))/sin(d\*x+c))\*(1/cos(d\*x+c))^(3/2)\*cos(d\*x+c)/(b/cos(d\*x+c))^(1/2)

**Maxima [B]** time = 1.98168, size = 88, normalized size = 2.67

$$\frac{\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2 \sin(dx+c) + 1)}{2\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(3/2)/(b\*sec(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/2\*(log(cos(d\*x + c)^2 + sin(d\*x + c)^2 + 2\*sin(d\*x + c) + 1) - log(cos(d\*x + c)^2 + sin(d\*x + c)^2 - 2\*sin(d\*x + c) + 1))/(sqrt(b)\*d)

**Fricas [A]** time = 1.90028, size = 298, normalized size = 9.03

$$\left[ \frac{\log\left(\frac{b \cos(dx+c)^2 - 2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)\sin(dx+c)} - 2b}{\cos(dx+c)^2}\right)}{2\sqrt{bd}}, \frac{\sqrt{-b} \arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)\sin(dx+c)}}{b}\right)}{bd} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(3/2)/(b\*sec(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/2\*log(-(b\*cos(d\*x + c)^2 - 2\*sqrt(b)\*sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c) - 2\*b)/cos(d\*x + c)^2)/(sqrt(b)\*d), -sqrt(-b)\*arctan(sqrt(-b)\*sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c)/b)/(b\*d)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*(3/2)/(b\*sec(d\*x+c))\*\*(1/2),x)

[Out] Integral(sec(c + d\*x)\*\*(3/2)/sqrt(b\*sec(c + d\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^3}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c)), x)
```

$$3.164 \quad \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b \sec(c+dx)}} dx$$

Optimal. Leaf size=24

$$\frac{x\sqrt{\sec(c+dx)}}{\sqrt{b \sec(c+dx)}}$$

[Out] (x\*Sqrt[Sec[c + d\*x]])/Sqrt[b\*Sec[c + d\*x]]

**Rubi [A]** time = 0.0022741, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d\*x]]/Sqrt[b\*Sec[c + d\*x]],x]

[Out] (x\*Sqrt[Sec[c + d\*x]])/Sqrt[b\*Sec[c + d\*x]]

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b \sec(c+dx)}} dx &= \frac{\sqrt{\sec(c+dx)} \int 1 dx}{\sqrt{b \sec(c+dx)}} \\ &= \frac{x\sqrt{\sec(c+dx)}}{\sqrt{b \sec(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0147899, size = 24, normalized size = 1.

$$\frac{x\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[c + d\*x]]/Sqrt[b\*Sec[c + d\*x]], x]

[Out] (x\*Sqrt[Sec[c + d\*x]])/Sqrt[b\*Sec[c + d\*x]]

---

**Maple [A]** time = 0.092, size = 32, normalized size = 1.3

$$\frac{dx+c}{d} \sqrt{(\cos(dx+c))^{-1}} \frac{1}{\sqrt{\frac{b}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(1/2)/(b\*sec(d\*x+c))^(1/2), x)

[Out] 1/d\*(d\*x+c)\*(1/cos(d\*x+c))^(1/2)/(b/cos(d\*x+c))^(1/2)

---

**Maxima [A]** time = 1.57971, size = 35, normalized size = 1.46

$$\frac{2 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(1/2)/(b\*sec(d\*x+c))^(1/2), x, algorithm="maxima")

[Out] 2\*arctan(sin(d\*x + c)/(cos(d\*x + c) + 1))/(sqrt(b)\*d)

---

**Fricas [A]** time = 1.97238, size = 274, normalized size = 11.42

$$\left[ \frac{\sqrt{-b} \log \left( 2 \sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 2b \cos(dx+c)^2 - b \right)}{2bd}, \frac{\arctan \left( \frac{\sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{b} \sqrt{\cos(dx+c)}} \right)}{\sqrt{bd}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(1/2)/(b\*sec(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/2\*sqrt(-b)\*log(2\*sqrt(-b)\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + 2\*b\*cos(d\*x + c)^2 - b)/(b\*d), arctan(sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/(sqrt(b)\*sqrt(cos(d\*x + c)))/sqrt(b)\*d)]

**Sympy [A]** time = 19.0768, size = 5, normalized size = 0.21

$$\frac{x}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*(1/2)/(b\*sec(d\*x+c))\*\*(1/2),x)

[Out] x/sqrt(b)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{b \sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(1/2)/(b\*sec(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(sec(d\*x + c))/sqrt(b\*sec(d\*x + c)), x)

$$3.165 \quad \int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=32

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{b \sec(c+dx)}}$$

[Out] (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0066139, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {18, 2637}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]),x]

[Out] (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 18

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(a^(m - 1/2)\*b^(n + 1/2)\*Sqrt[a\*v])/Sqrt[b\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{b \sec(c+dx)}} dx &= \frac{\sqrt{\sec(c+dx)} \int \cos(c+dx) dx}{\sqrt{b \sec(c+dx)}} \\ &= \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{d\sqrt{b \sec(c+dx)}} \end{aligned}$$



**Mathematica [A]** time = 0.0361462, size = 32, normalized size = 1.

$$\frac{\sin(c + dx)\sqrt{\sec(c + dx)}}{d\sqrt{b\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]]),x]

[Out] (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[b\*Sec[c + d\*x]])

**Maple [A]** time = 0.154, size = 41, normalized size = 1.3

$$\frac{\sin(dx + c)}{d \cos(dx + c)} \frac{1}{\sqrt{(\cos(dx + c))^{-1}}} \frac{1}{\sqrt{\frac{b}{\cos(dx + c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(d\*x+c)^(1/2)/(b\*sec(d\*x+c))^(1/2),x)

[Out] 1/d\*sin(d\*x+c)/(1/cos(d\*x+c))^(1/2)/(b/cos(d\*x+c))^(1/2)/cos(d\*x+c)

**Maxima [A]** time = 2.09274, size = 18, normalized size = 0.56

$$\frac{\sin(dx + c)}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d\*x+c)^(1/2)/(b\*sec(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] sin(d\*x + c)/(sqrt(b)\*d)

**Fricas [A]** time = 1.7484, size = 81, normalized size = 2.53

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d\*x+c)^(1/2)/(b\*sec(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c)/(b\*d)

**Sympy [A]** time = 25.9602, size = 36, normalized size = 1.12

$$\begin{cases} \frac{\tan(c+dx)}{\sqrt{bd}\sec(c+dx)} & \text{for } d \neq 0 \\ \frac{x}{\sqrt{b}\sec(c)\sqrt{\sec(c)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d\*x+c)\*\*(1/2)/(b\*sec(d\*x+c))\*\*(1/2),x)

[Out] Piecewise((tan(c + d\*x)/(sqrt(b)\*d\*sec(c + d\*x)), Ne(d, 0)), (x/(sqrt(b\*sec(c))\*sqrt(sec(c))), True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b}\sec(dx+c)\sqrt{\sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d\*x+c)^(1/2)/(b\*sec(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b\*sec(d\*x + c))\*sqrt(sec(d\*x + c))), x)

$$3.166 \quad \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=63

$$\frac{x\sqrt{\sec(c+dx)}}{2\sqrt{b \sec(c+dx)}} + \frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\sqrt{b \sec(c+dx)}}$$

[Out] (x\*Sqrt[Sec[c + d\*x]])/(2\*Sqrt[b\*Sec[c + d\*x]]) + Sin[c + d\*x]/(2\*d\*Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0135085, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {18, 2635, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{2\sqrt{b \sec(c+dx)}} + \frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sec[c + d\*x]^(3/2)\*Sqrt[b\*Sec[c + d\*x]]), x]

[Out] (x\*Sqrt[Sec[c + d\*x]])/(2\*Sqrt[b\*Sec[c + d\*x]]) + Sin[c + d\*x]/(2\*d\*Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

### Rule 18

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m - 1/2)\*b^(n + 1/2)\*Sqrt[a\*v])/Sqrt[b\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x])\*(b\*Sin[c + d\*x])^(n - 1)]/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}} dx &= \frac{\sqrt{\sec(c+dx)} \int \cos^2(c+dx) dx}{\sqrt{b\sec(c+dx)}} \\
&= \frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)} \int 1 dx}{2\sqrt{b\sec(c+dx)}} \\
&= \frac{x\sqrt{\sec(c+dx)}}{2\sqrt{b\sec(c+dx)}} + \frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0769065, size = 45, normalized size = 0.71

$$\frac{(2(c+dx) + \sin(2(c+dx)))\sqrt{\sec(c+dx)}}{4d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sec[c + d\*x]^(3/2)\*Sqrt[b\*Sec[c + d\*x]]), x]

[Out] (Sqrt[Sec[c + d\*x]]\*(2\*(c + d\*x) + Sin[2\*(c + d\*x)]))/(4\*d\*Sqrt[b\*Sec[c + d\*x]])

**Maple [A]** time = 0.135, size = 54, normalized size = 0.9

$$\frac{\cos(dx+c)\sin(dx+c)+dx+c}{2d(\cos(dx+c))^2} \left((\cos(dx+c))^{-1}\right)^{\frac{3}{2}} \frac{1}{\sqrt{\frac{b}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(d\*x+c)^(3/2)/(b\*sec(d\*x+c))^(1/2), x)

[Out] 1/2/d\*(cos(d\*x+c)\*sin(d\*x+c)+d\*x+c)/cos(d\*x+c)^2/(1/cos(d\*x+c))^(3/2)/(b/cos(d\*x+c))^(1/2)

**Maxima [A]** time = 2.05379, size = 34, normalized size = 0.54

$$\frac{2 dx + 2 c + \sin(2 dx + 2 c)}{4 \sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d\*x+c)^(3/2)/(b\*sec(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/4\*(2\*d\*x + 2\*c + sin(2\*d\*x + 2\*c))/(sqrt(b)\*d)

**Fricas [A]** time = 2.00488, size = 437, normalized size = 6.94

$$\left[ \frac{2 \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) - \sqrt{-b} \log\left(2 \sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 2b \cos(dx+c)^2 - b\right)}{4bd} \right],$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d\*x+c)^(3/2)/(b\*sec(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/4\*(2\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) - sqrt(-b)\*log(2\*sqrt(-b)\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + 2\*b\*cos(d\*x + c)^2 - b))/(b\*d), 1/2\*(sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + sqrt(b)\*arctan(sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/(sqrt(b)\*sqrt(cos(d\*x + c)))))/(b\*d)]

**Sympy [A]** time = 44.2609, size = 82, normalized size = 1.3

$$\begin{cases} \frac{x \tan^2(c+dx)}{2\sqrt{b} \sec^2(c+dx)} + \frac{x}{2\sqrt{b} \sec^2(c+dx)} + \frac{\tan(c+dx)}{2\sqrt{bd} \sec^2(c+dx)} & \text{for } d \neq 0 \\ \frac{3}{\sqrt{b} \sec(c) \sec^2(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d\*x+c)\*\*(3/2)/(b\*sec(d\*x+c))\*\*(1/2),x)

```
[Out] Piecewise((x*tan(c + d*x)**2/(2*sqrt(b)*sec(c + d*x)**2) + x/(2*sqrt(b)*sec
(c + d*x)**2) + tan(c + d*x)/(2*sqrt(b)*d*sec(c + d*x)**2), Ne(d, 0)), (x/(
sqrt(b*sec(c))*sec(c)**(3/2)), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sec(dx + c)} \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c))*sec(d*x + c)^(3/2)), x)
```

$$3.167 \quad \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=70

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{b \sec(c+dx)}} - \frac{\sin^3(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{b \sec(c+dx)}}$$

[Out] (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[b\*Sec[c + d\*x]]) - (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0157183, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {18, 2633}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{b \sec(c+dx)}} - \frac{\sin^3(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sec[c + d\*x]^(5/2)\*Sqrt[b\*Sec[c + d\*x]]), x]

[Out] (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x])/(d\*Sqrt[b\*Sec[c + d\*x]]) - (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 18

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m - 1/2)\*b^(n + 1/2)\*Sqrt[a\*v])/Sqrt[b\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

### Rule 2633

Int[sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

### Rubi steps

$$\int \frac{1}{\sec^2(c+dx)\sqrt{b\sec(c+dx)}} dx = \frac{\sqrt{\sec(c+dx)} \int \cos^3(c+dx) dx}{\sqrt{b\sec(c+dx)}}$$

$$= -\frac{\sqrt{\sec(c+dx)} \text{Subst}\left(\int (1-x^2) dx, x, -\sin(c+dx)\right)}{d\sqrt{b\sec(c+dx)}}$$

$$= \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{d\sqrt{b\sec(c+dx)}} - \frac{\sqrt{\sec(c+dx)} \sin^3(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

**Mathematica [A]** time = 0.0942708, size = 45, normalized size = 0.64

$$\frac{\sin(c+dx)(\cos(2(c+dx))+5)\sqrt{\sec(c+dx)}}{6d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sec[c + d\*x]^(5/2)\*Sqrt[b\*Sec[c + d\*x]]), x]

[Out] ((5 + Cos[2\*(c + d\*x)])\*Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x])/(6\*d\*Sqrt[b\*Sec[c + d\*x]])

**Maple [A]** time = 0.124, size = 52, normalized size = 0.7

$$\frac{\sin(dx+c)\left((\cos(dx+c))^2+2\right)}{3d(\cos(dx+c))^3} \left((\cos(dx+c))^{-1}\right)^{-\frac{5}{2}} \frac{1}{\sqrt{\frac{b}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(d\*x+c)^(5/2)/(b\*sec(d\*x+c))^(1/2), x)

[Out] 1/3/d\*sin(d\*x+c)\*(cos(d\*x+c)^2+2)/cos(d\*x+c)^3/(1/cos(d\*x+c))^(5/2)/(b/cos(d\*x+c))^(1/2)

**Maxima [A]** time = 2.12168, size = 57, normalized size = 0.81

$$\frac{\sin(3dx+3c)+9\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)}{12\sqrt{bd}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")`

[Out]  $1/12*(\sin(3*d*x + 3*c) + 9*\sin(1/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))))/(\sqrt{b}*d)$

**Fricas [A]** time = 1.6854, size = 132, normalized size = 1.89

$$\frac{(\cos(dx + c)^3 + 2 \cos(dx + c)) \sqrt{\frac{b}{\cos(dx+c)}} \sin(dx + c)}{3bd\sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")`

[Out]  $1/3*(\cos(d*x + c)^3 + 2*\cos(d*x + c))*\sqrt{b/\cos(d*x + c)}*\sin(d*x + c)/(b*d*\sqrt{\cos(d*x + c)})$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(1/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sec(dx + c)} \sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c))*sec(d*x + c)^(5/2)), x)
```

$$3.168 \quad \int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=78

$$\frac{\sin(c+dx) \sec^2(c+dx)}{2bd\sqrt{b \sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b \sec(c+dx)}}$$

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[Sec[c + d\*x]]/(2\*b\*d\*Sqrt[b\*Sec[c + d\*x]])) + (Sec[c + d\*x]^(5/2)\*Sin[c + d\*x])/(2\*b\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0201799, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 3768, 3770}

$$\frac{\sin(c+dx) \sec^2(c+dx)}{2bd\sqrt{b \sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(9/2)/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[Sec[c + d\*x]]/(2\*b\*d\*Sqrt[b\*Sec[c + d\*x]])) + (Sec[c + d\*x]^(5/2)\*Sin[c + d\*x])/(2\*b\*d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x])\*(b\*Csc[c + d\*x])^(n - 1)/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b \sec(c+dx))^{\frac{3}{2}}} dx &= \frac{\sqrt{\sec(c+dx)} \int \sec^3(c+dx) dx}{b\sqrt{b \sec(c+dx)}} \\ &= \frac{\sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2bd\sqrt{b \sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)} \int \sec(c+dx) dx}{2b\sqrt{b \sec(c+dx)}} \\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{2bd\sqrt{b \sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2bd\sqrt{b \sec(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0574739, size = 50, normalized size = 0.64

$$\frac{\sec^{\frac{3}{2}}(c+dx) \left( \tanh^{-1}(\sin(c+dx)) + \tan(c+dx) \sec(c+dx) \right)}{2d(b \sec(c+dx))^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(b*Sec[c + d*x])^(3/2), x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(2
*d*(b*Sec[c + d*x])^(3/2))
```

**Maple [A]** time = 0.117, size = 112, normalized size = 1.4

$$\frac{\cos(dx+c)}{2d} \left( \ln \left( -\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)} \right) (\cos(dx+c))^2 - \ln \left( -\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)} \right) (\cos(dx+c)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(3/2), x)
```

```
[Out] 1/2/d*(ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-ln(-(-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+sin(d*x+c))*(1/cos(d*x+c))^(9/2)
```

)\*cos(d\*x+c)/(b/cos(d\*x+c))^(3/2)

**Maxima [B]** time = 2.14968, size = 905, normalized size = 11.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(9/2)/(b\*sec(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] 
$$\frac{-1/4*(4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) + (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) - 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))}{((b*\cos(4*d*x + 4*c))^2 + 4*b*\cos(2*d*x + 2*c))^2 + b*\sin(4*d*x + 4*c)^2 + 4*b*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*b*\sin(2*d*x + 2*c)^2 + 2*(2*b*\cos(2*d*x + 2*c) + b)*\cos(4*d*x + 4*c) + 4*b*\cos(2*d*x + 2*c) + b)*\sqrt{b}*d}$$

**Fricas [A]** time = 1.95467, size = 539, normalized size = 6.91

$$\frac{\sqrt{b} \cos(dx+c) \log\left(\frac{b \cos(dx+c)^2 - 2\sqrt{b} \sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b}{\cos(dx+c)^2}\right) + \frac{2\sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4b^2d \cos(dx+c)}, \frac{\sqrt{-b} \arctan\left(\frac{\sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}}}{\sqrt{\cos(dx+c)}}\right)}{4b^2d \cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c))^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b^2*d*cos(d*x + c)), -1/2*(sqrt(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)*cos(d*x + c) - sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b^2*d*cos(d*x + c))]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^{\frac{9}{2}}}{(b \sec(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c))^(3/2), x)
```

$$3.169 \quad \int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=35

$$\frac{\sin(c+dx) \sec^2(c+dx)}{bd\sqrt{b \sec(c+dx)}}$$

[Out] (Sec[c + d\*x]^(3/2)\*Sin[c + d\*x])/(b\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0118306, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 3767, 8}

$$\frac{\sin(c+dx) \sec^2(c+dx)}{bd\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(7/2)/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (Sec[c + d\*x]^(3/2)\*Sin[c + d\*x])/(b\*d\*Sqrt[b\*Sec[c + d\*x]])

#### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

#### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

#### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b \sec(c+dx))^{\frac{3}{2}}} dx &= \frac{\sqrt{\sec(c+dx)} \int \sec^2(c+dx) dx}{b\sqrt{b \sec(c+dx)}} \\ &= -\frac{\sqrt{\sec(c+dx)} \text{Subst}\left(\int 1 dx, x, -\tan(c+dx)\right)}{bd\sqrt{b \sec(c+dx)}} \\ &= \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{bd\sqrt{b \sec(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0353204, size = 32, normalized size = 0.91

$$\frac{\sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{d(b \sec(c+dx))^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^(7/2)/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (Sec[c + d\*x]^(5/2)\*Sin[c + d\*x])/(d\*(b\*Sec[c + d\*x])^(3/2))

**Maple [A]** time = 0.101, size = 39, normalized size = 1.1

$$\frac{\cos(dx+c) \sin(dx+c)}{d} \left( (\cos(dx+c))^{-1} \right)^{\frac{7}{2}} \left( \frac{b}{\cos(dx+c)} \right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(7/2)/(b\*sec(d\*x+c))^(3/2), x)

[Out] 1/d\*(1/cos(d\*x+c))^(7/2)\*cos(d\*x+c)\*sin(d\*x+c)/(b/cos(d\*x+c))^(3/2)

**Maxima [B]** time = 2.1738, size = 90, normalized size = 2.57

$$\frac{2\sqrt{b} \sin(2dx+2c)}{(b^2 \cos(2dx+2c)^2 + b^2 \sin(2dx+2c)^2 + 2b^2 \cos(2dx+2c) + b^2)d}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out]  $2\sqrt{b}\sin(2dx + 2c)/((b^2\cos(2dx + 2c)^2 + b^2\sin(2dx + 2c)^2 + 2b^2\cos(2dx + 2c) + b^2)d$

**Fricas [A]** time = 1.67221, size = 84, normalized size = 2.4

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{b^2 d \sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `sqrt(b/cos(d*x + c))*sin(d*x + c)/(b^2*d*sqrt(cos(d*x + c)))`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(7/2)/(b*sec(d*x+c))**(3/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c))^(3/2), x)
```

$$3.170 \quad \int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=36

$$\frac{\sqrt{\sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{bd\sqrt{b \sec(c+dx)}}$$

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[Sec[c + d\*x]])/(b\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0074694, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 3770}

$$\frac{\sqrt{\sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{bd\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(5/2)/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[Sec[c + d\*x]])/(b\*d\*Sqrt[b\*Sec[c + d\*x]])

#### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

#### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\int \frac{\sec^5(c+dx)}{(b \sec(c+dx))^{3/2}} dx = \frac{\sqrt{\sec(c+dx)} \int \sec(c+dx) dx}{b\sqrt{b \sec(c+dx)}} = \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{bd\sqrt{b \sec(c+dx)}}$$

**Mathematica [A]** time = 0.0262157, size = 33, normalized size = 0.92

$$\frac{\sec^3(c+dx) \tanh^{-1}(\sin(c+dx))}{d(b \sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^(5/2)/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (ArcTanh[Sin[c + d\*x]]\*Sec[c + d\*x]^(3/2))/(d\*(b\*Sec[c + d\*x])^(3/2))

**Maple [A]** time = 0.092, size = 52, normalized size = 1.4

$$-2 \frac{\cos(dx+c) \left( (\cos(dx+c))^{-1} \right)^{5/2}}{d} \operatorname{Arctanh} \left( \frac{-1 + \cos(dx+c)}{\sin(dx+c)} \right) \left( \frac{b}{\cos(dx+c)} \right)^{-3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(5/2)/(b\*sec(d\*x+c))^(3/2), x)

[Out] -2/d\*arctanh((-1+cos(d\*x+c))/sin(d\*x+c))\*cos(d\*x+c)\*(1/cos(d\*x+c))^(5/2)/(b/cos(d\*x+c))^(3/2)

**Maxima [B]** time = 2.05257, size = 88, normalized size = 2.44

$$\frac{\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2 \sin(dx+c) + 1)}{2b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(5/2)/(b\*sec(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/2\*(log(cos(d\*x + c)^2 + sin(d\*x + c)^2 + 2\*sin(d\*x + c) + 1) - log(cos(d\*x + c)^2 + sin(d\*x + c)^2 - 2\*sin(d\*x + c) + 1))/(b^(3/2)\*d)

**Fricas [A]** time = 1.88568, size = 301, normalized size = 8.36

$$\left[ \frac{\log\left(\frac{b \cos(dx+c)^2 - 2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) - 2b}{\cos(dx+c)^2}\right)}{2b^{\frac{3}{2}}d}, \frac{\sqrt{-b} \arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b}\right)}{b^2d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(5/2)/(b\*sec(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/2\*log(-(b\*cos(d\*x + c)^2 - 2\*sqrt(b)\*sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c) - 2\*b)/cos(d\*x + c)^2)/(b^(3/2)\*d), -sqrt(-b)\*arctan(sqrt(-b)\*sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c)/b)/(b^2\*d)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*(5/2)/(b\*sec(d\*x+c))\*\*(3/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b \sec(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c))^(3/2), x)
```

$$3.171 \quad \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b \sec(c+dx))^{\frac{3}{2}}} dx$$

**Optimal.** Leaf size=27

$$\frac{x\sqrt{\sec(c+dx)}}{b\sqrt{b \sec(c+dx)}}$$

[Out] (x\*Sqrt[Sec[c + d\*x]])/(b\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0027357, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{b\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(3/2)/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (x\*Sqrt[Sec[c + d\*x]])/(b\*Sqrt[b\*Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned} \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b \sec(c+dx))^{\frac{3}{2}}} dx &= \frac{\sqrt{\sec(c+dx)} \int 1 dx}{b\sqrt{b \sec(c+dx)}} \\ &= \frac{x\sqrt{\sec(c+dx)}}{b\sqrt{b \sec(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.025473, size = 24, normalized size = 0.89

$$\frac{x \sec^2(c + dx)}{(b \sec(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^(3/2)/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (x\*Sec[c + d\*x]^(3/2))/(b\*Sec[c + d\*x])^(3/2)

---

**Maple [A]** time = 0.083, size = 32, normalized size = 1.2

$$\frac{dx + c}{d} \left( (\cos(dx + c))^{-1} \right)^{\frac{3}{2}} \left( \frac{b}{\cos(dx + c)} \right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(3/2)/(b\*sec(d\*x+c))^(3/2), x)

[Out] 1/d\*(d\*x+c)\*(1/cos(d\*x+c))^(3/2)/(b/cos(d\*x+c))^(3/2)

---

**Maxima [A]** time = 1.64326, size = 35, normalized size = 1.3

$$\frac{2 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(3/2)/(b\*sec(d\*x+c))^(3/2), x, algorithm="maxima")

[Out] 2\*arctan(sin(d\*x + c)/(cos(d\*x + c) + 1))/(b^(3/2)\*d)

---



**Fricas [A]** time = 1.93745, size = 277, normalized size = 10.26

$$\left[ \frac{\sqrt{-b} \log \left( 2 \sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 2b \cos(dx+c)^2 - b \right)}{2b^2d}, \frac{\arctan \left( \frac{\sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{b} \sqrt{\cos(dx+c)}} \right)}{b^{\frac{3}{2}}d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(3/2)/(b\*sec(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/2\*sqrt(-b)\*log(2\*sqrt(-b)\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + 2\*b\*cos(d\*x + c)^2 - b)/(b^2\*d), arctan(sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/(sqrt(b)\*sqrt(cos(d\*x + c))))/(b^(3/2)\*d)]

**Sympy [A]** time = 133.827, size = 5, normalized size = 0.19

$$\frac{x}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*(3/2)/(b\*sec(d\*x+c))\*\*(3/2),x)

[Out] x/b\*\*(3/2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b \sec(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(3/2)/(b\*sec(d\*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(sec(d\*x + c)^(3/2)/(b\*sec(d\*x + c))^(3/2), x)

$$3.172 \quad \int \frac{\sqrt{\sec(c+dx)}}{(b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=35

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\sqrt{b \sec(c+dx)}}$$

[Out] (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x])/(b\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.007553, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 2637}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d\*x]]/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x])/(b\*d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\sec(c+dx)}}{(b \sec(c+dx))^{3/2}} dx &= \frac{\sqrt{\sec(c+dx)} \int \cos(c+dx) dx}{b\sqrt{b \sec(c+dx)}} \\ &= \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{bd\sqrt{b \sec(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0410732, size = 32, normalized size = 0.91

$$\frac{\sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{d(b \sec(c + dx))^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[c + d\*x]]/(b\*Sec[c + d\*x])^(3/2), x]

[Out] (Sec[c + d\*x])^(3/2)\*Sin[c + d\*x]/(d\*(b\*Sec[c + d\*x])^(3/2))

**Maple [A]** time = 0.105, size = 41, normalized size = 1.2

$$\frac{\sin(dx + c)}{d \cos(dx + c)} \sqrt{(\cos(dx + c))^{-1} \left( \frac{b}{\cos(dx + c)} \right)^{-\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(1/2)/(b\*sec(d\*x+c))^(3/2), x)

[Out] 1/d\*sin(d\*x+c)\*(1/cos(d\*x+c))^(1/2)/(b/cos(d\*x+c))^(3/2)/cos(d\*x+c)

**Maxima [A]** time = 1.93042, size = 18, normalized size = 0.51

$$\frac{\sin(dx + c)}{b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(1/2)/(b\*sec(d\*x+c))^(3/2), x, algorithm="maxima")

[Out] sin(d\*x + c)/(b^(3/2)\*d)

**Fricas [A]** time = 1.65839, size = 84, normalized size = 2.4

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx + c)} \sin(dx + c)}{b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^2*d)
```

**Sympy [A]** time = 19.5518, size = 36, normalized size = 1.03

$$\begin{cases} \frac{\tan(c+dx)}{3} & \text{for } d \neq 0 \\ \frac{b^2 d \sec(c+dx)}{x \sqrt{\sec(c)}} & \text{otherwise} \\ (b \sec(c))^{\frac{3}{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((tan(c + d*x)/(b**(3/2)*d*sec(c + d*x)), Ne(d, 0)), (x*sqrt(sec(c))/(b*sec(c))**(3/2), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(b \sec(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c))^(3/2), x)
```

$$3.173 \quad \int \frac{1}{\sqrt{\sec(c+dx)}(b \sec(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=69

$$\frac{x\sqrt{\sec(c+dx)}}{2b\sqrt{b \sec(c+dx)}} + \frac{\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}\sqrt{b \sec(c+dx)}}$$

[Out] (x\*Sqrt[Sec[c + d\*x]])/(2\*b\*Sqrt[b\*Sec[c + d\*x]]) + Sin[c + d\*x]/(2\*b\*d\*Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0141337, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {18, 2635, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{2b\sqrt{b \sec(c+dx)}} + \frac{\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[Sec[c + d\*x]]\*(b\*Sec[c + d\*x])^(3/2)),x]

[Out] (x\*Sqrt[Sec[c + d\*x]])/(2\*b\*Sqrt[b\*Sec[c + d\*x]]) + Sin[c + d\*x]/(2\*b\*d\*Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

### Rule 18

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m - 1/2)\*b^(n + 1/2)\*Sqrt[a\*v])/Sqrt[b\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x])\*(b\*Sin[c + d\*x])^(n - 1)/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\sec(c+dx)}(b\sec(c+dx))^{3/2}} dx &= \frac{\sqrt{\sec(c+dx)} \int \cos^2(c+dx) dx}{b\sqrt{b\sec(c+dx)}} \\
&= \frac{\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)} \int 1 dx}{2b\sqrt{b\sec(c+dx)}} \\
&= \frac{x\sqrt{\sec(c+dx)}}{2b\sqrt{b\sec(c+dx)}} + \frac{\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0659757, size = 45, normalized size = 0.65

$$\frac{(2(c+dx) + \sin(2(c+dx))) \sec^{\frac{3}{2}}(c+dx)}{4d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[Sec[c + d\*x]]\*(b\*Sec[c + d\*x])^(3/2)), x]

[Out] (Sec[c + d\*x]^(3/2)\*(2\*(c + d\*x) + Sin[2\*(c + d\*x)]))/(4\*d\*(b\*Sec[c + d\*x])^(3/2))

**Maple [A]** time = 0.123, size = 54, normalized size = 0.8

$$\frac{\cos(dx+c)\sin(dx+c)+dx+c}{2d(\cos(dx+c))^2} \frac{1}{\sqrt{(\cos(dx+c))^{-1}}} \left(\frac{b}{\cos(dx+c)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(d\*x+c)^(1/2)/(b\*sec(d\*x+c))^(3/2), x)

[Out] 1/2/d\*(cos(d\*x+c)\*sin(d\*x+c)+d\*x+c)/cos(d\*x+c)^2/(1/cos(d\*x+c))^(1/2)/(b/cos(d\*x+c))^(3/2)

**Maxima [A]** time = 2.04361, size = 34, normalized size = 0.49

$$\frac{2dx + 2c + \sin(2dx + 2c)}{4b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d\*x+c)^(1/2)/(b\*sec(d\*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/4\*(2\*d\*x + 2\*c + sin(2\*d\*x + 2\*c))/(b^(3/2)\*d)

**Fricas [A]** time = 1.97708, size = 443, normalized size = 6.42

$$\left[ \frac{2\sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) - \sqrt{-b} \log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 2b \cos(dx+c)^2 - b\right)}{4b^2d} \right],$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d\*x+c)^(1/2)/(b\*sec(d\*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/4\*(2\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) - sqrt(-b)\*log(2\*sqrt(-b)\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + 2\*b\*cos(d\*x + c)^2 - b))/(b^2\*d), 1/2\*(sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + sqrt(b)\*arctan(sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/(sqrt(b)\*sqrt(cos(d\*x + c)))))/(b^2\*d)]

**Sympy [A]** time = 49.1731, size = 82, normalized size = 1.19

$$\begin{cases} \frac{x \tan^2(c+dx)}{2b^{\frac{3}{2}} \sec^2(c+dx)} + \frac{x}{2b^{\frac{3}{2}} \sec^2(c+dx)} + \frac{\tan(c+dx)}{2b^{\frac{3}{2}} d \sec^2(c+dx)} & \text{for } d \neq 0 \\ \frac{x}{(b \sec(c))^{\frac{3}{2}} \sqrt{\sec(c)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((x*tan(c + d*x)**2/(2*b**(3/2)*sec(c + d*x)**2) + x/(2*b**(3/2)*s
ec(c + d*x)**2) + tan(c + d*x)/(2*b**(3/2)*d*sec(c + d*x)**2), Ne(d, 0)), (
x/((b*sec(c))**(3/2)*sqrt(sec(c))), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(dx + c))^{\frac{3}{2}} \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c))^(3/2)*sqrt(sec(d*x + c))), x)
```



$$3.174 \quad \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b \sec(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=76

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\sqrt{b \sec(c+dx)}} - \frac{\sin^3(c+dx)\sqrt{\sec(c+dx)}}{3bd\sqrt{b \sec(c+dx)}}$$

[Out] (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x])/(b\*d\*Sqrt[b\*Sec[c + d\*x]]) - (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*b\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0168956, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {18, 2633}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\sqrt{b \sec(c+dx)}} - \frac{\sin^3(c+dx)\sqrt{\sec(c+dx)}}{3bd\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sec[c + d\*x]^(3/2)\*(b\*Sec[c + d\*x])^(3/2)),x]

[Out] (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x])/(b\*d\*Sqrt[b\*Sec[c + d\*x]]) - (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*b\*d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 18

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m - 1/2)\*b^(n + 1/2)\*Sqrt[a\*v])/Sqrt[b\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

### Rule 2633

Int[sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

### Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b \sec(c+dx))^{\frac{3}{2}}} dx = \frac{\sqrt{\sec(c+dx)} \int \cos^3(c+dx) dx}{b\sqrt{b \sec(c+dx)}}$$

$$= -\frac{\sqrt{\sec(c+dx)} \operatorname{Subst}\left(\int (1-x^2) dx, x, -\sin(c+dx)\right)}{bd\sqrt{b \sec(c+dx)}}$$

$$= \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{bd\sqrt{b \sec(c+dx)}} - \frac{\sqrt{\sec(c+dx)} \sin^3(c+dx)}{3bd\sqrt{b \sec(c+dx)}}$$

**Mathematica [A]** time = 0.0848518, size = 45, normalized size = 0.59

$$\frac{\sin(c+dx)(\cos(2(c+dx))+5)\sec^{\frac{3}{2}}(c+dx)}{6d(b \sec(c+dx))^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sec[c + d\*x]^(3/2)\*(b\*Sec[c + d\*x])^(3/2)), x]

[Out] ((5 + Cos[2\*(c + d\*x)])\*Sec[c + d\*x]^(3/2)\*Sin[c + d\*x])/(6\*d\*(b\*Sec[c + d\*x])^(3/2))

**Maple [A]** time = 0.107, size = 52, normalized size = 0.7

$$\frac{\sin(dx+c)\left((\cos(dx+c))^2+2\right)}{3d(\cos(dx+c))^3} \left((\cos(dx+c))^{-1}\right)^{-\frac{3}{2}} \left(\frac{b}{\cos(dx+c)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(d\*x+c)^(3/2)/(b\*sec(d\*x+c))^(3/2), x)

[Out] 1/3/d\*sin(d\*x+c)\*(cos(d\*x+c)^2+2)/cos(d\*x+c)^3/(1/cos(d\*x+c))^(3/2)/(b/cos(d\*x+c))^(3/2)

**Maxima [A]** time = 2.04891, size = 57, normalized size = 0.75

$$\frac{\sin(3dx+3c) + 9 \sin\left(\frac{1}{3} \arctan(\sin(3dx+3c), \cos(3dx+3c))\right)}{12b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out]  $1/12*(\sin(3*d*x + 3*c) + 9*\sin(1/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))))/(b^(3/2)*d)$

**Fricas [A]** time = 1.71806, size = 135, normalized size = 1.78

$$\frac{(\cos(dx + c)^3 + 2 \cos(dx + c)) \sqrt{\frac{b}{\cos(dx+c)}} \sin(dx + c)}{3 b^2 d \sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

[Out]  $1/3*(\cos(d*x + c)^3 + 2*\cos(d*x + c))*\sqrt{b/\cos(d*x + c)}*\sin(d*x + c)/(b^2*d*\sqrt{\cos(d*x + c)})$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(3/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(dx + c))^{\frac{3}{2}} \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c))^(3/2)*sec(d*x + c)^(3/2)), x)
```

$$3.175 \quad \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(b \sec(c+dx))^{3/2}} dx$$

**Optimal.** Leaf size=107

$$\frac{3x\sqrt{\sec(c+dx)}}{8b\sqrt{b \sec(c+dx)}} + \frac{\sin(c+dx)}{4bd \sec^{\frac{5}{2}}(c+dx)\sqrt{b \sec(c+dx)}} + \frac{3 \sin(c+dx)}{8bd\sqrt{\sec(c+dx)}\sqrt{b \sec(c+dx)}}$$

[Out] (3\*x\*Sqrt[Sec[c + d\*x]])/(8\*b\*Sqrt[b\*Sec[c + d\*x]]) + Sin[c + d\*x]/(4\*b\*d\*S  
ec[c + d\*x]^(5/2)\*Sqrt[b\*Sec[c + d\*x]]) + (3\*Sin[c + d\*x])/(8\*b\*d\*Sqrt[Sec[  
c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.027397, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {18, 2635, 8}

$$\frac{3x\sqrt{\sec(c+dx)}}{8b\sqrt{b \sec(c+dx)}} + \frac{\sin(c+dx)}{4bd \sec^{\frac{5}{2}}(c+dx)\sqrt{b \sec(c+dx)}} + \frac{3 \sin(c+dx)}{8bd\sqrt{\sec(c+dx)}\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sec[c + d\*x]^(5/2)\*(b\*Sec[c + d\*x])^(3/2)),x]

[Out] (3\*x\*Sqrt[Sec[c + d\*x]])/(8\*b\*Sqrt[b\*Sec[c + d\*x]]) + Sin[c + d\*x]/(4\*b\*d\*S  
ec[c + d\*x]^(5/2)\*Sqrt[b\*Sec[c + d\*x]]) + (3\*Sin[c + d\*x])/(8\*b\*d\*Sqrt[Sec[  
c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

### Rule 18

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(a^(m - 1/  
2)\*b^(n + 1/2)\*Sqrt[a\*v])/Sqrt[b\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b  
, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x  
]\* (b\*SIN[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*SIN[c  
+ d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n  
]

**Rule 8**

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

**Rubi steps**

$$\begin{aligned}
 \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(b \sec(c+dx))^{3/2}} dx &= \frac{\sqrt{\sec(c+dx)} \int \cos^4(c+dx) dx}{b\sqrt{b \sec(c+dx)}} \\
 &= \frac{\sin(c+dx)}{4bd \sec^{\frac{5}{2}}(c+dx)\sqrt{b \sec(c+dx)}} + \frac{(3\sqrt{\sec(c+dx)}) \int \cos^2(c+dx) dx}{4b\sqrt{b \sec(c+dx)}} \\
 &= \frac{\sin(c+dx)}{4bd \sec^{\frac{5}{2}}(c+dx)\sqrt{b \sec(c+dx)}} + \frac{3 \sin(c+dx)}{8bd\sqrt{\sec(c+dx)}\sqrt{b \sec(c+dx)}} + \frac{(3\sqrt{\sec(c+dx)}) \int \cos^2(c+dx) dx}{8b\sqrt{b \sec(c+dx)}} \\
 &= \frac{3x\sqrt{\sec(c+dx)}}{8b\sqrt{b \sec(c+dx)}} + \frac{\sin(c+dx)}{4bd \sec^{\frac{5}{2}}(c+dx)\sqrt{b \sec(c+dx)}} + \frac{3 \sin(c+dx)}{8bd\sqrt{\sec(c+dx)}\sqrt{b \sec(c+dx)}}
 \end{aligned}$$

**Mathematica [A]** time = 0.121046, size = 55, normalized size = 0.51

$$\frac{(12(c+dx) + 8 \sin(2(c+dx)) + \sin(4(c+dx))) \sec^{\frac{3}{2}}(c+dx)}{32d(b \sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^(3/2)),x]`

`[Out] (Sec[c + d*x]^(3/2)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])) / (32*d*(b*Sec[c + d*x])^(3/2))`

**Maple [A]** time = 0.139, size = 74, normalized size = 0.7

$$\frac{2 (\cos(dx+c))^3 \sin(dx+c) + 3 \cos(dx+c) \sin(dx+c) + 3 dx + 3c}{8d (\cos(dx+c))^4} \left( (\cos(dx+c))^{-1} \right)^{\frac{5}{2}} \left( \frac{b}{\cos(dx+c)} \right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x)`

[Out]  $1/8/d*(2*\cos(d*x+c)^3*\sin(d*x+c)+3*\cos(d*x+c)*\sin(d*x+c)+3*d*x+3*c)/(1/\cos(d*x+c))^{(5/2)}/(b/\cos(d*x+c))^{(3/2)}/\cos(d*x+c)^4$

**Maxima [A]** time = 1.87057, size = 66, normalized size = 0.62

$$\frac{12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin\left(\frac{1}{2} \arctan(\sin(4 dx + 4 c), \cos(4 dx + 4 c))\right)}{32 b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")`

[Out]  $1/32*(12*d*x + 12*c + \sin(4*d*x + 4*c) + 8*\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))))/(b^(3/2)*d)$

**Fricas [A]** time = 2.15391, size = 552, normalized size = 5.16

$$\frac{2 \left( 2 \cos(dx+c)^4 + 3 \cos(dx+c)^2 \right) \sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c) - 3 \sqrt{-b} \log \left( 2 \sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 2 b \cos(dx+c)^2 \right)}{16 b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")`

[Out]  $[1/16*(2*(2*\cos(d*x + c)^4 + 3*\cos(d*x + c)^2)*\sqrt{b/\cos(d*x + c)}*\sin(d*x + c)/\sqrt{\cos(d*x + c)} - 3*\sqrt{-b}*\log(2*\sqrt{-b}*\sqrt{b/\cos(d*x + c)}*\cos(d*x + c)^{(3/2)}*\sin(d*x + c) + 2*b*\cos(d*x + c)^2 - b))/(b^2*d), 1/8*((2*\cos(d*x + c)^4 + 3*\cos(d*x + c)^2)*\sqrt{b/\cos(d*x + c)}*\sin(d*x + c)/\sqrt{\cos(d*x + c)} + 3*\sqrt{b}*\arctan(\sqrt{b/\cos(d*x + c)}*\sin(d*x + c)/(\sqrt{b}*\sqrt{\cos(d*x + c)})))]/(b^2*d)]$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d\*x+c)\*\*(5/2)/(b\*sec(d\*x+c))\*\*(3/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(dx + c))^{\frac{3}{2}} \sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(d\*x+c)^(5/2)/(b\*sec(d\*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(1/((b\*sec(d\*x + c))^(3/2)\*sec(d\*x + c)^(5/2)), x)



$$3.176 \quad \int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b \sec(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=78

$$\frac{\sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{2b^2 d \sqrt{b \sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{2b^2 d \sqrt{b \sec(c+dx)}}$$

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[Sec[c + d\*x]]/(2\*b^2\*d\*Sqrt[b\*Sec[c + d\*x]]) + (Sec[c + d\*x]^(5/2)\*Sin[c + d\*x])/(2\*b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0202177, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 3768, 3770}

$$\frac{\sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{2b^2 d \sqrt{b \sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{2b^2 d \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(11/2)/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[Sec[c + d\*x]]/(2\*b^2\*d\*Sqrt[b\*Sec[c + d\*x]]) + (Sec[c + d\*x]^(5/2)\*Sin[c + d\*x])/(2\*b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x])\*(b\*Csc[c + d\*x])^(n - 1)/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b \sec(c+dx))^{5/2}} dx &= \frac{\sqrt{\sec(c+dx)} \int \sec^3(c+dx) dx}{b^2 \sqrt{b \sec(c+dx)}} \\ &= \frac{\sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2b^2 d \sqrt{b \sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)} \int \sec(c+dx) dx}{2b^2 \sqrt{b \sec(c+dx)}} \\ &= \frac{\tanh^{-1}(\sin(c+dx)) \sqrt{\sec(c+dx)}}{2b^2 d \sqrt{b \sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2b^2 d \sqrt{b \sec(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0506845, size = 53, normalized size = 0.68

$$\frac{\sqrt{\sec(c+dx)} \left( \tanh^{-1}(\sin(c+dx)) + \tan(c+dx) \sec(c+dx) \right)}{2b^2 d \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(11/2)/(b*Sec[c + d*x])^(5/2), x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(2
*b^2*d*Sqrt[b*Sec[c + d*x]])
```

**Maple [A]** time = 0.117, size = 112, normalized size = 1.4

$$\frac{\cos(dx+c)}{2d} \left( \ln \left( -\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)} \right) (\cos(dx+c))^2 - \ln \left( -\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)} \right) (\cos(dx+c)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(11/2)/(b*sec(d*x+c))^(5/2), x)
```

```
[Out] 1/2/d*(ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-ln(-(-1+cos(
d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+sin(d*x+c))*(1/cos(d*x+c))^(11/
```

$$2) \cdot \cos(dx+c) / (b/\cos(dx+c))^{5/2}$$

**Maxima [B]** time = 2.12037, size = 929, normalized size = 11.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(dx+c)^(11/2)/(b\*sec(dx+c))^(5/2),x, algorithm="maxima")

[Out] 
$$-1/4 \cdot (4 \cdot (\sin(4dx + 4c) + 2 \cdot \sin(2dx + 2c)) \cdot \cos(3/2 \cdot \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - 4 \cdot (\sin(4dx + 4c) + 2 \cdot \sin(2dx + 2c)) \cdot \cos(1/2 \cdot \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - (2 \cdot (2 \cdot \cos(2dx + 2c) + 1) \cdot \cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4 \cdot \cos(2dx + 2c)^2 + \sin(4dx + 4c)^2 + 4 \cdot \sin(4dx + 4c) \cdot \sin(2dx + 2c) + 4 \cdot \sin(2dx + 2c)^2 + 4 \cdot \cos(2dx + 2c) + 1) \cdot \log(\cos(1/2 \cdot \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))))^2 + \sin(1/2 \cdot \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + 2 \cdot \sin(1/2 \cdot \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 1) + (2 \cdot (2 \cdot \cos(2dx + 2c) + 1) \cdot \cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4 \cdot \cos(2dx + 2c)^2 + \sin(4dx + 4c)^2 + 4 \cdot \sin(4dx + 4c) \cdot \sin(2dx + 2c) + 4 \cdot \sin(2dx + 2c)^2 + 4 \cdot \cos(2dx + 2c) + 1) \cdot \log(\cos(1/2 \cdot \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))))^2 + \sin(1/2 \cdot \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 - 2 \cdot \sin(1/2 \cdot \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 1) - 4 \cdot (\cos(4dx + 4c) + 2 \cdot \cos(2dx + 2c) + 1) \cdot \sin(3/2 \cdot \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 4 \cdot (\cos(4dx + 4c) + 2 \cdot \cos(2dx + 2c) + 1) \cdot \sin(1/2 \cdot \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) / ((b^2 \cdot \cos(4dx + 4c)^2 + 4 \cdot b^2 \cdot \cos(2dx + 2c)^2 + b^2 \cdot \sin(4dx + 4c)^2 + 4 \cdot b^2 \cdot \sin(4dx + 4c) \cdot \sin(2dx + 2c) + 4 \cdot b^2 \cdot \sin(2dx + 2c)^2 + 4 \cdot b^2 \cdot \cos(2dx + 2c) + b^2 + 2 \cdot (2 \cdot b^2 \cdot \cos(2dx + 2c) + b^2) \cdot \cos(4dx + 4c)) \cdot \sqrt{b} \cdot d$$

**Fricas [A]** time = 1.95703, size = 539, normalized size = 6.91

$$\left[ \frac{\sqrt{b} \cos(dx+c) \log\left( \frac{b \cos(dx+c)^2 - 2\sqrt{b} \sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b}{\cos(dx+c)^2} \right) + \frac{2 \sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4b^3 d \cos(dx+c)}, -\sqrt{-b} \arctan\left( \frac{\sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}}}{\sqrt{\cos(dx+c)}} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(11/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c))^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b^3*d*cos(d*x + c)), -1/2*(sqrt(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)*cos(d*x + c) - sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b^3*d*cos(d*x + c))]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(11/2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^{\frac{11}{2}}}{(b \sec(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(11/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(11/2)/(b*sec(d*x + c))^(5/2), x)
```

$$3.177 \quad \int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=35

$$\frac{\sin(c+dx) \sec^2(c+dx)}{b^2 d \sqrt{b \sec(c+dx)}}$$

[Out] (Sec[c + d\*x]^(3/2)\*Sin[c + d\*x])/(b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0115766, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 3767, 8}

$$\frac{\sin(c+dx) \sec^2(c+dx)}{b^2 d \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(9/2)/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (Sec[c + d\*x]^(3/2)\*Sin[c + d\*x])/(b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned} \int \frac{\sec^9(c+dx)}{(b \sec(c+dx))^{5/2}} dx &= \frac{\sqrt{\sec(c+dx)} \int \sec^2(c+dx) dx}{b^2 \sqrt{b \sec(c+dx)}} \\ &= -\frac{\sqrt{\sec(c+dx)} \text{Subst}\left(\int 1 dx, x, -\tan(c+dx)\right)}{b^2 d \sqrt{b \sec(c+dx)}} \\ &= \frac{\sec^3(c+dx) \sin(c+dx)}{b^2 d \sqrt{b \sec(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0327365, size = 32, normalized size = 0.91

$$\frac{\sin(c+dx) \sec^7(c+dx)}{d(b \sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^(9/2)/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (Sec[c + d\*x]^(7/2)\*Sin[c + d\*x])/(d\*(b\*Sec[c + d\*x])^(5/2))

**Maple [A]** time = 0.106, size = 39, normalized size = 1.1

$$\frac{\cos(dx+c) \sin(dx+c)}{d} \left( (\cos(dx+c))^{-1} \right)^{\frac{9}{2}} \left( \frac{b}{\cos(dx+c)} \right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(9/2)/(b\*sec(d\*x+c))^(5/2), x)

[Out] 1/d\*(1/cos(d\*x+c))^(9/2)\*cos(d\*x+c)\*sin(d\*x+c)/(b/cos(d\*x+c))^(5/2)

**Maxima [B]** time = 1.85632, size = 90, normalized size = 2.57

$$\frac{2\sqrt{b} \sin(2dx+2c)}{(b^3 \cos(2dx+2c)^2 + b^3 \sin(2dx+2c)^2 + 2b^3 \cos(2dx+2c) + b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out]  $2\sqrt{b}\sin(2dx + 2c)/((b^3\cos(2dx + 2c))^2 + b^3\sin(2dx + 2c))^2 + 2b^3\cos(2dx + 2c) + b^3)d$

**Fricas [A]** time = 1.63829, size = 84, normalized size = 2.4

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{b^3 d \sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out]  $\sqrt{b/\cos(dx+c)}\sin(dx+c)/(b^3d\sqrt{\cos(dx+c)})$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(9/2)/(b*sec(d*x+c))**(5/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{9}{2}}}{(b\sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c))^(5/2), x)
```



$$3.178 \quad \int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=36

$$\frac{\sqrt{\sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{b^2 d \sqrt{b \sec(c+dx)}}$$

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[Sec[c + d\*x]])/(b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0077274, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 3770}

$$\frac{\sqrt{\sec(c+dx)} \tanh^{-1}(\sin(c+dx))}{b^2 d \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(7/2)/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (ArcTanh[Sin[c + d\*x]]\*Sqrt[Sec[c + d\*x]])/(b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b \sec(c+dx))^{\frac{5}{2}}} dx = \frac{\sqrt{\sec(c+dx)} \int \sec(c+dx) dx}{b^2 \sqrt{b \sec(c+dx)}} \\ = \frac{\tanh^{-1}(\sin(c+dx)) \sqrt{\sec(c+dx)}}{b^2 d \sqrt{b \sec(c+dx)}}$$

**Mathematica [A]** time = 0.0219341, size = 33, normalized size = 0.92

$$\frac{\sec^{\frac{5}{2}}(c+dx) \tanh^{-1}(\sin(c+dx))}{d(b \sec(c+dx))^{\frac{5}{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^(7/2)/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (ArcTanh[Sin[c + d\*x]]\*Sec[c + d\*x]^(5/2))/(d\*(b\*Sec[c + d\*x])^(5/2))

**Maple [A]** time = 0.088, size = 52, normalized size = 1.4

$$-2 \frac{((\cos(dx+c))^{-1})^{7/2} \cos(dx+c)}{d} \operatorname{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right) \left(\frac{b}{\cos(dx+c)}\right)^{-5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(7/2)/(b\*sec(d\*x+c))^(5/2), x)

[Out] -2/d\*(1/cos(d\*x+c))^(7/2)\*arctanh((-1+cos(d\*x+c))/sin(d\*x+c))\*cos(d\*x+c)/(b/cos(d\*x+c))^(5/2)

**Maxima [B]** time = 1.9491, size = 88, normalized size = 2.44

$$\frac{\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2 \sin(dx+c) + 1)}{2 b^{\frac{5}{2}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(7/2)/(b\*sec(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/2\*(log(cos(d\*x + c)^2 + sin(d\*x + c)^2 + 2\*sin(d\*x + c) + 1) - log(cos(d\*x + c)^2 + sin(d\*x + c)^2 - 2\*sin(d\*x + c) + 1))/(b^(5/2)\*d)

**Fricas [A]** time = 1.90422, size = 301, normalized size = 8.36

$$\left[ \frac{\log\left(\frac{b \cos(dx+c)^2 - 2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) - 2b}{\cos(dx+c)^2}\right)}{2b^{\frac{5}{2}}d}, \frac{\sqrt{-b} \arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b}\right)}{b^3d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(7/2)/(b\*sec(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] [1/2\*log(-(b\*cos(d\*x + c)^2 - 2\*sqrt(b)\*sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c) - 2\*b)/cos(d\*x + c)^2)/(b^(5/2)\*d), -sqrt(-b)\*arctan(sqrt(-b)\*sqrt(b/cos(d\*x + c))\*sqrt(cos(d\*x + c))\*sin(d\*x + c)/b)/(b^3\*d)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*(7/2)/(b\*sec(d\*x+c))\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c))^(5/2), x)
```

$$3.179 \quad \int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=27

$$\frac{x\sqrt{\sec(c+dx)}}{b^2\sqrt{b \sec(c+dx)}}$$

[Out] (x\*Sqrt[Sec[c + d\*x]])/(b^2\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0026661, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{b^2\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(5/2)/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (x\*Sqrt[Sec[c + d\*x]])/(b^2\*Sqrt[b\*Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{5/2}} dx &= \frac{\sqrt{\sec(c+dx)} \int 1 dx}{b^2 \sqrt{b \sec(c+dx)}} \\ &= \frac{x\sqrt{\sec(c+dx)}}{b^2 \sqrt{b \sec(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0261257, size = 24, normalized size = 0.89

$$\frac{x \sec^2(c + dx)}{(b \sec(c + dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^(5/2)/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (x\*Sec[c + d\*x]^(5/2))/(b\*Sec[c + d\*x])^(5/2)

**Maple [A]** time = 0.077, size = 32, normalized size = 1.2

$$\frac{dx + c}{d} \left( (\cos(dx + c))^{-1} \right)^{\frac{5}{2}} \left( \frac{b}{\cos(dx + c)} \right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(5/2)/(b\*sec(d\*x+c))^(5/2), x)

[Out] 1/d\*(d\*x+c)\*(1/cos(d\*x+c))^(5/2)/(b/cos(d\*x+c))^(5/2)

**Maxima [A]** time = 1.77235, size = 35, normalized size = 1.3

$$\frac{2 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(5/2)/(b\*sec(d\*x+c))^(5/2), x, algorithm="maxima")

[Out] 2\*arctan(sin(d\*x + c)/(cos(d\*x + c) + 1))/(b^(5/2)\*d)

**Fricas [A]** time = 1.95575, size = 277, normalized size = 10.26

$$\left[ \frac{\sqrt{-b} \log \left( 2 \sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 2b \cos(dx+c)^2 - b \right)}{2b^3d}, \frac{\arctan \left( \frac{\sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c)}{\sqrt{b} \sqrt{\cos(dx+c)}} \right)}{b^{\frac{5}{2}}d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(5/2)/(b\*sec(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/2\*sqrt(-b)\*log(2\*sqrt(-b)\*sqrt(b/cos(d\*x + c))\*cos(d\*x + c)^(3/2)\*sin(d\*x + c) + 2\*b\*cos(d\*x + c)^2 - b)/(b^3\*d), arctan(sqrt(b/cos(d\*x + c))\*sin(d\*x + c)/(sqrt(b)\*sqrt(cos(d\*x + c))))/(b^(5/2)\*d)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*(5/2)/(b\*sec(d\*x+c))\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(5/2)/(b\*sec(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(sec(d\*x + c)^(5/2)/(b\*sec(d\*x + c))^(5/2), x)

$$3.180 \quad \int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=35

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2 d \sqrt{b \sec(c+dx)}}$$

[Out] (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x])/(b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0069425, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {17, 2637}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2 d \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(3/2)/(b\*Sec[c + d\*x])^(5/2),x]

[Out] (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x])/(b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps



$$\int \frac{\sec^3(c+dx)}{(b \sec(c+dx))^{5/2}} dx = \frac{\sqrt{\sec(c+dx)} \int \cos(c+dx) dx}{b^2 \sqrt{b \sec(c+dx)}} \\ = \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{b^2 d \sqrt{b \sec(c+dx)}}$$

**Mathematica [A]** time = 0.0252233, size = 35, normalized size = 1.

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2 d \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^(3/2)/(b\*Sec[c + d\*x])^(5/2),x]

[Out] (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x])/(b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

**Maple [A]** time = 0.094, size = 41, normalized size = 1.2

$$\frac{\sin(dx+c)}{d \cos(dx+c)} \left( (\cos(dx+c))^{-1} \right)^{\frac{3}{2}} \left( \frac{b}{\cos(dx+c)} \right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(3/2)/(b\*sec(d\*x+c))^(5/2),x)

[Out] 1/d\*sin(d\*x+c)\*(1/cos(d\*x+c))^(3/2)/(b/cos(d\*x+c))^(5/2)/cos(d\*x+c)

**Maxima [A]** time = 1.94217, size = 18, normalized size = 0.51

$$\frac{\sin(dx+c)}{b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^(3/2)/(b\*sec(d\*x+c))^(5/2),x, algorithm="maxima")

[Out]  $\sin(dx + c)/(b^{5/2}d)$

---

**Fricas [A]** time = 1.64625, size = 84, normalized size = 2.4

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}} \sqrt{\cos(dx+c)} \sin(dx+c)}{b^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] `sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^3*d)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(5/2),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")`

[Out] `integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c))^(5/2), x)`

$$3.181 \quad \int \frac{\sqrt{\sec(c+dx)}}{(b \sec(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=69

$$\frac{x\sqrt{\sec(c+dx)}}{2b^2\sqrt{b \sec(c+dx)}} + \frac{\sin(c+dx)}{2b^2d\sqrt{\sec(c+dx)}\sqrt{b \sec(c+dx)}}$$

[Out] (x\*Sqrt[Sec[c + d\*x]])/(2\*b^2\*Sqrt[b\*Sec[c + d\*x]]) + Sin[c + d\*x]/(2\*b^2\*d\*Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0146361, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {17, 2635, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{2b^2\sqrt{b \sec(c+dx)}} + \frac{\sin(c+dx)}{2b^2d\sqrt{\sec(c+dx)}\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d\*x]]/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (x\*Sqrt[Sec[c + d\*x]])/(2\*b^2\*Sqrt[b\*Sec[c + d\*x]]) + Sin[c + d\*x]/(2\*b^2\*d\*Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

### Rule 17

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m + 1/2)\*b^(n - 1/2)\*Sqrt[b\*v])/Sqrt[a\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x])\*(b\*Sin[c + d\*x])^(n - 1)/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}}{(b \sec(c+dx))^{5/2}} dx &= \frac{\sqrt{\sec(c+dx)} \int \cos^2(c+dx) dx}{b^2 \sqrt{b \sec(c+dx)}} \\
&= \frac{\sin(c+dx)}{2b^2 d \sqrt{\sec(c+dx)} \sqrt{b \sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)} \int 1 dx}{2b^2 \sqrt{b \sec(c+dx)}} \\
&= \frac{x \sqrt{\sec(c+dx)}}{2b^2 \sqrt{b \sec(c+dx)}} + \frac{\sin(c+dx)}{2b^2 d \sqrt{\sec(c+dx)} \sqrt{b \sec(c+dx)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0516438, size = 48, normalized size = 0.7

$$\frac{(2(c+dx) + \sin(2(c+dx)))\sqrt{\sec(c+dx)}}{4b^2 d \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[c + d\*x]]/(b\*Sec[c + d\*x])^(5/2), x]

[Out] (Sqrt[Sec[c + d\*x]]\*(2\*(c + d\*x) + Sin[2\*(c + d\*x)]))/(4\*b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

**Maple [A]** time = 0.127, size = 54, normalized size = 0.8

$$\frac{\cos(dx+c)\sin(dx+c)+dx+c}{2d(\cos(dx+c))^2} \sqrt{(\cos(dx+c))^{-1} \left(\frac{b}{\cos(dx+c)}\right)^{-\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^(1/2)/(b\*sec(d\*x+c))^(5/2), x)

[Out] 1/2/d\*(cos(d\*x+c)\*sin(d\*x+c)+d\*x+c)\*(1/cos(d\*x+c))^(1/2)/cos(d\*x+c)^2/(b/cos(d\*x+c))^(5/2)

**Maxima [A]** time = 2.1739, size = 34, normalized size = 0.49

$$\frac{2dx + 2c + \sin(2dx + 2c)}{4b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out]  $1/4*(2*d*x + 2*c + \sin(2*d*x + 2*c))/(b^{(5/2)*d})$

**Fricas [A]** time = 1.98352, size = 443, normalized size = 6.42

$$\left[ \frac{2 \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) - \sqrt{-b} \log\left(2 \sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 2b \cos(dx+c)^2 - b\right)}{4b^3d}, \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out]  $[1/4*(2*\sqrt{b/\cos(d*x + c)}*\cos(d*x + c)^{(3/2)}*\sin(d*x + c) - \sqrt{-b}*\log(2*\sqrt{-b}*\sqrt{b/\cos(d*x + c)}*\cos(d*x + c)^{(3/2)}*\sin(d*x + c) + 2*b*\cos(d*x + c)^2 - b))/(b^3*d), 1/2*(\sqrt{b/\cos(d*x + c)}*\cos(d*x + c)^{(3/2)}*\sin(d*x + c) + \sqrt{b}*\arctan(\sqrt{b/\cos(d*x + c)}*\sin(d*x + c)/(\sqrt{b}*\sqrt{\cos(d*x + c)})))/(b^3*d)]$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(5/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c))^(5/2), x)
```

$$3.182 \quad \int \frac{1}{\sqrt{\sec(c+dx)}(b \sec(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=76

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2 d \sqrt{b \sec(c+dx)}} - \frac{\sin^3(c+dx)\sqrt{\sec(c+dx)}}{3b^2 d \sqrt{b \sec(c+dx)}}$$

[Out] (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x])/(b^2\*d\*Sqrt[b\*Sec[c + d\*x]]) - (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0167266, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {18, 2633}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2 d \sqrt{b \sec(c+dx)}} - \frac{\sin^3(c+dx)\sqrt{\sec(c+dx)}}{3b^2 d \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[Sec[c + d\*x]]\*(b\*Sec[c + d\*x])^(5/2)),x]

[Out] (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x])/(b^2\*d\*Sqrt[b\*Sec[c + d\*x]]) - (Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x]^3)/(3\*b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

### Rule 18

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(a^(m - 1/2)\*b^(n + 1/2)\*Sqrt[a\*v])/Sqrt[b\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

### Rule 2633

Int[sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

### Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}(b \sec(c+dx))^{5/2}} dx = \frac{\sqrt{\sec(c+dx)} \int \cos^3(c+dx) dx}{b^2 \sqrt{b \sec(c+dx)}} \\ = -\frac{\sqrt{\sec(c+dx)} \operatorname{Subst}\left(\int (1-x^2) dx, x, -\sin(c+dx)\right)}{b^2 d \sqrt{b \sec(c+dx)}} \\ = \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{b^2 d \sqrt{b \sec(c+dx)}} - \frac{\sqrt{\sec(c+dx)} \sin^3(c+dx)}{3b^2 d \sqrt{b \sec(c+dx)}}$$

**Mathematica [A]** time = 0.0724163, size = 48, normalized size = 0.63

$$\frac{\sin(c+dx)(\cos(2(c+dx))+5)\sqrt{\sec(c+dx)}}{6b^2 d \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[Sec[c + d\*x]]\*(b\*Sec[c + d\*x])^(5/2)), x]

[Out] ((5 + Cos[2\*(c + d\*x)])\*Sqrt[Sec[c + d\*x]]\*Sin[c + d\*x])/(6\*b^2\*d\*Sqrt[b\*Sec[c + d\*x]])

**Maple [A]** time = 0.134, size = 52, normalized size = 0.7

$$\frac{\sin(dx+c)((\cos(dx+c))^2+2)}{3d(\cos(dx+c))^3} \frac{1}{\sqrt{(\cos(dx+c))^{-1}}} \left(\frac{b}{\cos(dx+c)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(d\*x+c)^(1/2)/(b\*sec(d\*x+c))^(5/2), x)

[Out] 1/3/d\*sin(d\*x+c)\*(cos(d\*x+c)^2+2)/cos(d\*x+c)^3/(1/cos(d\*x+c))^(1/2)/(b/cos(d\*x+c))^(5/2)

**Maxima [A]** time = 2.16079, size = 57, normalized size = 0.75

$$\frac{\sin(3dx+3c)+9\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)}{12b^{\frac{5}{2}}d}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out]  $1/12*(\sin(3*d*x + 3*c) + 9*\sin(1/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))))/(b^(5/2)*d)$

**Fricas [A]** time = 1.69798, size = 135, normalized size = 1.78

$$\frac{(\cos(dx + c)^3 + 2 \cos(dx + c)) \sqrt{\frac{b}{\cos(dx+c)}} \sin(dx + c)}{3 b^3 d \sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out]  $1/3*(\cos(d*x + c)^3 + 2*\cos(d*x + c))*\sqrt{b/\cos(d*x + c)}*\sin(d*x + c)/(b^3*d*\sqrt{\cos(d*x + c)})$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(5/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(dx + c))^{\frac{5}{2}} \sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c))^(5/2)*sqrt(sec(d*x + c))), x)
```

$$3.183 \quad \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b \sec(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=107

$$\frac{3x\sqrt{\sec(c+dx)}}{8b^2\sqrt{b \sec(c+dx)}} + \frac{\sin(c+dx)}{4b^2d \sec^{\frac{5}{2}}(c+dx)\sqrt{b \sec(c+dx)}} + \frac{3 \sin(c+dx)}{8b^2d\sqrt{\sec(c+dx)}\sqrt{b \sec(c+dx)}}$$

[Out] (3\*x\*Sqrt[Sec[c + d\*x]])/(8\*b^2\*Sqrt[b\*Sec[c + d\*x]]) + Sin[c + d\*x]/(4\*b^2\*d\*Sec[c + d\*x]^(5/2)\*Sqrt[b\*Sec[c + d\*x]]) + (3\*Sin[c + d\*x])/(8\*b^2\*d\*Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

**Rubi [A]** time = 0.0284181, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {18, 2635, 8}

$$\frac{3x\sqrt{\sec(c+dx)}}{8b^2\sqrt{b \sec(c+dx)}} + \frac{\sin(c+dx)}{4b^2d \sec^{\frac{5}{2}}(c+dx)\sqrt{b \sec(c+dx)}} + \frac{3 \sin(c+dx)}{8b^2d\sqrt{\sec(c+dx)}\sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sec[c + d\*x]^(3/2)\*(b\*Sec[c + d\*x])^(5/2)),x]

[Out] (3\*x\*Sqrt[Sec[c + d\*x]])/(8\*b^2\*Sqrt[b\*Sec[c + d\*x]]) + Sin[c + d\*x]/(4\*b^2\*d\*Sec[c + d\*x]^(5/2)\*Sqrt[b\*Sec[c + d\*x]]) + (3\*Sin[c + d\*x])/(8\*b^2\*d\*Sqrt[Sec[c + d\*x]]\*Sqrt[b\*Sec[c + d\*x]])

### Rule 18

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(a^(m - 1/2)\*b^(n + 1/2)\*Sqrt[a\*v])/Sqrt[b\*v], Int[u\*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x])\*(b\*Sin[c + d\*x])^(n - 1)]/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

**Rule 8**

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

**Rubi steps**

$$\begin{aligned} \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b \sec(c+dx))^{5/2}} dx &= \frac{\sqrt{\sec(c+dx)} \int \cos^4(c+dx) dx}{b^2 \sqrt{b \sec(c+dx)}} \\ &= \frac{\sin(c+dx)}{4b^2 d \sec^{\frac{5}{2}}(c+dx) \sqrt{b \sec(c+dx)}} + \frac{(3\sqrt{\sec(c+dx)}) \int \cos^2(c+dx) dx}{4b^2 \sqrt{b \sec(c+dx)}} \\ &= \frac{\sin(c+dx)}{4b^2 d \sec^{\frac{5}{2}}(c+dx) \sqrt{b \sec(c+dx)}} + \frac{3 \sin(c+dx)}{8b^2 d \sqrt{\sec(c+dx)} \sqrt{b \sec(c+dx)}} + \frac{(3\sqrt{\sec(c+dx)})}{8b^2 \sqrt{b \sec(c+dx)}} \\ &= \frac{3x\sqrt{\sec(c+dx)}}{8b^2 \sqrt{b \sec(c+dx)}} + \frac{\sin(c+dx)}{4b^2 d \sec^{\frac{5}{2}}(c+dx) \sqrt{b \sec(c+dx)}} + \frac{3 \sin(c+dx)}{8b^2 d \sqrt{\sec(c+dx)} \sqrt{b \sec(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0575848, size = 58, normalized size = 0.54

$$\frac{(12(c+dx) + 8 \sin(2(c+dx)) + \sin(4(c+dx)))\sqrt{\sec(c+dx)}}{32b^2 d \sqrt{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

`[In] Integrate[1/(Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(5/2)),x]`

`[Out] (Sqrt[Sec[c + d*x]]*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])) / (32*b^2*d*Sqrt[b*Sec[c + d*x]])`

**Maple [A]** time = 0.127, size = 74, normalized size = 0.7

$$\frac{2 (\cos(dx+c))^3 \sin(dx+c) + 3 \cos(dx+c) \sin(dx+c) + 3 dx + 3c}{8 d (\cos(dx+c))^4} \left( (\cos(dx+c))^{-1} \right)^{-\frac{3}{2}} \left( \frac{b}{\cos(dx+c)} \right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x)`

[Out]  $1/8/d*(2*\cos(d*x+c)^3*\sin(d*x+c)+3*\cos(d*x+c)*\sin(d*x+c)+3*d*x+3*c)/\cos(d*x+c)^4/(1/\cos(d*x+c))^(3/2)/(b/\cos(d*x+c))^(5/2)$

**Maxima [A]** time = 1.97597, size = 66, normalized size = 0.62

$$\frac{12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin\left(\frac{1}{2} \arctan(\sin(4 dx + 4 c), \cos(4 dx + 4 c))\right)}{32 b^2 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")`

[Out]  $1/32*(12*d*x + 12*c + \sin(4*d*x + 4*c) + 8*\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))))/(b^(5/2)*d)$

**Fricas [A]** time = 2.46266, size = 552, normalized size = 5.16

$$\frac{2 \left( 2 \cos(dx+c)^4 + 3 \cos(dx+c)^2 \right) \sqrt{\frac{b}{\cos(dx+c)}} \sin(dx+c) - 3 \sqrt{-b} \log \left( 2 \sqrt{-b} \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)^{\frac{3}{2}} \sin(dx+c) + 2 b \cos(dx+c)^2 \right)}{16 b^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")`

[Out]  $[1/16*(2*(2*\cos(d*x + c)^4 + 3*\cos(d*x + c)^2)*\sqrt{b/\cos(d*x + c)}*\sin(d*x + c)/\sqrt{\cos(d*x + c)} - 3*\sqrt{-b}*\log(2*\sqrt{-b}*\sqrt{b/\cos(d*x + c)}*\cos(d*x + c)^{(3/2)}*\sin(d*x + c) + 2*b*\cos(d*x + c)^2 - b))/(b^3*d), 1/8*((2*\cos(d*x + c)^4 + 3*\cos(d*x + c)^2)*\sqrt{b/\cos(d*x + c)}*\sin(d*x + c)/\sqrt{\cos(d*x + c)} + 3*\sqrt{b}*\arctan(\sqrt{b/\cos(d*x + c)}*\sin(d*x + c)/(\sqrt{b}*\sqrt{\cos(d*x + c)})))]/(b^3*d)]$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(5/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(dx + c))^{\frac{5}{2}} \sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")`

[Out] `integrate(1/((b*sec(d*x + c))^(5/2)*sec(d*x + c)^(3/2)), x)`

### 3.184 $\int \sec^2(c + dx) \sqrt[3]{b \sec(c + dx)} dx$

**Optimal.** Leaf size=58

$$\frac{3 \sin(c + dx)(b \sec(c + dx))^{4/3} \text{Hypergeometric2F1}\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \cos^2(c + dx)\right)}{4bd \sqrt{\sin^2(c + dx)}}$$

[Out] (3\*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(4/3)\*Sin[c + d\*x])/(4\*b\*d\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0325862, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16, 3772, 2643}

$$\frac{3 \sin(c + dx)(b \sec(c + dx))^{4/3} {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right)}{4bd \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^2\*(b\*Sec[c + d\*x])^(1/3), x]

[Out] (3\*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(4/3)\*Sin[c + d\*x])/(4\*b\*d\*Sqrt[Sin[c + d\*x]^2])

#### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] :> Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c

+ d\*x]^2))/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x]  
&& !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \sec^2(c + dx) \sqrt[3]{b \sec(c + dx)} dx &= \frac{\int (b \sec(c + dx))^{7/3} dx}{b^2} \\ &= \frac{\left( \sqrt[3]{\frac{\cos(c+dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{7/3}} dx}{b^2} \\ &= \frac{3 {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right) (b \sec(c + dx))^{4/3} \sin(c + dx)}{4bd \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0749997, size = 60, normalized size = 1.03

$$\frac{3 \sqrt{-\tan^2(c + dx)} \csc(c + dx) (b \sec(c + dx))^{4/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \sec^2(c + dx)\right)}{7bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^2\*(b\*Sec[c + d\*x])^(1/3), x]

[Out] (3\*Csc[c + d\*x]\*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(4/3)\*Sqrt[-Tan[c + d\*x]^2])/(7\*b\*d)

**Maple [F]** time = 0.067, size = 0, normalized size = 0.

$$\int (\sec(dx + c))^2 \sqrt[3]{b \sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^2\*(b\*sec(d\*x+c))^(1/3), x)

[Out] int(sec(d\*x+c)^2\*(b\*sec(d\*x+c))^(1/3), x)



---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{1}{3}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^2\*(b\*sec(d\*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^(1/3)\*sec(d\*x + c)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec(dx + c))^{\frac{1}{3}} \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^2\*(b\*sec(d\*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(1/3)\*sec(d\*x + c)^2, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b \sec(c + dx)} \sec^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*2\*(b\*sec(d\*x+c))\*\*(1/3),x)

[Out] Integral((b\*sec(c + d\*x))\*\*(1/3)\*sec(c + d\*x)\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{1}{3}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

### 3.185 $\int \sec(c + dx) \sqrt[3]{b \sec(c + dx)} dx$

**Optimal.** Leaf size=53

$$\frac{3 \sin(c + dx) \sqrt[3]{b \sec(c + dx)} \operatorname{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \cos^2(c + dx)\right)}{d \sqrt{\sin^2(c + dx)}}$$

[Out] (3\*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(1/3)\*Sin[c + d\*x])/(d\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0308292, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {16, 3772, 2643}

$$\frac{3 \sin(c + dx) \sqrt[3]{b \sec(c + dx)} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right)}{d \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]\*(b\*Sec[c + d\*x])^(1/3), x]

[Out] (3\*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(1/3)\*Sin[c + d\*x])/(d\*Sqrt[Sin[c + d\*x]^2])

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c

+ d\*x]^2))/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x]  
 && !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \sec(c + dx) \sqrt[3]{b \sec(c + dx)} dx &= \frac{\int (b \sec(c + dx))^{4/3} dx}{b} \\ &= \frac{\left( \sqrt[3]{\frac{\cos(c+dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{4/3}} dx}{b} \\ &= \frac{3 {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0473121, size = 60, normalized size = 1.13

$$\frac{3 \sqrt{-\tan^2(c + dx)} \cot(c + dx) (b \sec(c + dx))^{4/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \sec^2(c + dx)\right)}{4bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]\*(b\*Sec[c + d\*x])^(1/3), x]

[Out] (3\*Cot[c + d\*x]\*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(4/3)\*Sqrt[-Tan[c + d\*x]^2])/(4\*b\*d)

**Maple [F]** time = 0.063, size = 0, normalized size = 0.

$$\int \sec(dx + c) \sqrt[3]{b \sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)\*(b\*sec(d\*x+c))^(1/3), x)

[Out] int(sec(d\*x+c)\*(b\*sec(d\*x+c))^(1/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{1}{3}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*(b\*sec(d\*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^(1/3)\*sec(d\*x + c), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec(dx + c))^{\frac{1}{3}} \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*(b\*sec(d\*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(1/3)\*sec(d\*x + c), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b \sec(c + dx)} \sec(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*(b\*sec(d\*x+c))\*\*(1/3),x)

[Out] Integral((b\*sec(c + d\*x))\*\*(1/3)\*sec(c + d\*x), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{1}{3}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c), x)
```

### 3.186 $\int \sqrt[3]{b \sec(c + dx)} dx$

**Optimal.** Leaf size=56

$$-\frac{3b \sin(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos^2(c + dx)\right)}{2d \sqrt{\sin^2(c + dx)} (b \sec(c + dx))^{2/3}}$$

[Out]  $(-3*b*\operatorname{Hypergeometric2F1}[1/3, 1/2, 4/3, \operatorname{Cos}[c + d*x]^2]*\operatorname{Sin}[c + d*x])/(2*d*(b*\operatorname{Sec}[c + d*x])^{(2/3)}*\operatorname{Sqrt}[\operatorname{Sin}[c + d*x]^2])$

**Rubi [A]** time = 0.0249474, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3772, 2643}

$$-\frac{3b \sin(c + dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{2d \sqrt{\sin^2(c + dx)} (b \sec(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(b*\operatorname{Sec}[c + d*x])^{(1/3)}, x]$

[Out]  $(-3*b*\operatorname{Hypergeometric2F1}[1/3, 1/2, 4/3, \operatorname{Cos}[c + d*x]^2]*\operatorname{Sin}[c + d*x])/(2*d*(b*\operatorname{Sec}[c + d*x])^{(2/3)}*\operatorname{Sqrt}[\operatorname{Sin}[c + d*x]^2])$

#### Rule 3772

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(b*\operatorname{Csc}[c + d*x])^{(n - 1)}*((\operatorname{Sin}[c + d*x]/b)^{(n - 1)}*\operatorname{Int}[1/(\operatorname{Sin}[c + d*x]/b)^n, x]), x] /; \operatorname{FreeQ}[\{b, c, d, n\}, x] \ \&\amp; \ !\operatorname{IntegerQ}[n]$

#### Rule 2643

$\operatorname{Int}[(b_.)*\operatorname{sin}[(c_.) + (d_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Cos}[c + d*x])*(b*\operatorname{Sin}[c + d*x])^{(n + 1)}*\operatorname{Hypergeometric2F1}[1/2, (n + 1)/2, (n + 3)/2, \operatorname{Sin}[c + d*x]^2]/(b*d*(n + 1)*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]^2]), x] /; \operatorname{FreeQ}[\{b, c, d, n\}, x] \ \&\amp; \ !\operatorname{IntegerQ}[2*n]$

#### Rubi steps

$$\int \sqrt[3]{b \sec(c + dx)} dx = \sqrt[3]{\frac{\cos(c + dx)}{b}} \sqrt[3]{b \sec(c + dx)} \int \frac{1}{\sqrt[3]{\frac{\cos(c + dx)}{b}}} dx$$

$$= -\frac{3 \cos(c + dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{2d \sqrt{\sin^2(c + dx)}}$$

**Mathematica [A]** time = 0.0415476, size = 55, normalized size = 0.98

$$\frac{3 \sqrt{-\tan^2(c + dx)} \cot(c + dx) \sqrt[3]{b \sec(c + dx)} \text{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \sec^2(c + dx)\right)}{d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(b\*Sec[c + d\*x])^(1/3), x]

[Out] (3\*Cot[c + d\*x]\*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(1/3)\*Sqrt[-Tan[c + d\*x]^2])/d

**Maple [F]** time = 0.131, size = 0, normalized size = 0.

$$\int \sqrt[3]{b \sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(1/3), x)

[Out] int((b\*sec(d\*x+c))^(1/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((b\*sec(d\*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^(1/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec(dx + c))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(1/3), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b \sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(1/3),x)

[Out] Integral((b\*sec(c + d\*x))\*\*(1/3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(1/3), x)

$$3.187 \quad \int \cos(c + dx) \sqrt[3]{b \sec(c + dx)} dx$$

**Optimal.** Leaf size=58

$$\frac{3b^2 \sin(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos^2(c + dx)\right)}{5d \sqrt{\sin^2(c + dx)} (b \sec(c + dx))^{5/3}}$$

[Out]  $(-3*b^2*\operatorname{Hypergeometric2F1}[1/2, 5/6, 11/6, \operatorname{Cos}[c + d*x]^2]*\operatorname{Sin}[c + d*x])/(5*d*(b*\operatorname{Sec}[c + d*x])^{(5/3)}*\operatorname{Sqrt}[\operatorname{Sin}[c + d*x]^2])$

**Rubi [A]** time = 0.0358247, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {16, 3772, 2643}

$$\frac{3b^2 \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5d \sqrt{\sin^2(c + dx)} (b \sec(c + dx))^{5/3}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cos}[c + d*x]*(b*\operatorname{Sec}[c + d*x])^{(1/3)}, x]$

[Out]  $(-3*b^2*\operatorname{Hypergeometric2F1}[1/2, 5/6, 11/6, \operatorname{Cos}[c + d*x]^2]*\operatorname{Sin}[c + d*x])/(5*d*(b*\operatorname{Sec}[c + d*x])^{(5/3)}*\operatorname{Sqrt}[\operatorname{Sin}[c + d*x]^2])$

### Rule 16

$\operatorname{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[1/b^m, \operatorname{Int}[u*(b*v)^{(m+n)}, x], x] /;$   $\operatorname{FreeQ}\{b, n\}, x \ \&\& \ \operatorname{IntegerQ}[m]$

### Rule 3772

$\operatorname{Int}[(\operatorname{csc}[(c_*) + (d_*)*(x_*)]*(b_*))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(b*\operatorname{Csc}[c + d*x])^{(n-1)}*((\operatorname{Sin}[c + d*x]/b)^{(n-1)}*\operatorname{Int}[1/(\operatorname{Sin}[c + d*x]/b)^n, x]), x] /;$   $\operatorname{FreeQ}\{b, c, d, n\}, x \ \&\& \ !\operatorname{IntegerQ}[n]$

### Rule 2643

$\operatorname{Int}[(b_**\operatorname{sin}[(c_*) + (d_*)*(x_*)])^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Cos}[c + d*x]*(b*\operatorname{Sin}[c + d*x])^{(n+1)}*\operatorname{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \operatorname{Sin}[c$

+ d\*x]^2))/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x]  
 && !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \cos(c + dx) \sqrt[3]{b \sec(c + dx)} dx &= b \int \frac{1}{(b \sec(c + dx))^{2/3}} dx \\ &= \left( b \sqrt[3]{\frac{\cos(c + dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \left( \frac{\cos(c + dx)}{b} \right)^{2/3} dx \\ &= -\frac{3 \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{5d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0669936, size = 58, normalized size = 1.

$$-\frac{3b \sqrt{-\tan^2(c + dx)} \cot(c + dx) \text{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \sec^2(c + dx)\right)}{2d(b \sec(c + dx))^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d\*x]\*(b\*Sec[c + d\*x])^(1/3), x]

[Out] (-3\*b\*Cot[c + d\*x]\*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[c + d\*x]^2]\*Sqrt[-Tan[c + d\*x]^2])/(2\*d\*(b\*Sec[c + d\*x])^(2/3))

**Maple [F]** time = 0.11, size = 0, normalized size = 0.

$$\int \cos(dx + c) \sqrt[3]{b \sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)\*(b\*sec(d\*x+c))^(1/3), x)

[Out] int(cos(d\*x+c)\*(b\*sec(d\*x+c))^(1/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{1}{3}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*(b\*sec(d\*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^(1/3)\*cos(d\*x + c), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec(dx + c))^{\frac{1}{3}} \cos(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*(b\*sec(d\*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(1/3)\*cos(d\*x + c), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b \sec(c + dx)} \cos(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*(b\*sec(d\*x+c))\*\*(1/3),x)

[Out] Integral((b\*sec(c + d\*x))\*\*(1/3)\*cos(c + d\*x), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{1}{3}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*cos(d*x + c), x)
```

### 3.188 $\int \cos^2(c + dx) \sqrt[3]{b \sec(c + dx)} dx$

**Optimal.** Leaf size=58

$$\frac{3b^3 \sin(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{4}{3}, \frac{7}{3}, \cos^2(c + dx)\right)}{8d \sqrt{\sin^2(c + dx)} (b \sec(c + dx))^{8/3}}$$

[Out]  $(-3*b^3*\operatorname{Hypergeometric2F1}[1/2, 4/3, 7/3, \operatorname{Cos}[c + d*x]^2]*\operatorname{Sin}[c + d*x])/(8*d*(b*\operatorname{Sec}[c + d*x])^{(8/3)}*\operatorname{Sqrt}[\operatorname{Sin}[c + d*x]^2])$

**Rubi [A]** time = 0.0432759, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16, 3772, 2643}

$$\frac{3b^3 \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right)}{8d \sqrt{\sin^2(c + dx)} (b \sec(c + dx))^{8/3}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cos}[c + d*x]^2*(b*\operatorname{Sec}[c + d*x])^{(1/3)}, x]$

[Out]  $(-3*b^3*\operatorname{Hypergeometric2F1}[1/2, 4/3, 7/3, \operatorname{Cos}[c + d*x]^2]*\operatorname{Sin}[c + d*x])/(8*d*(b*\operatorname{Sec}[c + d*x])^{(8/3)}*\operatorname{Sqrt}[\operatorname{Sin}[c + d*x]^2])$

#### Rule 16

$\operatorname{Int}[(u_*)*(v_)^{(m_*)}*((b_*)*(v_))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[1/b^m, \operatorname{Int}[u*(b*v)^{(m+n)}, x], x] /;$  FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3772

$\operatorname{Int}[(\operatorname{csc}[(c_*) + (d_*)*(x_*)]*(b_*)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{Simp}[(b*\operatorname{Csc}[c + d*x])^{(n-1)}*((\operatorname{Sin}[c + d*x]/b)^{(n-1)}*\operatorname{Int}[1/(\operatorname{Sin}[c + d*x]/b)^n, x]), x] /;$  FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

$\operatorname{Int}[(b_*)*\operatorname{sin}[(c_*) + (d_*)*(x_*)]^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Cos}[c + d*x]*(b*\operatorname{Sin}[c + d*x])^{(n+1)}*\operatorname{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \operatorname{Sin}[c$

+ d\*x]^2)]/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x]  
 && !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \cos^2(c + dx) \sqrt[3]{b \sec(c + dx)} dx &= b^2 \int \frac{1}{(b \sec(c + dx))^{5/3}} dx \\ &= \left( b^2 \sqrt[3]{\frac{\cos(c + dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \left( \frac{\cos(c + dx)}{b} \right)^{5/3} dx \\ &= \frac{3 \cos^3(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{8d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.143094, size = 59, normalized size = 1.02

$$\frac{3 \sin(2(c + dx)) \sqrt[3]{b \sec(c + dx)} \text{Hypergeometric2F1}\left(-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \sec^2(c + dx)\right)}{10d \sqrt{-\tan^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d\*x]^2\*(b\*Sec[c + d\*x])^(1/3), x]

[Out] (3\*Hypergeometric2F1[-5/6, 1/2, 1/6, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(1/3)\*Sin[2\*(c + d\*x)])/(10\*d\*Sqrt[-Tan[c + d\*x]^2])

**Maple [F]** time = 0.166, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^2 \sqrt[3]{b \sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)^2\*(b\*sec(d\*x+c))^(1/3), x)

[Out] int(cos(d\*x+c)^2\*(b\*sec(d\*x+c))^(1/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{1}{3}} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^2\*(b\*sec(d\*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^(1/3)\*cos(d\*x + c)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sec(dx + c)\right)^{\frac{1}{3}} \cos(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^2\*(b\*sec(d\*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(1/3)\*cos(d\*x + c)^2, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*\*2\*(b\*sec(d\*x+c))\*\*(1/3),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{1}{3}} \cos(dx + c)^2 dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

### 3.189 $\int \sec^2(c + dx)(b \sec(c + dx))^{4/3} dx$

**Optimal.** Leaf size=58

$$\frac{3 \sin(c + dx)(b \sec(c + dx))^{7/3} \text{Hypergeometric2F1}\left(-\frac{7}{6}, \frac{1}{2}, -\frac{1}{6}, \cos^2(c + dx)\right)}{7bd\sqrt{\sin^2(c + dx)}}$$

[Out] (3\*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(7/3)\*Sin[c + d\*x])/(7\*b\*d\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0309513, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16, 3772, 2643}

$$\frac{3 \sin(c + dx)(b \sec(c + dx))^{7/3} {}_2F_1\left(-\frac{7}{6}, \frac{1}{2}; -\frac{1}{6}; \cos^2(c + dx)\right)}{7bd\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^2\*(b\*Sec[c + d\*x])^(4/3),x]

[Out] (3\*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(7/3)\*Sin[c + d\*x])/(7\*b\*d\*Sqrt[Sin[c + d\*x]^2])

#### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3772

Int[(csc[(c\_.) + (d\_)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_)\*sin[(c\_.) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c

+ d\*x]^2))/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x]  
 && !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \sec^2(c + dx)(b \sec(c + dx))^{4/3} dx &= \frac{\int (b \sec(c + dx))^{10/3} dx}{b^2} \\ &= \frac{\left( \sqrt[3]{\frac{\cos(c+dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{10/3}} dx}{b^2} \\ &= \frac{3b {}_2F_1\left(-\frac{7}{6}, \frac{1}{2}; -\frac{1}{6}; \cos^2(c + dx)\right) \sec(c + dx) \sqrt[3]{b \sec(c + dx)} \tan(c + dx)}{7d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0796494, size = 60, normalized size = 1.03

$$\frac{3\sqrt{-\tan^2(c + dx)} \csc(c + dx) (b \sec(c + dx))^{7/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \sec^2(c + dx)\right)}{10bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^2\*(b\*Sec[c + d\*x])^(4/3), x]

[Out] (3\*Csc[c + d\*x]\*Hypergeometric2F1[1/2, 5/3, 8/3, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(7/3)\*Sqrt[-Tan[c + d\*x]^2])/(10\*b\*d)

**Maple [F]** time = 0.068, size = 0, normalized size = 0.

$$\int (\sec(dx + c))^2 (b \sec(dx + c))^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^2\*(b\*sec(d\*x+c))^(4/3), x)

[Out] int(sec(d\*x+c)^2\*(b\*sec(d\*x+c))^(4/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{4}{3}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^2\*(b\*sec(d\*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^(4/3)\*sec(d\*x + c)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sec(dx + c)\right)^{\frac{1}{3}} b \sec(dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^2\*(b\*sec(d\*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(1/3)\*b\*sec(d\*x + c)^3, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*2\*(b\*sec(d\*x+c))\*\*(4/3),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{4}{3}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

### 3.190 $\int \sec(c + dx)(b \sec(c + dx))^{4/3} dx$

**Optimal.** Leaf size=55

$$\frac{3 \sin(c + dx)(b \sec(c + dx))^{4/3} \text{Hypergeometric2F1}\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \cos^2(c + dx)\right)}{4d\sqrt{\sin^2(c + dx)}}$$

[Out] (3\*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(4/3)\*Sin[c + d\*x])/(4\*d\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0302989, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {16, 3772, 2643}

$$\frac{3 \sin(c + dx)(b \sec(c + dx))^{4/3} {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right)}{4d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]\*(b\*Sec[c + d\*x])^(4/3),x]

[Out] (3\*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(4/3)\*Sin[c + d\*x])/(4\*d\*Sqrt[Sin[c + d\*x]^2])

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c

+ d\*x]^2))/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x]  
 && !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \sec(c + dx)(b \sec(c + dx))^{4/3} dx &= \frac{\int (b \sec(c + dx))^{7/3} dx}{b} \\ &= \frac{\left( \sqrt[3]{\frac{\cos(c+dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{7/3}} dx}{b} \\ &= \frac{{}_3F_2\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right) (b \sec(c + dx))^{4/3} \sin(c + dx)}{4d\sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0490058, size = 60, normalized size = 1.09

$$\frac{3\sqrt{-\tan^2(c + dx)} \cot(c + dx)(b \sec(c + dx))^{7/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \sec^2(c + dx)\right)}{7bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]\*(b\*Sec[c + d\*x])^(4/3), x]

[Out] (3\*Cot[c + d\*x]\*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(7/3)\*Sqrt[-Tan[c + d\*x]^2])/(7\*b\*d)

**Maple [F]** time = 0.064, size = 0, normalized size = 0.

$$\int \sec(dx + c) (b \sec(dx + c))^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)\*(b\*sec(d\*x+c))^(4/3), x)

[Out] int(sec(d\*x+c)\*(b\*sec(d\*x+c))^(4/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{4}{3}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*(b\*sec(d\*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^(4/3)\*sec(d\*x + c), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sec(dx + c)\right)^{\frac{1}{3}} b \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*(b\*sec(d\*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(1/3)\*b\*sec(d\*x + c)^2, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*(b\*sec(d\*x+c))\*\*(4/3),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{4}{3}} \sec(dx + c) dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c), x)
```

### 3.191 $\int (b \sec(c + dx))^{4/3} dx$

**Optimal.** Leaf size=54

$$\frac{3b \sin(c + dx) \sqrt[3]{b \sec(c + dx)} \operatorname{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \cos^2(c + dx)\right)}{d \sqrt{\sin^2(c + dx)}}$$

[Out] (3\*b\*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(1/3)\*Sin[c + d\*x])/(d\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.026934, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3772, 2643}

$$\frac{3b \sin(c + dx) \sqrt[3]{b \sec(c + dx)} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right)}{d \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(4/3), x]

[Out] (3\*b\*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(1/3)\*Sin[c + d\*x])/(d\*Sqrt[Sin[c + d\*x]^2])

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*SIN[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

#### Rubi steps

$$\int (b \sec(c + dx))^{4/3} dx = \sqrt[3]{\frac{\cos(c + dx)}{b}} \sqrt[3]{b \sec(c + dx)} \int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{4/3}} dx$$

$$= \frac{3b {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{d \sqrt{\sin^2(c + dx)}}$$

**Mathematica [A]** time = 0.0082638, size = 57, normalized size = 1.06

$$\frac{3\sqrt{-\tan^2(c + dx)} \cot(c + dx) (b \sec(c + dx))^{4/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \sec^2(c + dx)\right)}{4d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(b\*Sec[c + d\*x])^(4/3),x]

[Out] (3\*Cot[c + d\*x]\*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(4/3)\*Sqrt[-Tan[c + d\*x]^2])/(4\*d)

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{4/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^(4/3),x)

[Out] int((b\*sec(d\*x+c))^(4/3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{4/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^(4/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sec(dx + c)\right)^{\frac{1}{3}} b \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(1/3)\*b\*sec(d\*x + c), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*(4/3),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(4/3), x)

### 3.192 $\int \cos(c + dx)(b \sec(c + dx))^{4/3} dx$

**Optimal.** Leaf size=58

$$\frac{3b^2 \sin(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos^2(c + dx)\right)}{2d \sqrt{\sin^2(c + dx)} (b \sec(c + dx))^{2/3}}$$

[Out]  $(-3*b^2*\operatorname{Hypergeometric2F1}[1/3, 1/2, 4/3, \operatorname{Cos}[c + d*x]^2]*\operatorname{Sin}[c + d*x])/(2*d*(b*\operatorname{Sec}[c + d*x])^{(2/3)}*\operatorname{Sqrt}[\operatorname{Sin}[c + d*x]^2])$

**Rubi [A]** time = 0.0381933, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {16, 3772, 2643}

$$\frac{3b^2 \sin(c + dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{2d \sqrt{\sin^2(c + dx)} (b \sec(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cos}[c + d*x]*(b*\operatorname{Sec}[c + d*x])^{(4/3)}, x]$

[Out]  $(-3*b^2*\operatorname{Hypergeometric2F1}[1/3, 1/2, 4/3, \operatorname{Cos}[c + d*x]^2]*\operatorname{Sin}[c + d*x])/(2*d*(b*\operatorname{Sec}[c + d*x])^{(2/3)}*\operatorname{Sqrt}[\operatorname{Sin}[c + d*x]^2])$

#### Rule 16

$\operatorname{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[1/b^m, \operatorname{Int}[u*(b*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{b, n\}, x \ \&\& \operatorname{IntegerQ}[m]$

#### Rule 3772

$\operatorname{Int}[(\operatorname{csc}[(c_*) + (d_*)*(x_*)])*(b_*)^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(b*\operatorname{Csc}[c + d*x])^{(n-1)}*((\operatorname{Sin}[c + d*x]/b)^{(n-1)}*\operatorname{Int}[1/(\operatorname{Sin}[c + d*x]/b)^n, x]), x] /; \operatorname{FreeQ}\{b, c, d, n\}, x \ \&\& \operatorname{IntegerQ}[n]$

#### Rule 2643

$\operatorname{Int}[(b_*)*\operatorname{sin}[(c_*) + (d_*)*(x_*)]^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Cos}[c + d*x]*(b*\operatorname{Sin}[c + d*x])^{(n+1)}*\operatorname{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \operatorname{Sin}[c$

$+ d*x]^2] / (b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]$   
 $\&\& \text{!IntegerQ}[2*n]$

### Rubi steps

$$\begin{aligned} \int \cos(c + dx)(b \sec(c + dx))^{4/3} dx &= b \int \sqrt[3]{b \sec(c + dx)} dx \\ &= \left( b \sqrt[3]{\frac{\cos(c + dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \frac{1}{\sqrt[3]{\frac{\cos(c + dx)}{b}}} dx \\ &= -\frac{3b \cos(c + dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{2d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.009011, size = 56, normalized size = 0.97

$$\frac{3b \sqrt{-\tan^2(c + dx)} \cot(c + dx) \sqrt[3]{b \sec(c + dx)} \text{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \sec^2(c + dx)\right)}{d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d\*x]\*(b\*Sec[c + d\*x])^(4/3), x]

[Out] (3\*b\*Cot[c + d\*x]\*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(1/3)\*Sqrt[-Tan[c + d\*x]^2])/d

**Maple [F]** time = 0.106, size = 0, normalized size = 0.

$$\int \cos(dx + c) (b \sec(dx + c))^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)\*(b\*sec(d\*x+c))^(4/3), x)

[Out] int(cos(d\*x+c)\*(b\*sec(d\*x+c))^(4/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{4}{3}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*(b\*sec(d\*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^(4/3)\*cos(d\*x + c), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sec(dx + c)\right)^{\frac{1}{3}} b \cos(dx + c) \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*(b\*sec(d\*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(1/3)\*b\*cos(d\*x + c)\*sec(d\*x + c), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*(b\*sec(d\*x+c))\*\*(4/3),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{4}{3}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*cos(d*x + c), x)
```



$$3.193 \quad \int \cos^2(c + dx)(b \sec(c + dx))^{4/3} dx$$

**Optimal.** Leaf size=58

$$\frac{3b^3 \sin(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos^2(c + dx)\right)}{5d \sqrt{\sin^2(c + dx)} (b \sec(c + dx))^{5/3}}$$

[Out]  $(-3*b^3*\text{Hypergeometric2F1}[1/2, 5/6, 11/6, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(5*d*(b*\text{Sec}[c + d*x])^{(5/3)}*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

**Rubi [A]** time = 0.0460992, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16, 3772, 2643}

$$\frac{3b^3 \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5d \sqrt{\sin^2(c + dx)} (b \sec(c + dx))^{5/3}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cos}[c + d*x]^2*(b*\text{Sec}[c + d*x])^{(4/3)}, x]$

[Out]  $(-3*b^3*\text{Hypergeometric2F1}[1/2, 5/6, 11/6, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(5*d*(b*\text{Sec}[c + d*x])^{(5/3)}*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

### Rule 16

$\text{Int}[(u_*)*(v_)^{(m_*)}*((b_*)*(v_))^{(n_)}, x\_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n\}, x \ \&\& \ \text{IntegerQ}[m]$

### Rule 3772

$\text{Int}[(\text{csc}[(c_*) + (d_*)*(x_)]*(b_*)^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[(b*\text{Csc}[c + d*x])^{(n-1)}*((\text{Sin}[c + d*x]/b)^{(n-1)}*\text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /; \text{FreeQ}\{b, c, d, n\}, x \ \&\& \ !\text{IntegerQ}[n]$

### Rule 2643

$\text{Int}[(b_*)*\text{sin}[(c_*) + (d_*)*(x_)]^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c$

$+ d*x]^2] / (b*d*(n + 1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}[\{b, c, d, n\}, x]$   
 $\&\& \text{!IntegerQ}[2*n]$

### Rubi steps

$$\begin{aligned} \int \cos^2(c + dx)(b \sec(c + dx))^{4/3} dx &= b^2 \int \frac{1}{(b \sec(c + dx))^{2/3}} dx \\ &= \left( b^2 \sqrt[3]{\frac{\cos(c + dx)}{b}} \sqrt[3]{b \sec(c + dx)} \right) \int \left( \frac{\cos(c + dx)}{b} \right)^{2/3} dx \\ &= -\frac{3b \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{5d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.054224, size = 60, normalized size = 1.03

$$-\frac{3b^2 \sqrt{-\tan^2(c + dx)} \cot(c + dx) \text{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \sec^2(c + dx)\right)}{2d(b \sec(c + dx))^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d\*x]^2\*(b\*Sec[c + d\*x])^(4/3), x]

[Out]  $(-3*b^2*\text{Cot}[c + d*x]*\text{Hypergeometric2F1}[-1/3, 1/2, 2/3, \text{Sec}[c + d*x]^2]*\text{Sqrt}[-\text{Tan}[c + d*x]^2]) / (2*d*(b*\text{Sec}[c + d*x])^{(2/3)})$

**Maple [F]** time = 0.194, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^2 (b \sec(dx + c))^{4/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)^2\*(b\*sec(d\*x+c))^(4/3), x)

[Out] int(cos(d\*x+c)^2\*(b\*sec(d\*x+c))^(4/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{4}{3}} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^2\*(b\*sec(d\*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^(4/3)\*cos(d\*x + c)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sec(dx + c)\right)^{\frac{1}{3}} b \cos(dx + c)^2 \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^2\*(b\*sec(d\*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(1/3)\*b\*cos(d\*x + c)^2\*sec(d\*x + c), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*\*2\*(b\*sec(d\*x+c))\*\*(4/3),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{4}{3}} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

$$3.194 \quad \int \frac{\sec^2(c+dx)}{\sqrt[3]{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=58

$$\frac{3 \sin(c+dx)(b \sec(c+dx))^{2/3} \text{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c+dx)\right)}{2bd \sqrt{\sin^2(c+dx)}}$$

[Out] (3\*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(2/3)\*Sin[c + d\*x])/(2\*b\*d\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0311336, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16, 3772, 2643}

$$\frac{3 \sin(c+dx)(b \sec(c+dx))^{2/3} {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c+dx)\right)}{2bd \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^2/(b\*Sec[c + d\*x])^(1/3), x]

[Out] (3\*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(2/3)\*Sin[c + d\*x])/(2\*b\*d\*Sqrt[Sin[c + d\*x]^2])

### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n-1)\*((Sin[c + d\*x]/b)^(n-1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 2643

Int[((b\_)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n+1)\*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c

+ d\*x]^2))/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x]  
&& !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c + dx)}{\sqrt[3]{b \sec(c + dx)}} dx &= \frac{\int (b \sec(c + dx))^{5/3} dx}{b^2} \\ &= \frac{\left( \left( \frac{\cos(c+dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \right) \int \frac{1}{\left( \frac{\cos(c+dx)}{b} \right)^{5/3}} dx}{b^2} \\ &= \frac{{}_3F_1\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}; \cos^2(c + dx)\right) (b \sec(c + dx))^{2/3} \sin(c + dx)}{2bd\sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0595676, size = 60, normalized size = 1.03

$$\frac{3\sqrt{-\tan^2(c + dx) \csc(c + dx)} (b \sec(c + dx))^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \sec^2(c + dx)\right)}{5bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^2/(b\*Sec[c + d\*x])^(1/3), x]

[Out] (3\*Csc[c + d\*x]\*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(2/3)\*Sqrt[-Tan[c + d\*x]^2])/(5\*b\*d)

**Maple [F]** time = 0.065, size = 0, normalized size = 0.

$$\int (\sec(dx + c))^2 \frac{1}{\sqrt[3]{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^2/(b\*sec(d\*x+c))^(1/3), x)

[Out] int(sec(d\*x+c)^2/(b\*sec(d\*x+c))^(1/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b \sec(dx+c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^2/(b\*sec(d\*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate(sec(d\*x + c)^2/(b\*sec(d\*x + c))^(1/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec(dx+c))^{\frac{2}{3}} \sec(dx+c)}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^2/(b\*sec(d\*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(2/3)\*sec(d\*x + c)/b, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b \sec(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*2/(b\*sec(d\*x+c))\*\*(1/3),x)

[Out] Integral(sec(c + d\*x)\*\*2/(b\*sec(c + d\*x))\*\*(1/3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b \sec(dx+c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```



$$3.195 \quad \int \frac{\sec(c+dx)}{\sqrt[3]{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=53

$$\frac{3 \sin(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c+dx)\right)}{d \sqrt{\sin^2(c+dx)} \sqrt[3]{b \sec(c+dx)}}$$

[Out] (-3\*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d\*x]^2]\*Sin[c + d\*x])/(d\*(b\*Sec[c + d\*x])^(1/3)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0313008, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {16, 3772, 2643}

$$\frac{3 \sin(c+dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right)}{d \sqrt{\sin^2(c+dx)} \sqrt[3]{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]/(b\*Sec[c + d\*x])^(1/3), x]

[Out] (-3\*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d\*x]^2]\*Sin[c + d\*x])/(d\*(b\*Sec[c + d\*x])^(1/3)\*Sqrt[Sin[c + d\*x]^2])

### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3772

Int[(csc[(c\_)+(d\_)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(b\*Csc[c + d\*x])^(n-1)\*((Sin[c + d\*x]/b)^(n-1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 2643

Int[((b\_)\*sin[(c\_)+(d\_)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n+1)\*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c

+ d\*x]^2))/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x]  
 && !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \frac{\sec(c+dx)}{\sqrt[3]{b \sec(c+dx)}} dx &= \frac{\int (b \sec(c+dx))^{2/3} dx}{b} \\ &= \frac{\left(\left(\frac{\cos(c+dx)}{b}\right)^{2/3} (b \sec(c+dx))^{2/3}\right) \int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}} dx}{b} \\ &= \frac{3 \cos(c+dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right) (b \sec(c+dx))^{2/3} \sin(c+dx)}{bd \sqrt{\sin^2(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0474672, size = 60, normalized size = 1.13

$$\frac{3\sqrt{-\tan^2(c+dx)} \cot(c+dx) (b \sec(c+dx))^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \sec^2(c+dx)\right)}{2bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]/(b\*Sec[c + d\*x])^(1/3), x]

[Out] (3\*Cot[c + d\*x]\*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(2/3)\*Sqrt[-Tan[c + d\*x]^2])/(2\*b\*d)

**Maple [F]** time = 0.092, size = 0, normalized size = 0.

$$\int \sec(dx+c) \frac{1}{\sqrt[3]{b \sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)/(b\*sec(d\*x+c))^(1/3), x)

[Out] int(sec(d\*x+c)/(b\*sec(d\*x+c))^(1/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)}{(b \sec(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)/(b\*sec(d\*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate(sec(d\*x + c)/(b\*sec(d\*x + c))^(1/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec(dx + c))^{\frac{2}{3}}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)/(b\*sec(d\*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(2/3)/b, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c + dx)}{\sqrt[3]{b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)/(b\*sec(d\*x+c))\*\*(1/3),x)

[Out] Integral(sec(c + d\*x)/(b\*sec(c + d\*x))\*\*(1/3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)}{(b \sec(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

$$3.196 \quad \int \frac{1}{\sqrt[3]{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=56

$$\frac{3b \sin(c+dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c+dx)\right)}{4d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{4/3}}$$

[Out] (-3\*b\*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d\*x]^2]\*Sin[c + d\*x])/(4\*d\*(b\*Sec[c + d\*x])^(4/3)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.025305, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3772, 2643}

$$\frac{3b \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right)}{4d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(-1/3), x]

[Out] (-3\*b\*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d\*x]^2]\*Sin[c + d\*x])/(4\*d\*(b\*Sec[c + d\*x])^(4/3)\*Sqrt[Sin[c + d\*x]^2])

### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

### Rubi steps

$$\int \frac{1}{\sqrt[3]{b \sec(c+dx)}} dx = \left( \frac{\cos(c+dx)}{b} \right)^{2/3} (b \sec(c+dx))^{2/3} \int \sqrt[3]{\frac{\cos(c+dx)}{b}} dx$$

$$= -\frac{3 \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right) (b \sec(c+dx))^{2/3} \sin(c+dx)}{4bd \sqrt{\sin^2(c+dx)}}$$

**Mathematica [A]** time = 0.0542083, size = 55, normalized size = 0.98

$$\frac{3\sqrt{-\tan^2(c+dx)} \cot(c+dx) \text{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \sec^2(c+dx)\right)}{d\sqrt[3]{b \sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(b\*Sec[c + d\*x])^(-1/3),x]

[Out] (-3\*Cot[c + d\*x]\*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d\*x]^2]\*Sqrt[-Tan[c + d\*x]^2])/(d\*(b\*Sec[c + d\*x])^(1/3))

**Maple [F]** time = 0.091, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b \sec(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*sec(d\*x+c))^(1/3),x)

[Out] int(1/(b\*sec(d\*x+c))^(1/3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(dx+c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*sec(d\*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^(1/3), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec(dx + c))^{\frac{2}{3}}}{b \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*sec(d\*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(2/3)/(b\*sec(d\*x + c)), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*sec(d\*x+c))\*\*(1/3),x)

[Out] Integral((b\*sec(c + d\*x))\*\*(-1/3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*sec(d\*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(1/3), x)

$$3.197 \quad \int \frac{\cos(c+dx)}{\sqrt[3]{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=58

$$\frac{3b^2 \sin(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c+dx)\right)}{7d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{7/3}}$$

[Out]  $(-3*b^2*\operatorname{Hypergeometric2F1}[1/2, 7/6, 13/6, \operatorname{Cos}[c + d*x]^2]*\operatorname{Sin}[c + d*x])/(7*d*(b*\operatorname{Sec}[c + d*x])^{(7/3)}*\operatorname{Sqrt}[\operatorname{Sin}[c + d*x]^2])$

**Rubi [A]** time = 0.0374952, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {16, 3772, 2643}

$$\frac{3b^2 \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c+dx)\right)}{7d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cos}[c + d*x]/(b*\operatorname{Sec}[c + d*x])^{(1/3)}, x]$

[Out]  $(-3*b^2*\operatorname{Hypergeometric2F1}[1/2, 7/6, 13/6, \operatorname{Cos}[c + d*x]^2]*\operatorname{Sin}[c + d*x])/(7*d*(b*\operatorname{Sec}[c + d*x])^{(7/3)}*\operatorname{Sqrt}[\operatorname{Sin}[c + d*x]^2])$

### Rule 16

$\operatorname{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[1/b^m, \operatorname{Int}[u*(b*v)^{(m+n)}, x], x] /;$  FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3772

$\operatorname{Int}[(\operatorname{csc}[(c_*) + (d_*)*(x_*)]*(b_*))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(b*\operatorname{Csc}[c + d*x])^{(n-1)}*((\operatorname{Sin}[c + d*x]/b)^{(n-1)}*\operatorname{Int}[1/(\operatorname{Sin}[c + d*x]/b)^n, x]), x] /;$  FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 2643

$\operatorname{Int}[(b_**\sin[(c_*) + (d_*)*(x_*)])^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Cos}[c + d*x]*(b*\operatorname{Sin}[c + d*x])^{(n+1)}*\operatorname{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \operatorname{Sin}[c$



+ d\*x]^2))/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x]  
 && !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \frac{\cos(c + dx)}{\sqrt[3]{b \sec(c + dx)}} dx &= b \int \frac{1}{(b \sec(c + dx))^{4/3}} dx \\ &= \left( b \left( \frac{\cos(c + dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \right) \int \left( \frac{\cos(c + dx)}{b} \right)^{4/3} dx \\ &= -\frac{3 \cos^3(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c + dx)\right) (b \sec(c + dx))^{2/3} \sin(c + dx)}{7bd \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.114559, size = 58, normalized size = 1.

$$-\frac{3b \sqrt{-\tan^2(c + dx)} \cot(c + dx) \text{Hypergeometric2F1}\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \sec^2(c + dx)\right)}{4d(b \sec(c + dx))^{4/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d\*x]/(b\*Sec[c + d\*x])^(1/3), x]

[Out] (-3\*b\*Cot[c + d\*x]\*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[c + d\*x]^2]\*Sqrt[-Tan[c + d\*x]^2])/(4\*d\*(b\*Sec[c + d\*x])^(4/3))

**Maple [F]** time = 0.068, size = 0, normalized size = 0.

$$\int \cos(dx + c) \frac{1}{\sqrt[3]{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)/(b\*sec(d\*x+c))^(1/3), x)

[Out] int(cos(d\*x+c)/(b\*sec(d\*x+c))^(1/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c)}{(b \sec(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)/(b\*sec(d\*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate(cos(d\*x + c)/(b\*sec(d\*x + c))^(1/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec(dx + c))^{\frac{2}{3}} \cos(dx + c)}{b \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)/(b\*sec(d\*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(2/3)\*cos(d\*x + c)/(b\*sec(d\*x + c)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c + dx)}{\sqrt[3]{b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)/(b\*sec(d\*x+c))\*\*(1/3),x)

[Out] Integral(cos(c + d\*x)/(b\*sec(c + d\*x))\*\*(1/3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c)}{(b \sec(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)/(b\*sec(d\*x+c))^(1/3),x, algorithm="giac")

[Out] integrate(cos(d\*x + c)/(b\*sec(d\*x + c))^(1/3), x)

$$3.198 \quad \int \frac{\cos^2(c+dx)}{\sqrt[3]{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=58

$$\frac{3b^3 \sin(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c+dx)\right)}{10d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{10/3}}$$

[Out] (-3\*b^3\*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d\*x]^2]\*Sin[c + d\*x])/(10\*d\*(b\*Sec[c + d\*x])^(10/3)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.04462, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16, 3772, 2643}

$$\frac{3b^3 \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c+dx)\right)}{10d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^2/(b\*Sec[c + d\*x])^(1/3), x]

[Out] (-3\*b^3\*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d\*x]^2]\*Sin[c + d\*x])/(10\*d\*(b\*Sec[c + d\*x])^(10/3)\*Sqrt[Sin[c + d\*x]^2])

### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3772

Int[(csc[(c\_)+(d\_)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := Simp[(b\*Csc[c+d\*x])^(n-1)\*((Sin[c+d\*x]/b)^(n-1)\*Int[1/(Sin[c+d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 2643

Int[((b\_)\*sin[(c\_)+(d\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c+d\*x]\*(b\*Sin[c+d\*x])^(n+1)\*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c

+ d\*x]^2))/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x]  
 && !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \frac{\cos^2(c + dx)}{\sqrt[3]{b \sec(c + dx)}} dx &= b^2 \int \frac{1}{(b \sec(c + dx))^{7/3}} dx \\ &= \left( b^2 \left( \frac{\cos(c + dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \right) \int \left( \frac{\cos(c + dx)}{b} \right)^{7/3} dx \\ &= -\frac{3 \cos^4(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c + dx)\right) (b \sec(c + dx))^{2/3} \sin(c + dx)}{10bd \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0697015, size = 60, normalized size = 1.03

$$-\frac{3b^2 \sqrt{-\tan^2(c + dx)} \cot(c + dx) \text{Hypergeometric2F1}\left(-\frac{7}{6}, \frac{1}{2}, -\frac{1}{6}, \sec^2(c + dx)\right)}{7d(b \sec(c + dx))^{7/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d\*x]^2/(b\*Sec[c + d\*x])^(1/3), x]

[Out] (-3\*b^2\*Cot[c + d\*x]\*Hypergeometric2F1[-7/6, 1/2, -1/6, Sec[c + d\*x]^2]\*Sqrt[-Tan[c + d\*x]^2])/(7\*d\*(b\*Sec[c + d\*x])^(7/3))

**Maple [F]** time = 0.142, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^2 \frac{1}{\sqrt[3]{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)^2/(b\*sec(d\*x+c))^(1/3), x)

[Out] int(cos(d\*x+c)^2/(b\*sec(d\*x+c))^(1/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b \sec(dx+c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^2/(b\*sec(d\*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate(cos(d\*x + c)^2/(b\*sec(d\*x + c))^(1/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec(dx+c))^{\frac{2}{3}} \cos(dx+c)^2}{b \sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^2/(b\*sec(d\*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(2/3)\*cos(d\*x + c)^2/(b\*sec(d\*x + c)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b \sec(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*\*2/(b\*sec(d\*x+c))\*\*(1/3),x)

[Out] Integral(cos(c + d\*x)\*\*2/(b\*sec(c + d\*x))\*\*(1/3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b \sec(dx+c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

$$3.199 \quad \int \frac{\sec^2(c+dx)}{(b \sec(c+dx))^{4/3}} dx$$

**Optimal.** Leaf size=56

$$\frac{3 \sin(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)} \sqrt[3]{b \sec(c+dx)}}$$

[Out] (-3\*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d\*x]^2]\*Sin[c + d\*x])/(b\*d\*(b\*Sec[c + d\*x])^(1/3)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.031286, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16, 3772, 2643}

$$\frac{3 \sin(c+dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)} \sqrt[3]{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^2/(b\*Sec[c + d\*x])^(4/3), x]

[Out] (-3\*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d\*x]^2]\*Sin[c + d\*x])/(b\*d\*(b\*Sec[c + d\*x])^(1/3)\*Sqrt[Sin[c + d\*x]^2])

### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3772

Int[(csc[(c\_)+(d\_)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := Simp[(b\*Csc[c+d\*x])^(n-1)\*((Sin[c+d\*x]/b)^(n-1)\*Int[1/(Sin[c+d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 2643

Int[((b\_)\*sin[(c\_)+(d\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c+d\*x]\*(b\*Sin[c+d\*x])^(n+1)\*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c



$+ d*x]^2) / (b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]$   
 $\&\& !IntegerQ[2*n]$

### Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c + dx)}{(b \sec(c + dx))^{4/3}} dx &= \frac{\int (b \sec(c + dx))^{2/3} dx}{b^2} \\ &= \frac{\left( \left( \frac{\cos(c+dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \right) \int \frac{1}{\left( \frac{\cos(c+dx)}{b} \right)^{2/3}} dx}{b^2} \\ &= -\frac{3 \cos(c + dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right) (b \sec(c + dx))^{2/3} \sin(c + dx)}{b^2 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0426375, size = 60, normalized size = 1.07

$$\frac{3\sqrt{-\tan^2(c + dx) \cot(c + dx)} (b \sec(c + dx))^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \sec^2(c + dx)\right)}{2b^2 d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[c + d\*x]^2/(b\*Sec[c + d\*x])^(4/3), x]

[Out] (3\*Cot[c + d\*x]\*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(2/3)\*Sqrt[-Tan[c + d\*x]^2])/(2\*b^2\*d)

**Maple [F]** time = 0.087, size = 0, normalized size = 0.

$$\int (\sec(dx + c))^2 (b \sec(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^2/(b\*sec(d\*x+c))^(4/3), x)

[Out] int(sec(d\*x+c)^2/(b\*sec(d\*x+c))^(4/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^2}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^2/(b\*sec(d\*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate(sec(d\*x + c)^2/(b\*sec(d\*x + c))^(4/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec(dx + c))^{\frac{2}{3}}}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^2/(b\*sec(d\*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(2/3)/b^2, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c + dx)}{(b \sec(c + dx))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*2/(b\*sec(d\*x+c))\*\*(4/3),x)

[Out] Integral(sec(c + d\*x)\*\*2/(b\*sec(c + d\*x))\*\*(4/3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b \sec(dx+c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

$$3.200 \quad \int \frac{\sec(c+dx)}{(b \sec(c+dx))^{4/3}} dx$$

**Optimal.** Leaf size=55

$$\frac{3 \sin(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c+dx)\right)}{4d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{4/3}}$$

[Out] (-3\*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d\*x]^2]\*Sin[c + d\*x])/(4\*d\*(b\*Sec[c + d\*x])^(4/3)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.02957, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {16, 3772, 2643}

$$\frac{3 \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right)}{4d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]/(b\*Sec[c + d\*x])^(4/3), x]

[Out] (-3\*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d\*x]^2]\*Sin[c + d\*x])/(4\*d\*(b\*Sec[c + d\*x])^(4/3)\*Sqrt[Sin[c + d\*x]^2])

### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 3772

Int[(csc[(c\_)+(d\_)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := Simp[(b\*Csc[c+d\*x])^(n-1)\*((Sin[c+d\*x]/b)^(n-1)\*Int[1/(Sin[c+d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 2643

Int[((b\_)\*sin[(c\_)+(d\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c+d\*x]\*(b\*Sin[c+d\*x])^(n+1)\*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c

$+ d*x]^2] / (b*d*(n + 1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}[\{b, c, d, n\}, x]$   
 $\&\& \text{IntegerQ}[2*n]$

### Rubi steps

$$\begin{aligned} \int \frac{\sec(c+dx)}{(b \sec(c+dx))^{4/3}} dx &= \frac{\int \frac{1}{\sqrt[3]{b \sec(c+dx)}} dx}{b} \\ &= \frac{\left( \left( \frac{\cos(c+dx)}{b} \right)^{2/3} (b \sec(c+dx))^{2/3} \right) \int \sqrt[3]{\frac{\cos(c+dx)}{b}} dx}{b} \\ &= -\frac{3 \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right) (b \sec(c+dx))^{2/3} \sin(c+dx)}{4b^2 d \sqrt{\sin^2(c+dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0151685, size = 58, normalized size = 1.05

$$-\frac{3\sqrt{-\tan^2(c+dx)} \cot(c+dx) \text{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \sec^2(c+dx)\right)}{bd \sqrt[3]{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]/(b\*Sec[c + d\*x])^(4/3), x]

[Out] (-3\*Cot[c + d\*x]\*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d\*x]^2]\*Sqrt[-Tan[c + d\*x]^2])/(b\*d\*(b\*Sec[c + d\*x])^(1/3))

**Maple [F]** time = 0.085, size = 0, normalized size = 0.

$$\int \sec(dx+c) (b \sec(dx+c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)/(b\*sec(d\*x+c))^(4/3), x)

[Out] int(sec(d\*x+c)/(b\*sec(d\*x+c))^(4/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)/(b\*sec(d\*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate(sec(d\*x + c)/(b\*sec(d\*x + c))^(4/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec(dx + c))^{\frac{2}{3}}}{b^2 \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)/(b\*sec(d\*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(2/3)/(b^2\*sec(d\*x + c)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c + dx)}{(b \sec(c + dx))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)/(b\*sec(d\*x+c))\*\*(4/3),x)

[Out] Integral(sec(c + d\*x)/(b\*sec(c + d\*x))\*\*(4/3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)/(b\*sec(d\*x+c))^(4/3),x, algorithm="giac")

[Out] integrate(sec(d\*x + c)/(b\*sec(d\*x + c))^(4/3), x)

$$3.201 \quad \int \frac{1}{(b \sec(c+dx))^{4/3}} dx$$

**Optimal.** Leaf size=56

$$\frac{3b \sin(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c+dx)\right)}{7d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{7/3}}$$

[Out] (-3\*b\*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d\*x]^2]\*Sin[c + d\*x])/(7\*d\*(b\*Sec[c + d\*x])^(7/3)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0259363, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3772, 2643}

$$\frac{3b \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c+dx)\right)}{7d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^(-4/3), x]

[Out] (-3\*b\*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d\*x]^2]\*Sin[c + d\*x])/(7\*d\*(b\*Sec[c + d\*x])^(7/3)\*Sqrt[Sin[c + d\*x]^2])

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_.)])^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

#### Rubi steps



$$\int \frac{1}{(b \sec(c + dx))^{4/3}} dx = \left( \frac{\cos(c + dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \int \left( \frac{\cos(c + dx)}{b} \right)^{4/3} dx$$

$$= -\frac{3 \cos^3(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c + dx)\right) (b \sec(c + dx))^{2/3} \sin(c + dx)}{7b^2 d \sqrt{\sin^2(c + dx)}}$$

**Mathematica [A]** time = 0.0035631, size = 57, normalized size = 1.02

$$-\frac{3\sqrt{-\tan^2(c + dx) \cot(c + dx)} \operatorname{Hypergeometric2F1}\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \sec^2(c + dx)\right)}{4d(b \sec(c + dx))^{4/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(b\*Sec[c + d\*x])^(-4/3), x]

[Out] (-3\*Cot[c + d\*x]\*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[c + d\*x]^2]\*Sqrt[-Tan[c + d\*x]^2])/(4\*d\*(b\*Sec[c + d\*x])^(4/3))

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*sec(d\*x+c))^(4/3), x)

[Out] int(1/(b\*sec(d\*x+c))^(4/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(b \sec(dx + c))^{\frac{2}{3}}}{b^2 \sec(dx + c)^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(c + dx))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*sec(c + d*x))**(-4/3), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3), x)
```

$$3.202 \quad \int \frac{\cos(c+dx)}{(b \sec(c+dx))^{4/3}} dx$$

**Optimal.** Leaf size=58

$$\frac{3b^2 \sin(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c+dx)\right)}{10d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{10/3}}$$

[Out]  $(-3*b^2*\operatorname{Hypergeometric2F1}[1/2, 5/3, 8/3, \operatorname{Cos}[c + d*x]^2]*\operatorname{Sin}[c + d*x])/(10*d*(b*\operatorname{Sec}[c + d*x])^{(10/3)}*\operatorname{Sqrt}[\operatorname{Sin}[c + d*x]^2])$

**Rubi [A]** time = 0.0380579, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {16, 3772, 2643}

$$\frac{3b^2 \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c+dx)\right)}{10d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cos}[c + d*x]/(b*\operatorname{Sec}[c + d*x])^{(4/3)}, x]$

[Out]  $(-3*b^2*\operatorname{Hypergeometric2F1}[1/2, 5/3, 8/3, \operatorname{Cos}[c + d*x]^2]*\operatorname{Sin}[c + d*x])/(10*d*(b*\operatorname{Sec}[c + d*x])^{(10/3)}*\operatorname{Sqrt}[\operatorname{Sin}[c + d*x]^2])$

### Rule 16

$\operatorname{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[1/b^m, \operatorname{Int}[u*(b*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{b, n\}, x \ \&\& \ \operatorname{IntegerQ}[m]$

### Rule 3772

$\operatorname{Int}[(\operatorname{csc}[(c_*) + (d_*)*(x_*)])*(b_*)^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(b*\operatorname{Csc}[c + d*x])^{(n-1)}*((\operatorname{Sin}[c + d*x]/b)^{(n-1)}*\operatorname{Int}[1/(\operatorname{Sin}[c + d*x]/b)^n, x]), x] /; \operatorname{FreeQ}\{b, c, d, n\}, x \ \&\& \ !\operatorname{IntegerQ}[n]$

### Rule 2643

$\operatorname{Int}[(b_*)*\operatorname{sin}[(c_*) + (d_*)*(x_*)]^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Cos}[c + d*x]*(b*\operatorname{Sin}[c + d*x])^{(n+1)}*\operatorname{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \operatorname{Sin}[c$

+ d\*x]^2))/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x]  
&& !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \frac{\cos(c + dx)}{(b \sec(c + dx))^{4/3}} dx &= b \int \frac{1}{(b \sec(c + dx))^{7/3}} dx \\ &= \left( b \left( \frac{\cos(c + dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \right) \int \left( \frac{\cos(c + dx)}{b} \right)^{7/3} dx \\ &= -\frac{3 \cos^4(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c + dx)\right) (b \sec(c + dx))^{2/3} \sin(c + dx)}{10b^2 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0170913, size = 58, normalized size = 1.

$$\frac{3b\sqrt{-\tan^2(c + dx)} \cot(c + dx) \text{Hypergeometric2F1}\left(-\frac{7}{6}, \frac{1}{2}, -\frac{1}{6}, \sec^2(c + dx)\right)}{7d(b \sec(c + dx))^{7/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d\*x]/(b\*Sec[c + d\*x])^(4/3), x]

[Out] (-3\*b\*Cot[c + d\*x]\*Hypergeometric2F1[-7/6, 1/2, -1/6, Sec[c + d\*x]^2]\*Sqrt[-Tan[c + d\*x]^2])/(7\*d\*(b\*Sec[c + d\*x])^(7/3))

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int \cos(dx + c) (b \sec(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)/(b\*sec(d\*x+c))^(4/3), x)

[Out] int(cos(d\*x+c)/(b\*sec(d\*x+c))^(4/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c)}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)/(b\*sec(d\*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate(cos(d\*x + c)/(b\*sec(d\*x + c))^(4/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec(dx + c))^{\frac{2}{3}} \cos(dx + c)}{b^2 \sec(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)/(b\*sec(d\*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(2/3)\*cos(d\*x + c)/(b^2\*sec(d\*x + c)^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)/(b\*sec(d\*x+c))\*\*(4/3),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx + c)}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

$$3.203 \quad \int \frac{\cos^2(c+dx)}{(b \sec(c+dx))^{4/3}} dx$$

**Optimal.** Leaf size=58

$$\frac{3b^3 \sin(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{13}{6}, \frac{19}{6}, \cos^2(c+dx)\right)}{13d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{13/3}}$$

[Out]  $(-3*b^3*\operatorname{Hypergeometric2F1}[1/2, 13/6, 19/6, \operatorname{Cos}[c + d*x]^2]*\operatorname{Sin}[c + d*x])/(13*d*(b*\operatorname{Sec}[c + d*x])^{(13/3)}*\operatorname{Sqrt}[\operatorname{Sin}[c + d*x]^2])$

**Rubi [A]** time = 0.0472666, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {16, 3772, 2643}

$$\frac{3b^3 \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{13}{6}; \frac{19}{6}; \cos^2(c+dx)\right)}{13d \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{13/3}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cos}[c + d*x]^2/(b*\operatorname{Sec}[c + d*x])^{(4/3)}, x]$

[Out]  $(-3*b^3*\operatorname{Hypergeometric2F1}[1/2, 13/6, 19/6, \operatorname{Cos}[c + d*x]^2]*\operatorname{Sin}[c + d*x])/(13*d*(b*\operatorname{Sec}[c + d*x])^{(13/3)}*\operatorname{Sqrt}[\operatorname{Sin}[c + d*x]^2])$

### Rule 16

$\operatorname{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[1/b^m, \operatorname{Int}[u*(b*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{b, n\}, x] \ \&\& \ \operatorname{IntegerQ}[m]$

### Rule 3772

$\operatorname{Int}[(\operatorname{csc}[(c_*) + (d_*)*(x_*)])*(b_*)^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(b*\operatorname{Csc}[c + d*x])^{(n-1)}*((\operatorname{Sin}[c + d*x]/b)^{(n-1)}*\operatorname{Int}[1/(\operatorname{Sin}[c + d*x]/b)^n, x]), x] /; \operatorname{FreeQ}\{b, c, d, n\}, x] \ \&\& \ !\operatorname{IntegerQ}[n]$

### Rule 2643

$\operatorname{Int}[(b_*)*\operatorname{sin}[(c_*) + (d_*)*(x_*)]^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Cos}[c + d*x])*(b*\operatorname{Sin}[c + d*x])^{(n+1)}*\operatorname{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \operatorname{Sin}[c + d*x]]^{(n+1)}], x]$

+ d\*x]^2)]/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x]  
&& !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \frac{\cos^2(c + dx)}{(b \sec(c + dx))^{4/3}} dx &= b^2 \int \frac{1}{(b \sec(c + dx))^{10/3}} dx \\ &= \left( b^2 \left( \frac{\cos(c + dx)}{b} \right)^{2/3} (b \sec(c + dx))^{2/3} \right) \int \left( \frac{\cos(c + dx)}{b} \right)^{10/3} dx \\ &= -\frac{3 \cos^5(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{13}{6}; \frac{19}{6}; \cos^2(c + dx)\right) (b \sec(c + dx))^{2/3} \sin(c + dx)}{13b^2 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0993077, size = 60, normalized size = 1.03

$$-\frac{3b^2 \sqrt{-\tan^2(c + dx)} \cot(c + dx) \text{Hypergeometric2F1}\left(-\frac{5}{3}, \frac{1}{2}, -\frac{2}{3}, \sec^2(c + dx)\right)}{10d(b \sec(c + dx))^{10/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d\*x]^2/(b\*Sec[c + d\*x])^(4/3), x]

[Out] (-3\*b^2\*Cot[c + d\*x]\*Hypergeometric2F1[-5/3, 1/2, -2/3, Sec[c + d\*x]^2]\*Sqrt[-Tan[c + d\*x]^2])/(10\*d\*(b\*Sec[c + d\*x])^(10/3))

**Maple [F]** time = 0.138, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^2 (b \sec(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)^2/(b\*sec(d\*x+c))^(4/3), x)

[Out] int(cos(d\*x+c)^2/(b\*sec(d\*x+c))^(4/3), x)



**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b \sec(dx+c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^2/(b\*sec(d\*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate(cos(d\*x + c)^2/(b\*sec(d\*x + c))^(4/3), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec(dx+c))^{\frac{2}{3}} \cos(dx+c)^2}{b^2 \sec(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^2/(b\*sec(d\*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^(2/3)\*cos(d\*x + c)^2/(b^2\*sec(d\*x + c)^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*\*2/(b\*sec(d\*x+c))\*\*(4/3),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b \sec(dx+c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

### 3.204 $\int \sec^m(c + dx)(b \sec(c + dx))^{4/3} dx$

**Optimal.** Leaf size=81

$$\frac{3b \sin(c + dx) \sqrt[3]{b \sec(c + dx)} \sec^m(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(-3m - 1), \frac{1}{6}(5 - 3m), \cos^2(c + dx)\right)}{d(3m + 1) \sqrt{\sin^2(c + dx)}}$$

[Out] (3\*b\*Hypergeometric2F1[1/2, (-1 - 3\*m)/6, (5 - 3\*m)/6, Cos[c + d\*x]^2]\*Sec[c + d\*x]^m\*(b\*Sec[c + d\*x])^(1/3)\*Sin[c + d\*x])/(d\*(1 + 3\*m)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0402926, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {20, 3772, 2643}

$$\frac{3b \sin(c + dx) \sqrt[3]{b \sec(c + dx)} \sec^m(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(-3m - 1); \frac{1}{6}(5 - 3m); \cos^2(c + dx)\right)}{d(3m + 1) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^m\*(b\*Sec[c + d\*x])^(4/3), x]

[Out] (3\*b\*Hypergeometric2F1[1/2, (-1 - 3\*m)/6, (5 - 3\*m)/6, Cos[c + d\*x]^2]\*Sec[c + d\*x]^m\*(b\*Sec[c + d\*x])^(1/3)\*Sin[c + d\*x])/(d\*(1 + 3\*m)\*Sqrt[Sin[c + d\*x]^2])

#### Rule 20

Int[(u\_)\*((a\_)\*(v\_))^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/((Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \sec^m(c + dx)(b \sec(c + dx))^{4/3} dx &= \frac{(b \sqrt[3]{b \sec(c + dx)}) \int \sec^{\frac{4}{3}+m}(c + dx) dx}{\sqrt[3]{\sec(c + dx)}} \\ &= \left( b \cos^{\frac{1}{3}+m}(c + dx) \sec^m(c + dx) \sqrt[3]{b \sec(c + dx)} \right) \int \cos^{-\frac{4}{3}-m}(c + dx) dx \\ &= \frac{3b {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(-1 - 3m); \frac{1}{6}(5 - 3m); \cos^2(c + dx)\right) \sec^m(c + dx) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{d(1 + 3m)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.092348, size = 83, normalized size = 1.02

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) (b \sec(c + dx))^{4/3} \sec^{m-1}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}\left(m + \frac{4}{3}\right), \frac{1}{2}\left(m + \frac{10}{3}\right), \sec^2(c + dx)\right)}{d\left(m + \frac{4}{3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3), x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (4/3 + m)/2, (10/3 + m)/2, Sec[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(d*(4/3 + m))
```

**Maple [F]** time = 0.102, size = 0, normalized size = 0.

$$\int (\sec(dx + c))^m (b \sec(dx + c))^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x)`

[Out] `int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{4}{3}} \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c)^m, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec(dx + c))^{\frac{1}{3}} b \sec(dx + c)^m \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x, algorithm="fricas")`

[Out] `integral((b*sec(d*x + c))^(1/3)*b*sec(d*x + c)^m*sec(d*x + c), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(4/3),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{4}{3}} \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c)^m, x)
```

### 3.205 $\int \sec^m(c + dx)(b \sec(c + dx))^{2/3} dx$

**Optimal.** Leaf size=82

$$\frac{3 \sin(c + dx)(b \sec(c + dx))^{2/3} \sec^{m-1}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(1 - 3m), \frac{1}{6}(7 - 3m), \cos^2(c + dx)\right)}{d(1 - 3m)\sqrt{\sin^2(c + dx)}}$$

[Out] (-3\*Hypergeometric2F1[1/2, (1 - 3\*m)/6, (7 - 3\*m)/6, Cos[c + d\*x]^2]\*Sec[c + d\*x]^(-1 + m)\*(b\*Sec[c + d\*x])^(2/3)\*Sin[c + d\*x])/(d\*(1 - 3\*m)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0386954, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {20, 3772, 2643}

$$\frac{3 \sin(c + dx)(b \sec(c + dx))^{2/3} \sec^{m-1}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(1 - 3m); \frac{1}{6}(7 - 3m); \cos^2(c + dx)\right)}{d(1 - 3m)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^m\*(b\*Sec[c + d\*x])^(2/3), x]

[Out] (-3\*Hypergeometric2F1[1/2, (1 - 3\*m)/6, (7 - 3\*m)/6, Cos[c + d\*x]^2]\*Sec[c + d\*x]^(-1 + m)\*(b\*Sec[c + d\*x])^(2/3)\*Sin[c + d\*x])/(d\*(1 - 3\*m)\*Sqrt[Sin[c + d\*x]^2])

#### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/((Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \sec^m(c + dx)(b \sec(c + dx))^{2/3} dx &= \frac{(b \sec(c + dx))^{2/3} \int \sec^{\frac{2}{3}+m}(c + dx) dx}{\sec^{\frac{2}{3}}(c + dx)} \\ &= \left( \cos^{\frac{2}{3}+m}(c + dx) \sec^m(c + dx)(b \sec(c + dx))^{2/3} \right) \int \cos^{-\frac{2}{3}-m}(c + dx) dx \\ &= -\frac{{}_3F_2\left(\frac{1}{2}, \frac{1}{6}(1-3m); \frac{1}{6}(7-3m); \cos^2(c + dx)\right) \sec^{-1+m}(c + dx)(b \sec(c + dx))^{2/3}}{d(1-3m)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.138423, size = 83, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx)(b \sec(c + dx))^{2/3} \sec^{m-1}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}\left(m + \frac{2}{3}\right), \frac{1}{2}\left(m + \frac{8}{3}\right), \sec^2(c + dx)\right)}{d\left(m + \frac{2}{3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3), x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (2/3 + m)/2, (8/3 + m)/2, Sec[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(d*(2/3 + m))
```

**Maple [F]** time = 0.094, size = 0, normalized size = 0.

$$\int (\sec(dx + c))^m (b \sec(dx + c))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x)`

[Out] `int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{2}{3}} \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sec(dx + c)\right)^{\frac{2}{3}} \sec(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x, algorithm="fricas")`

[Out] `integral((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(c + dx))^{\frac{2}{3}} \sec^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(2/3),x)`

[Out] `Integral((b*sec(c + d*x))**(2/3)*sec(c + d*x)**m, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{2}{3}} \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

### 3.206 $\int \sec^m(c + dx) \sqrt[3]{b \sec(c + dx)} dx$

**Optimal.** Leaf size=82

$$\frac{3 \sin(c + dx) \sqrt[3]{b \sec(c + dx)} \sec^{m-1}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(2 - 3m), \frac{1}{6}(8 - 3m), \cos^2(c + dx)\right)}{d(2 - 3m) \sqrt{\sin^2(c + dx)}}$$

[Out] (-3\*Hypergeometric2F1[1/2, (2 - 3\*m)/6, (8 - 3\*m)/6, Cos[c + d\*x]^2]\*Sec[c + d\*x]^(-1 + m)\*(b\*Sec[c + d\*x])^(1/3)\*Sin[c + d\*x])/(d\*(2 - 3\*m)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0406077, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {20, 3772, 2643}

$$\frac{3 \sin(c + dx) \sqrt[3]{b \sec(c + dx)} \sec^{m-1}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(2 - 3m); \frac{1}{6}(8 - 3m); \cos^2(c + dx)\right)}{d(2 - 3m) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^m\*(b\*Sec[c + d\*x])^(1/3), x]

[Out] (-3\*Hypergeometric2F1[1/2, (2 - 3\*m)/6, (8 - 3\*m)/6, Cos[c + d\*x]^2]\*Sec[c + d\*x]^(-1 + m)\*(b\*Sec[c + d\*x])^(1/3)\*Sin[c + d\*x])/(d\*(2 - 3\*m)\*Sqrt[Sin[c + d\*x]^2])

#### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/((Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \sec^m(c + dx) \sqrt[3]{b \sec(c + dx)} dx &= \frac{\sqrt[3]{b \sec(c + dx)} \int \sec^{\frac{1}{3}+m}(c + dx) dx}{\sqrt[3]{\sec(c + dx)}} \\ &= \left( \cos^{\frac{1}{3}+m}(c + dx) \sec^m(c + dx) \sqrt[3]{b \sec(c + dx)} \right) \int \cos^{-\frac{1}{3}-m}(c + dx) dx \\ &= -\frac{{}_3F_2\left(\frac{1}{2}, \frac{1}{6}(2-3m); \frac{1}{6}(8-3m); \cos^2(c + dx)\right) \sec^{-1+m}(c + dx) \sqrt[3]{b \sec(c + dx)} \sin(c + dx)}{d(2-3m)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.140574, size = 83, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sqrt[3]{b \sec(c + dx)} \sec^{m-1}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}\left(m + \frac{1}{3}\right), \frac{1}{2}\left(m + \frac{7}{3}\right), \sec^2(c + dx)\right)}{d\left(m + \frac{1}{3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3), x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (1/3 + m)/2, (7/3 + m)/2, Sec[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/(d*(1/3 + m))
```

**Maple [F]** time = 0.086, size = 0, normalized size = 0.

$$\int (\sec(dx + c))^m \sqrt[3]{b \sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x)`

[Out] `int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{1}{3}} \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sec(dx + c)\right)^{\frac{1}{3}} \sec(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x, algorithm="fricas")`

[Out] `integral((b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{b \sec(c + dx)} \sec^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(1/3),x)`

[Out] `Integral((b*sec(c + d*x))**(1/3)*sec(c + d*x)**m, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^{\frac{1}{3}} \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^m\*(b\*sec(d\*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^(1/3)\*sec(d\*x + c)^m, x)

$$3.207 \quad \int \frac{\sec^m(c+dx)}{\sqrt[3]{b \sec(c+dx)}} dx$$

**Optimal.** Leaf size=82

$$\frac{3 \sin(c+dx) \sec^{m-1}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(4-3m), \frac{1}{6}(10-3m), \cos^2(c+dx)\right)}{d(4-3m) \sqrt{\sin^2(c+dx)} \sqrt[3]{b \sec(c+dx)}}$$

[Out] (-3\*Hypergeometric2F1[1/2, (4 - 3\*m)/6, (10 - 3\*m)/6, Cos[c + d\*x]^2]\*Sec[c + d\*x]^(-1 + m)\*Sin[c + d\*x])/(d\*(4 - 3\*m)\*(b\*Sec[c + d\*x])^(1/3)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0390033, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {20, 3772, 2643}

$$\frac{3 \sin(c+dx) \sec^{m-1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(4-3m); \frac{1}{6}(10-3m); \cos^2(c+dx)\right)}{d(4-3m) \sqrt{\sin^2(c+dx)} \sqrt[3]{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^m/(b\*Sec[c + d\*x])^(1/3), x]

[Out] (-3\*Hypergeometric2F1[1/2, (4 - 3\*m)/6, (10 - 3\*m)/6, Cos[c + d\*x]^2]\*Sec[c + d\*x]^(-1 + m)\*Sin[c + d\*x])/(d\*(4 - 3\*m)\*(b\*Sec[c + d\*x])^(1/3)\*Sqrt[Sin[c + d\*x]^2])

### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n-1)\*((Sin[c + d\*x]/b)^(n-1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{\sec^m(c + dx)}{\sqrt[3]{b \sec(c + dx)}} dx &= \frac{\sqrt[3]{\sec(c + dx)} \int \sec^{-\frac{1}{3}+m}(c + dx) dx}{\sqrt[3]{b \sec(c + dx)}} \\ &= \frac{\left(\cos^{\frac{2}{3}+m}(c + dx) \sec^{1+m}(c + dx)\right) \int \cos^{\frac{1}{3}-m}(c + dx) dx}{\sqrt[3]{b \sec(c + dx)}} \\ &= -\frac{{}_3F_2\left(\frac{1}{2}, \frac{1}{6}(4 - 3m); \frac{1}{6}(10 - 3m); \cos^2(c + dx)\right) \sec^{-1+m}(c + dx) \sin(c + dx)}{d(4 - 3m) \sqrt[3]{b \sec(c + dx)} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.140716, size = 83, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sec^{m-1}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}\left(m - \frac{1}{3}\right), \frac{1}{2}\left(m + \frac{5}{3}\right), \sec^2(c + dx)\right)}{d\left(m - \frac{1}{3}\right) \sqrt[3]{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m/(b*Sec[c + d*x])^(1/3), x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (-1/3 + m)/2, (5/3 + m)/2, Sec[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*(-1/3 + m)*(b*Sec[c + d*x])^(1/3))
```

**Maple [F]** time = 0.082, size = 0, normalized size = 0.

$$\int (\sec(dx + c))^m \frac{1}{\sqrt[3]{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x)`

[Out] `int(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^m}{(b \sec(dx+c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")`

[Out] `integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(1/3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec(dx+c))^{\frac{2}{3}} \sec(dx+c)^m}{b \sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")`

[Out] `integral((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m/(b*sec(d*x + c)), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^m(c+dx)}{\sqrt[3]{b \sec(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**m/(b*sec(d*x+c))**(1/3),x)`

[Out] Integral(sec(c + d\*x)\*\*m/(b\*sec(c + d\*x))\*\*(1/3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^m}{(b \sec(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^m/(b\*sec(d\*x+c))^(1/3),x, algorithm="giac")

[Out] integrate(sec(d\*x + c)^m/(b\*sec(d\*x + c))^(1/3), x)

$$3.208 \quad \int \frac{\sec^m(c+dx)}{(b \sec(c+dx))^{2/3}} dx$$

**Optimal.** Leaf size=82

$$\frac{3 \sin(c+dx) \sec^{m-1}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(5-3m), \frac{1}{6}(11-3m), \cos^2(c+dx)\right)}{d(5-3m) \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{2/3}}$$

[Out] (-3\*Hypergeometric2F1[1/2, (5 - 3\*m)/6, (11 - 3\*m)/6, Cos[c + d\*x]^2]\*Sec[c + d\*x]^(-1 + m)\*Sin[c + d\*x])/(d\*(5 - 3\*m)\*(b\*Sec[c + d\*x])^(2/3)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0406255, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {20, 3772, 2643}

$$\frac{3 \sin(c+dx) \sec^{m-1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(5-3m); \frac{1}{6}(11-3m); \cos^2(c+dx)\right)}{d(5-3m) \sqrt{\sin^2(c+dx)} (b \sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^m/(b\*Sec[c + d\*x])^(2/3), x]

[Out] (-3\*Hypergeometric2F1[1/2, (5 - 3\*m)/6, (11 - 3\*m)/6, Cos[c + d\*x]^2]\*Sec[c + d\*x]^(-1 + m)\*Sin[c + d\*x])/(d\*(5 - 3\*m)\*(b\*Sec[c + d\*x])^(2/3)\*Sqrt[Sin[c + d\*x]^2])

### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n-1)\*((Sin[c + d\*x]/b)^(n-1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{\sec^m(c + dx)}{(b \sec(c + dx))^{2/3}} dx &= \frac{\sec^{\frac{2}{3}}(c + dx) \int \sec^{-\frac{2}{3}+m}(c + dx) dx}{(b \sec(c + dx))^{2/3}} \\ &= \frac{\left(\cos^{\frac{1}{3}+m}(c + dx) \sec^{1+m}(c + dx)\right) \int \cos^{\frac{2}{3}-m}(c + dx) dx}{(b \sec(c + dx))^{2/3}} \\ &= -\frac{{}_3F_1\left(\frac{1}{2}, \frac{1}{6}(5-3m); \frac{1}{6}(11-3m); \cos^2(c + dx)\right) \sec^{-1+m}(c + dx) \sin(c + dx)}{d(5-3m)(b \sec(c + dx))^{2/3} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.130089, size = 83, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sec^{m-1}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}\left(m - \frac{2}{3}\right), \frac{1}{2}\left(m + \frac{4}{3}\right), \sec^2(c + dx)\right)}{d\left(m - \frac{2}{3}\right)(b \sec(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m/(b*Sec[c + d*x])^(2/3), x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (-2/3 + m)/2, (4/3 + m)/2, Sec[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*(-2/3 + m)*(b*Sec[c + d*x])^(2/3))
```

**Maple [F]** time = 0.091, size = 0, normalized size = 0.

$$\int (\sec(dx + c))^m (b \sec(dx + c))^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x)`

[Out] `int(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^m}{(b \sec(dx+c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x, algorithm="maxima")`

[Out] `integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(2/3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec(dx+c))^{\frac{1}{3}} \sec(dx+c)^m}{b \sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x, algorithm="fricas")`

[Out] `integral((b*sec(d*x + c))^(1/3)*sec(d*x + c)^m/(b*sec(d*x + c)), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^m(c+dx)}{(b \sec(c+dx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**m/(b*sec(d*x+c))**(2/3),x)`

[Out] Integral(sec(c + d\*x)\*\*m/(b\*sec(c + d\*x))\*\*(2/3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^m}{(b \sec(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^m/(b\*sec(d\*x+c))^(2/3),x, algorithm="giac")

[Out] integrate(sec(d\*x + c)^m/(b\*sec(d\*x + c))^(2/3), x)

$$3.209 \quad \int \frac{\sec^m(c+dx)}{(b \sec(c+dx))^{4/3}} dx$$

**Optimal.** Leaf size=85

$$\frac{3 \sin(c+dx) \sec^{m-2}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(7-3m), \frac{1}{6}(13-3m), \cos^2(c+dx)\right)}{bd(7-3m) \sqrt{\sin^2(c+dx)} \sqrt[3]{b \sec(c+dx)}}$$

[Out] (-3\*Hypergeometric2F1[1/2, (7 - 3\*m)/6, (13 - 3\*m)/6, Cos[c + d\*x]^2]\*Sec[c + d\*x]^(-2 + m)\*Sin[c + d\*x])/(b\*d\*(7 - 3\*m)\*(b\*Sec[c + d\*x])^(1/3)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0429287, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {20, 3772, 2643}

$$\frac{3 \sin(c+dx) \sec^{m-2}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(7-3m); \frac{1}{6}(13-3m); \cos^2(c+dx)\right)}{bd(7-3m) \sqrt{\sin^2(c+dx)} \sqrt[3]{b \sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^m/(b\*Sec[c + d\*x])^(4/3), x]

[Out] (-3\*Hypergeometric2F1[1/2, (7 - 3\*m)/6, (13 - 3\*m)/6, Cos[c + d\*x]^2]\*Sec[c + d\*x]^(-2 + m)\*Sin[c + d\*x])/(b\*d\*(7 - 3\*m)\*(b\*Sec[c + d\*x])^(1/3)\*Sqrt[Sin[c + d\*x]^2])

### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n-1)\*((Sin[c + d\*x]/b)^(n-1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{\sec^m(c + dx)}{(b \sec(c + dx))^{4/3}} dx &= \frac{\sqrt[3]{\sec(c + dx)} \int \sec^{-\frac{4}{3}+m}(c + dx) dx}{b \sqrt[3]{b \sec(c + dx)}} \\ &= \frac{\left( \cos^{\frac{2}{3}+m}(c + dx) \sec^{1+m}(c + dx) \right) \int \cos^{\frac{4}{3}-m}(c + dx) dx}{b \sqrt[3]{b \sec(c + dx)}} \\ &= \frac{{}_3F_1\left(\frac{1}{2}, \frac{1}{6}(7 - 3m); \frac{1}{6}(13 - 3m); \cos^2(c + dx)\right) \sec^{-2+m}(c + dx) \sin(c + dx)}{bd(7 - 3m) \sqrt[3]{b \sec(c + dx)} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.183404, size = 83, normalized size = 0.98

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sec^{m-1}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}\left(m - \frac{4}{3}\right), \frac{1}{2}\left(m + \frac{2}{3}\right), \sec^2(c + dx)\right)}{d\left(m - \frac{4}{3}\right) (b \sec(c + dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m/(b*Sec[c + d*x])^(4/3), x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (-4/3 + m)/2, (2/3 + m)/2, Sec[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*(-4/3 + m)*(b*Sec[c + d*x])^(4/3))
```

**Maple [F]** time = 0.082, size = 0, normalized size = 0.

$$\int (\sec(dx + c))^m (b \sec(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x)`

[Out] `int(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^m}{(b \sec(dx+c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")`

[Out] `integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(4/3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec(dx+c))^{\frac{2}{3}} \sec(dx+c)^m}{b^2 \sec(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")`

[Out] `integral((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m/(b^2*sec(d*x + c)^2), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^m(c+dx)}{(b \sec(c+dx))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**m/(b*sec(d*x+c))**(4/3),x)`

[Out] Integral(sec(c + d\*x)\*\*m/(b\*sec(c + d\*x))\*\*(4/3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^m}{(b \sec(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^m/(b\*sec(d\*x+c))^(4/3),x, algorithm="giac")

[Out] integrate(sec(d\*x + c)^m/(b\*sec(d\*x + c))^(4/3), x)

### 3.210 $\int \sec^m(c + dx)(b \sec(c + dx))^n dx$

**Optimal.** Leaf size=89

$$\frac{\sin(c + dx) \sec^{m-1}(c + dx)(b \sec(c + dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-m - n + 1), \frac{1}{2}(-m - n + 3), \cos^2(c + dx)\right)}{d(-m - n + 1)\sqrt{\sin^2(c + dx)}}$$

```
[Out] -((Hypergeometric2F1[1/2, (1 - m - n)/2, (3 - m - n)/2, Cos[c + d*x]^2]*Sec
[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - m - n)*Sqrt[Sin
[c + d*x]^2]))
```

**Rubi [A]** time = 0.0385914, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {20, 3772, 2643}

$$\frac{\sin(c + dx) \sec^{m-1}(c + dx)(b \sec(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-m - n + 1); \frac{1}{2}(-m - n + 3); \cos^2(c + dx)\right)}{d(-m - n + 1)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^n,x]
```

```
[Out] -((Hypergeometric2F1[1/2, (1 - m - n)/2, (3 - m - n)/2, Cos[c + d*x]^2]*Sec
[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - m - n)*Sqrt[Sin
[c + d*x]^2]))
```

#### Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

#### Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/((Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \sec^m(c + dx)(b \sec(c + dx))^n dx &= (\sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{m+n}(c + dx) dx \\ &= (\cos^{m+n}(c + dx) \sec^m(c + dx)(b \sec(c + dx))^n) \int \cos^{-m-n}(c + dx) dx \\ &= -\frac{{}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1 - m - n); \frac{1}{2}(3 - m - n); \cos^2(c + dx)\right) \sec^{-1+m}(c + dx)(b \sec(c + dx))^n}{d(1 - m - n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0650672, size = 76, normalized size = 0.85

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sec^{m-1}(c + dx)(b \sec(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+n}{2}, \frac{1}{2}(m+n+2), \sec^2(c + dx)\right)}{d(m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (m + n)/2, (2 + m + n)/2, Sec[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(m + n))
```

**Maple [F]** time = 0.594, size = 0, normalized size = 0.

$$\int (\sec(dx + c))^m (b \sec(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^n,x)
```

[Out] `int(sec(d*x+c)^m*(b*sec(d*x+c))^n,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n,x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^n*sec(d*x + c)^m, x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec(dx + c))^n \sec(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n,x, algorithm="fricas")`

[Out] `integral((b*sec(d*x + c))^n*sec(d*x + c)^m, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(c + dx))^n \sec^m(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**m*(b*sec(d*x+c))**n,x)`

[Out] `Integral((b*sec(c + d*x))**n*sec(c + d*x)**m, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \sec(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

### 3.211 $\int \sec^2(c + dx)(b \sec(c + dx))^n dx$

**Optimal.** Leaf size=72

$$\frac{\sin(c + dx)(b \sec(c + dx))^{n+1} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-n - 1), \frac{1-n}{2}, \cos^2(c + dx)\right)}{bd(n + 1)\sqrt{\sin^2(c + dx)}}$$

[Out] (Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(1 + n)\*Sin[c + d\*x])/(b\*d\*(1 + n)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0442525, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {16, 3772, 2643}

$$\frac{\sin(c + dx)(b \sec(c + dx))^{n+1} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-n - 1); \frac{1-n}{2}; \cos^2(c + dx)\right)}{bd(n + 1)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^2\*(b\*Sec[c + d\*x])^n,x]

[Out] (Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(1 + n)\*Sin[c + d\*x])/(b\*d\*(1 + n)\*Sqrt[Sin[c + d\*x]^2])

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c

$+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]$   
 $\&\& \text{!IntegerQ}[2*n]$

### Rubi steps

$$\begin{aligned} \int \sec^2(c + dx)(b \sec(c + dx))^n dx &= \frac{\int (b \sec(c + dx))^{2+n} dx}{b^2} \\ &= \frac{\left(\left(\frac{\cos(c+dx)}{b}\right)^n (b \sec(c + dx))^n\right) \int \left(\frac{\cos(c+dx)}{b}\right)^{-2-n} dx}{b^2} \\ &= \frac{{}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-1 - n); \frac{1-n}{2}; \cos^2(c + dx)\right) (b \sec(c + dx))^{1+n} \sin(c + dx)}{bd(1 + n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.047058, size = 71, normalized size = 0.99

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sec(c + dx) (b \sec(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n+2}{2}, \frac{n+4}{2}, \sec^2(c + dx)\right)}{d(n + 2)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]^2\*(b\*Sec[c + d\*x])^n,x]

[Out] (Csc[c + d\*x]\*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sec[c + d\*x]^2]\*  
 Sec[c + d\*x]\*(b\*Sec[c + d\*x])^n\*Sqrt[-Tan[c + d\*x]^2])/(d\*(2 + n))

**Maple [F]** time = 0.328, size = 0, normalized size = 0.

$$\int (\sec(dx + c))^2 (b \sec(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)^2\*(b\*sec(d\*x+c))^n,x)

[Out] int(sec(d\*x+c)^2\*(b\*sec(d\*x+c))^n,x)



---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^2\*(b\*sec(d\*x+c))^n,x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^n\*sec(d\*x + c)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec(dx + c))^n \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)^2\*(b\*sec(d\*x+c))^n,x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^n\*sec(d\*x + c)^2, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(c + dx))^n \sec^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*\*2\*(b\*sec(d\*x+c))\*\*n,x)

[Out] Integral((b\*sec(c + d\*x))\*\*n\*sec(c + d\*x)\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^2, x)
```

### 3.212 $\int \sec(c + dx)(b \sec(c + dx))^n dx$

**Optimal.** Leaf size=61

$$\frac{\sin(c + dx)(b \sec(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{n}{2}, \frac{2-n}{2}, \cos^2(c + dx)\right)}{dn\sqrt{\sin^2(c + dx)}}$$

[Out] (Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^n \*Sin[c + d\*x])/(d\*n\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0366607, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {16, 3772, 2643}

$$\frac{\sin(c + dx)(b \sec(c + dx))^n {}_2F_1\left(\frac{1}{2}, -\frac{n}{2}; \frac{2-n}{2}; \cos^2(c + dx)\right)}{dn\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]\*(b\*Sec[c + d\*x])^n,x]

[Out] (Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^n \*Sin[c + d\*x])/(d\*n\*Sqrt[Sin[c + d\*x]^2])

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c

$+ d*x]^2] / (b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]$   
 $\&\& \text{IntegerQ}[2*n]$

### Rubi steps

$$\begin{aligned} \int \sec(c + dx)(b \sec(c + dx))^n dx &= \frac{\int (b \sec(c + dx))^{1+n} dx}{b} \\ &= \frac{\left(\frac{\cos(c+dx)}{b}\right)^n (b \sec(c + dx))^n \int \left(\frac{\cos(c+dx)}{b}\right)^{-1-n} dx}{b} \\ &= \frac{{}_2F_1\left(\frac{1}{2}, -\frac{n}{2}; \frac{2-n}{2}; \cos^2(c + dx)\right) (b \sec(c + dx))^n \sin(c + dx)}{dn \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0401088, size = 65, normalized size = 1.07

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) (b \sec(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n+1}{2}, \frac{n+3}{2}, \sec^2(c + dx)\right)}{d(n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d\*x]\*(b\*Sec[c + d\*x])^n,x]

[Out] (Csc[c + d\*x]\*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^n\*Sqrt[-Tan[c + d\*x]^2])/(d\*(1 + n))

**Maple [F]** time = 0.396, size = 0, normalized size = 0.

$$\int \sec(dx + c) (b \sec(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d\*x+c)\*(b\*sec(d\*x+c))^n,x)

[Out] int(sec(d\*x+c)\*(b\*sec(d\*x+c))^n,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*(b\*sec(d\*x+c))^n,x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^n\*sec(d\*x + c), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec(dx + c))^n \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*(b\*sec(d\*x+c))^n,x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^n\*sec(d\*x + c), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(c + dx))^n \sec(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d\*x+c)\*(b\*sec(d\*x+c))\*\*n,x)

[Out] Integral((b\*sec(c + d\*x))\*\*n\*sec(c + d\*x), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c), x)
```

### 3.213 $\int (b \sec(c + dx))^n dx$

**Optimal.** Leaf size=73

$$\frac{b \sin(c + dx)(b \sec(c + dx))^{n-1} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \cos^2(c + dx)\right)}{d(1-n)\sqrt{\sin^2(c + dx)}}$$

[Out]  $-\left(\frac{b \text{Hypergeometric2F1}\left[\frac{1}{2}, (1-n)/2, (3-n)/2, \text{Cos}[c + d*x]^2\right] * (b \text{Sec}[c + d*x])^{-1+n} * \text{Sin}[c + d*x]}{d*(1-n)*\text{Sqrt}[\text{Sin}[c + d*x]^2]}\right)$

**Rubi [A]** time = 0.0336746, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {3772, 2643}

$$\frac{b \sin(c + dx)(b \sec(c + dx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \cos^2(c + dx)\right)}{d(1-n)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^n,x]

[Out]  $-\left(\frac{b \text{Hypergeometric2F1}\left[\frac{1}{2}, (1-n)/2, (3-n)/2, \text{Cos}[c + d*x]^2\right] * (b \text{Sec}[c + d*x])^{-1+n} * \text{Sin}[c + d*x]}{d*(1-n)*\text{Sqrt}[\text{Sin}[c + d*x]^2]}\right)$

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n-1)\*((Sin[c + d\*x]/b)^(n-1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*SIN[c + d\*x])^(n+1)\*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d\*x]^2])/(b\*d\*(n+1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

#### Rubi steps

$$\int (b \sec(c + dx))^n dx = \left(\frac{\cos(c + dx)}{b}\right)^n (b \sec(c + dx))^n \int \left(\frac{\cos(c + dx)}{b}\right)^{-n} dx$$

$$= -\frac{\cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \cos^2(c + dx)\right) (b \sec(c + dx))^n \sin(c + dx)}{d(1-n)\sqrt{\sin^2(c + dx)}}$$

**Mathematica [A]** time = 0.0421139, size = 61, normalized size = 0.84

$$\frac{\sqrt{-\tan^2(c + dx)} \cot(c + dx) (b \sec(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n}{2}, \frac{n+2}{2}, \sec^2(c + dx)\right)}{dn}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(b\*Sec[c + d\*x])^n, x]

[Out] (Cot[c + d\*x]\*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^n\*Sqrt[-Tan[c + d\*x]^2])/(d\*n)

**Maple [F]** time = 0.243, size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*sec(d\*x+c))^n, x)

[Out] int((b\*sec(d\*x+c))^n, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((b\*sec(d\*x+c))^n,x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^n, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec(dx + c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^n,x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^n, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))\*\*n,x)

[Out] Integral((b\*sec(c + d\*x))\*\*n, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^n,x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^n, x)

### 3.214 $\int \cos(c + dx)(b \sec(c + dx))^n dx$

**Optimal.** Leaf size=75

$$\frac{b^2 \sin(c + dx)(b \sec(c + dx))^{n-2} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2-n}{2}, \frac{4-n}{2}, \cos^2(c + dx)\right)}{d(2-n)\sqrt{\sin^2(c + dx)}}$$

[Out]  $-\left((b^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(2-n)}{2}, \frac{(4-n)}{2}, \text{Cos}[c + d*x]^2\right] * (b \text{Sec}[c + d*x])^{-(2+n)} * \text{Sin}[c + d*x]\right) / (d * (2-n) * \text{Sqrt}[\text{Sin}[c + d*x]^2])\right)$

**Rubi [A]** time = 0.0492554, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {16, 3772, 2643}

$$\frac{b^2 \sin(c + dx)(b \sec(c + dx))^{n-2} {}_2F_1\left(\frac{1}{2}, \frac{2-n}{2}; \frac{4-n}{2}; \cos^2(c + dx)\right)}{d(2-n)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cos}[c + d*x] * (b \text{Sec}[c + d*x])^n, x]$

[Out]  $-\left((b^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(2-n)}{2}, \frac{(4-n)}{2}, \text{Cos}[c + d*x]^2\right] * (b \text{Sec}[c + d*x])^{-(2+n)} * \text{Sin}[c + d*x]\right) / (d * (2-n) * \text{Sqrt}[\text{Sin}[c + d*x]^2])\right)$

#### Rule 16

$\text{Int}[(u_.) * (v_.)^{(m_.)} * ((b_.) * (v_))^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u * (b*v)^{(m+n)}, x], x] /;$  FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3772

$\text{Int}[(\text{csc}[(c_.) + (d_.) * (x_)] * (b_.)^{(n_.)}), x\_Symbol] \rightarrow \text{Simp}[(b * \text{Csc}[c + d*x])^{(n-1)} * ((\text{Sin}[c + d*x]/b)^{(n-1)} * \text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /;$  FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

$\text{Int}[(b_.) * \text{sin}[(c_.) + (d_.) * (x_)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x] * (b * \text{Sin}[c + d*x])^{(n+1)} * \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(n+1)}{2}, \frac{(n+3)}{2}, \text{Sin}[c + d*x]^2\right]) / (d * (n+1) * \text{Sqrt}[\text{Sin}[c + d*x]^2]), x]$

+ d\*x]^2)]/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x]  
 && !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \cos(c + dx)(b \sec(c + dx))^n dx &= b \int (b \sec(c + dx))^{-1+n} dx \\ &= \left( b \left( \frac{\cos(c + dx)}{b} \right)^n (b \sec(c + dx))^n \right) \int \left( \frac{\cos(c + dx)}{b} \right)^{1-n} dx \\ &= \frac{\cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{2-n}{2}; \frac{4-n}{2}; \cos^2(c + dx)\right) (b \sec(c + dx))^n \sin(c + dx)}{d(2-n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0938096, size = 68, normalized size = 0.91

$$\frac{b\sqrt{-\tan^2(c + dx)} \cot(c + dx)(b \sec(c + dx))^{n-1} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n-1}{2}, \frac{n+1}{2}, \sec^2(c + dx)\right)}{d(n-1)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d\*x]\*(b\*Sec[c + d\*x])^n,x]

[Out] (b\*Cot[c + d\*x]\*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(-1 + n)\*Sqrt[-Tan[c + d\*x]^2])/(d\*(-1 + n))

**Maple [F]** time = 0.859, size = 0, normalized size = 0.

$$\int \cos(dx + c)(b \sec(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)\*(b\*sec(d\*x+c))^n,x)

[Out] int(cos(d\*x+c)\*(b\*sec(d\*x+c))^n,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*(b\*sec(d\*x+c))^n,x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^n\*cos(d\*x + c), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec(dx + c))^n \cos(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*(b\*sec(d\*x+c))^n,x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^n\*cos(d\*x + c), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(c + dx))^n \cos(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*(b\*sec(d\*x+c))\*\*n,x)

[Out] Integral((b\*sec(c + d\*x))\*\*n\*cos(c + d\*x), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*cos(d*x + c), x)
```

### 3.215 $\int \cos^2(c + dx)(b \sec(c + dx))^n dx$

**Optimal.** Leaf size=75

$$\frac{b^3 \sin(c + dx)(b \sec(c + dx))^{n-3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3-n}{2}, \frac{5-n}{2}, \cos^2(c + dx)\right)}{d(3-n)\sqrt{\sin^2(c + dx)}}$$

[Out] -((b^3\*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(-3 + n)\*Sin[c + d\*x])/(d\*(3 - n)\*Sqrt[Sin[c + d\*x]^2]))

**Rubi [A]** time = 0.0553529, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {16, 3772, 2643}

$$\frac{b^3 \sin(c + dx)(b \sec(c + dx))^{n-3} {}_2F_1\left(\frac{1}{2}, \frac{3-n}{2}; \frac{5-n}{2}; \cos^2(c + dx)\right)}{d(3-n)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d\*x]^2\*(b\*Sec[c + d\*x])^n,x]

[Out] -((b^3\*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^(-3 + n)\*Sin[c + d\*x])/(d\*(3 - n)\*Sqrt[Sin[c + d\*x]^2]))

#### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c

+ d\*x]^2)]/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x]  
 && !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \cos^2(c + dx)(b \sec(c + dx))^n dx &= b^2 \int (b \sec(c + dx))^{-2+n} dx \\ &= \left( b^2 \left( \frac{\cos(c + dx)}{b} \right)^n (b \sec(c + dx))^n \right) \int \left( \frac{\cos(c + dx)}{b} \right)^{2-n} dx \\ &= -\frac{\cos^3(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{3-n}{2}; \frac{5-n}{2}; \cos^2(c + dx)\right) (b \sec(c + dx))^n \sin(c + dx)}{d(3-n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0951624, size = 71, normalized size = 0.95

$$\frac{\cos^2(c + dx)\sqrt{-\tan^2(c + dx)} \cot(c + dx)(b \sec(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n-2}{2}, \frac{n}{2}, \sec^2(c + dx)\right)}{d(n-2)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d\*x]^2\*(b\*Sec[c + d\*x])^n,x]

[Out] (Cos[c + d\*x]^2\*Cot[c + d\*x]\*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^n\*Sqrt[-Tan[c + d\*x]^2])/(d\*(-2 + n))

**Maple [F]** time = 0.968, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^2 (b \sec(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)^2\*(b\*sec(d\*x+c))^n,x)

[Out] int(cos(d\*x+c)^2\*(b\*sec(d\*x+c))^n,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^2\*(b\*sec(d\*x+c))^n,x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^n\*cos(d\*x + c)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec(dx + c))^n \cos(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^2\*(b\*sec(d\*x+c))^n,x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^n\*cos(d\*x + c)^2, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(c + dx))^n \cos^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*\*2\*(b\*sec(d\*x+c))\*\*n,x)

[Out] Integral((b\*sec(c + d\*x))\*\*n\*cos(c + d\*x)\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*cos(d*x + c)^2, x)
```

### 3.216 $\int \cos^3(c + dx)(b \sec(c + dx))^n dx$

**Optimal.** Leaf size=75

$$\frac{b^4 \sin(c + dx)(b \sec(c + dx))^{n-4} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{4-n}{2}, \frac{6-n}{2}, \cos^2(c + dx)\right)}{d(4-n)\sqrt{\sin^2(c + dx)}}$$

[Out]  $-\left((b^4 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(4-n)}{2}, \frac{(6-n)}{2}, \text{Cos}[c + d*x]^2\right] * (b * \text{Sec}[c + d*x])^{-(4+n)} * \text{Sin}[c + d*x]\right) / (d * (4-n) * \text{Sqrt}[\text{Sin}[c + d*x]^2])\right)$

**Rubi [A]** time = 0.0558594, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {16, 3772, 2643}

$$\frac{b^4 \sin(c + dx)(b \sec(c + dx))^{n-4} {}_2F_1\left(\frac{1}{2}, \frac{4-n}{2}; \frac{6-n}{2}; \cos^2(c + dx)\right)}{d(4-n)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cos}[c + d*x]^3 * (b * \text{Sec}[c + d*x])^n, x]$

[Out]  $-\left((b^4 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(4-n)}{2}, \frac{(6-n)}{2}, \text{Cos}[c + d*x]^2\right] * (b * \text{Sec}[c + d*x])^{-(4+n)} * \text{Sin}[c + d*x]\right) / (d * (4-n) * \text{Sqrt}[\text{Sin}[c + d*x]^2])\right)$

#### Rule 16

$\text{Int}[(u_.) * (v_.)^{(m_.)} * ((b_.) * (v_))^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u * (b*v)^{(m+n)}, x], x] /;$  FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 3772

$\text{Int}[(\text{csc}[(c_.) + (d_.) * (x_)] * (b_.)^{(n_.)}), x\_Symbol] \rightarrow \text{Simp}[(b * \text{Csc}[c + d*x])^{(n-1)} * ((\text{Sin}[c + d*x]/b)^{(n-1)} * \text{Int}[1/(\text{Sin}[c + d*x]/b)^n, x]), x] /;$  FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

$\text{Int}[(b_.) * \text{sin}[(c_.) + (d_.) * (x_)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x] * (b * \text{Sin}[c + d*x])^{(n+1)} * \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(n+1)}{2}, \frac{(n+3)}{2}, \text{Sin}[c + d*x]^2\right]) / (d * (n+1) * \text{Sqrt}[\text{Sin}[c + d*x]^2]), x]$

+ d\*x]^2))/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x]  
 && !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \cos^3(c + dx)(b \sec(c + dx))^n dx &= b^3 \int (b \sec(c + dx))^{-3+n} dx \\ &= \left( b^3 \left( \frac{\cos(c + dx)}{b} \right)^n (b \sec(c + dx))^n \right) \int \left( \frac{\cos(c + dx)}{b} \right)^{3-n} dx \\ &= -\frac{\cos^4(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{4-n}{2}; \frac{6-n}{2}; \cos^2(c + dx)\right) (b \sec(c + dx))^n \sin(c + dx)}{d(4-n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.107992, size = 73, normalized size = 0.97

$$\frac{\cos^3(c + dx)\sqrt{-\tan^2(c + dx)} \cot(c + dx)(b \sec(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n-3}{2}, \frac{n-1}{2}, \sec^2(c + dx)\right)}{d(n-3)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d\*x]^3\*(b\*Sec[c + d\*x])^n,x]

[Out] (Cos[c + d\*x]^3\*Cot[c + d\*x]\*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n)/2, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^n\*Sqrt[-Tan[c + d\*x]^2])/(d\*(-3 + n))

**Maple [F]** time = 1.35, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^3 (b \sec(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d\*x+c)^3\*(b\*sec(d\*x+c))^n,x)

[Out] int(cos(d\*x+c)^3\*(b\*sec(d\*x+c))^n,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \cos(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^3\*(b\*sec(d\*x+c))^n,x, algorithm="maxima")

[Out] integrate((b\*sec(d\*x + c))^n\*cos(d\*x + c)^3, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec(dx + c))^n \cos(dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)^3\*(b\*sec(d\*x+c))^n,x, algorithm="fricas")

[Out] integral((b\*sec(d\*x + c))^n\*cos(d\*x + c)^3, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d\*x+c)\*\*3\*(b\*sec(d\*x+c))\*\*n,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \cos(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*cos(d*x + c)^3, x)
```

### 3.217 $\int \sec^{\frac{5}{2}}(c + dx)(b \sec(c + dx))^n dx$

**Optimal.** Leaf size=80

$$\frac{2 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)(b \sec(c + dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-2n - 3), \frac{1}{4}(1 - 2n), \cos^2(c + dx)\right)}{d(2n + 3)\sqrt{\sin^2(c + dx)}}$$

[Out] (2\*Hypergeometric2F1[1/2, (-3 - 2\*n)/4, (1 - 2\*n)/4, Cos[c + d\*x]^2]\*Sec[c + d\*x]^(3/2)\*(b\*Sec[c + d\*x])^n\*Sin[c + d\*x])/(d\*(3 + 2\*n)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0400818, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {20, 3772, 2643}

$$\frac{2 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)(b \sec(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(-2n - 3); \frac{1}{4}(1 - 2n); \cos^2(c + dx)\right)}{d(2n + 3)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(5/2)\*(b\*Sec[c + d\*x])^n,x]

[Out] (2\*Hypergeometric2F1[1/2, (-3 - 2\*n)/4, (1 - 2\*n)/4, Cos[c + d\*x]^2]\*Sec[c + d\*x]^(3/2)\*(b\*Sec[c + d\*x])^n\*Sin[c + d\*x])/(d\*(3 + 2\*n)\*Sqrt[Sin[c + d\*x]^2])

#### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n-1)\*((Sin[c + d\*x]/b)^(n-1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \sec^{\frac{5}{2}}(c + dx)(b \sec(c + dx))^n dx &= (\sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{\frac{5}{2}+n}(c + dx) dx \\ &= \left( \cos^{\frac{1}{2}+n}(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n \right) \int \cos^{-\frac{5}{2}-n}(c + dx) dx \\ &= \frac{{}_2F_1\left(\frac{1}{2}, \frac{1}{4}(-3 - 2n); \frac{1}{4}(1 - 2n); \cos^2(c + dx)\right) \sec^{\frac{3}{2}}(c + dx)(b \sec(c + dx))^n \sin(c + dx)}{d(3 + 2n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.103719, size = 81, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sec^{\frac{3}{2}}(c + dx) (b \sec(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}\left(n + \frac{5}{2}\right), \frac{1}{2}\left(n + \frac{9}{2}\right), \sec^2(c + dx)\right)}{d\left(n + \frac{5}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (5/2 + n)/2, (9/2 + n)/2, Sec[c + d*x]^2]*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(5/2 + n))
```

**Maple [F]** time = 0.12, size = 0, normalized size = 0.

$$\int (\sec(dx + c))^{\frac{5}{2}} (b \sec(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x)
```

[Out] `int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \sec(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^n*sec(d*x + c)^(5/2), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec(dx + c))^n \sec(dx + c)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x, algorithm="fricas")`

[Out] `integral((b*sec(d*x + c))^n*sec(d*x + c)^(5/2), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(5/2)*(b*sec(d*x+c))**n,x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \sec(dx + c)^{\frac{5}{2}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^(5/2), x)
```

### 3.218 $\int \sec^{\frac{3}{2}}(c + dx)(b \sec(c + dx))^n dx$

**Optimal.** Leaf size=80

$$\frac{2 \sin(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-2n - 1), \frac{1}{4}(3 - 2n), \cos^2(c + dx)\right)}{d(2n + 1) \sqrt{\sin^2(c + dx)}}$$

[Out] (2\*Hypergeometric2F1[1/2, (-1 - 2\*n)/4, (3 - 2\*n)/4, Cos[c + d\*x]^2]\*Sqrt[Sec[c + d\*x]]\*(b\*Sec[c + d\*x])^n\*Sin[c + d\*x])/(d\*(1 + 2\*n)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0382578, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {20, 3772, 2643}

$$\frac{2 \sin(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(-2n - 1); \frac{1}{4}(3 - 2n); \cos^2(c + dx)\right)}{d(2n + 1) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d\*x]^(3/2)\*(b\*Sec[c + d\*x])^n,x]

[Out] (2\*Hypergeometric2F1[1/2, (-1 - 2\*n)/4, (3 - 2\*n)/4, Cos[c + d\*x]^2]\*Sqrt[Sec[c + d\*x]]\*(b\*Sec[c + d\*x])^n\*Sin[c + d\*x])/(d\*(1 + 2\*n)\*Sqrt[Sin[c + d\*x]^2])

#### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n-1)\*((Sin[c + d\*x]/b)^(n-1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \sec^{\frac{3}{2}}(c + dx)(b \sec(c + dx))^n dx &= (\sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{\frac{3}{2}+n}(c + dx) dx \\ &= \left( \cos^{\frac{1}{2}+n}(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n \right) \int \cos^{-\frac{3}{2}-n}(c + dx) dx \\ &= \frac{{}_2F_1\left(\frac{1}{2}, \frac{1}{4}(-1 - 2n); \frac{1}{4}(3 - 2n); \cos^2(c + dx)\right) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n \sin(c + dx)}{d(1 + 2n) \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0922359, size = 81, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}\left(n + \frac{3}{2}\right), \frac{1}{2}\left(n + \frac{7}{2}\right), \sec^2(c + dx)\right)}{d\left(n + \frac{3}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (3/2 + n)/2, (7/2 + n)/2, Sec[c + d*x]^2]*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(3/2 + n))
```

**Maple [F]** time = 0.117, size = 0, normalized size = 0.

$$\int (\sec(dx + c))^{\frac{3}{2}} (b \sec(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x)
```

[Out] `int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec(dx + c))^n \sec(dx + c)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x, algorithm="fricas")`

[Out] `integral((b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**n,x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \sec(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)
```

### 3.219 $\int \sqrt{\sec(c + dx)}(b \sec(c + dx))^n dx$

**Optimal.** Leaf size=80

$$\frac{2 \sin(c + dx)(b \sec(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(1 - 2n), \frac{1}{4}(5 - 2n), \cos^2(c + dx)\right)}{d(1 - 2n)\sqrt{\sin^2(c + dx)}\sqrt{\sec(c + dx)}}$$

[Out] (-2\*Hypergeometric2F1[1/2, (1 - 2\*n)/4, (5 - 2\*n)/4, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^n\*Sin[c + d\*x])/(d\*(1 - 2\*n)\*Sqrt[Sec[c + d\*x]]\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0381516, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {20, 3772, 2643}

$$\frac{2 \sin(c + dx)(b \sec(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(1 - 2n); \frac{1}{4}(5 - 2n); \cos^2(c + dx)\right)}{d(1 - 2n)\sqrt{\sin^2(c + dx)}\sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[c + d\*x]]\*(b\*Sec[c + d\*x])^n,x]

[Out] (-2\*Hypergeometric2F1[1/2, (1 - 2\*n)/4, (5 - 2\*n)/4, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^n\*Sin[c + d\*x])/(d\*(1 - 2\*n)\*Sqrt[Sec[c + d\*x]]\*Sqrt[Sin[c + d\*x]^2])

#### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \sqrt{\sec(c + dx)} (b \sec(c + dx))^n dx &= (\sec^{-n}(c + dx) (b \sec(c + dx))^n) \int \sec^{\frac{1}{2}+n}(c + dx) dx \\ &= \left( \cos^{\frac{1}{2}+n}(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n \right) \int \cos^{-\frac{1}{2}-n}(c + dx) dx \\ &= \frac{{}_2F_1\left(\frac{1}{2}, \frac{1}{4}(1 - 2n); \frac{1}{4}(5 - 2n); \cos^2(c + dx)\right) (b \sec(c + dx))^n \sin(c + dx)}{d(1 - 2n) \sqrt{\sec(c + dx)} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.101078, size = 81, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) (b \sec(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}\left(n + \frac{1}{2}\right), \frac{1}{2}\left(n + \frac{5}{2}\right), \sec^2(c + dx)\right)}{d\left(n + \frac{1}{2}\right) \sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (1/2 + n)/2, (5/2 + n)/2, Sec[c + d*x]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(1/2 + n)*Sqrt[Sec[c + d*x]])
```

**Maple [F]** time = 0.128, size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*sec(d*x+c)^(1/2),x)
```

[Out] `int((b*sec(d*x+c))^n*sec(d*x+c)^(1/2),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \sqrt{\sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^n*sec(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( (b \sec(dx + c))^n \sqrt{\sec(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^n*sec(d*x+c)^(1/2),x, algorithm="fricas")`

[Out] `integral((b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(c + dx))^n \sqrt{\sec(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))**n*sec(d*x+c)**(1/2),x)`

[Out] `Integral((b*sec(c + d*x))**n*sqrt(sec(c + d*x)), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \sqrt{\sec(dx + c)} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

$$3.220 \quad \int \frac{(b \sec(c+dx))^n}{\sqrt{\sec(c+dx)}} dx$$

**Optimal.** Leaf size=80

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(3-2n), \frac{1}{4}(7-2n), \cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{3}{2}}(c+dx)}$$

[Out] (-2\*Hypergeometric2F1[1/2, (3 - 2\*n)/4, (7 - 2\*n)/4, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^n\*Sin[c + d\*x])/(d\*(3 - 2\*n)\*Sec[c + d\*x]^(3/2)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0389261, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {20, 3772, 2643}

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(3-2n); \frac{1}{4}(7-2n); \cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^n/Sqrt[Sec[c + d\*x]],x]

[Out] (-2\*Hypergeometric2F1[1/2, (3 - 2\*n)/4, (7 - 2\*n)/4, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^n\*Sin[c + d\*x])/(d\*(3 - 2\*n)\*Sec[c + d\*x]^(3/2)\*Sqrt[Sin[c + d\*x]^2])

### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n-1)\*((Sin[c + d\*x]/b)^(n-1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{(b \sec(c + dx))^n}{\sqrt{\sec(c + dx)}} dx &= (\sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{-\frac{1}{2}+n}(c + dx) dx \\ &= \left( \cos^{\frac{1}{2}+n}(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n \right) \int \cos^{\frac{1}{2}-n}(c + dx) dx \\ &= -\frac{{}_2F_1\left(\frac{1}{2}, \frac{1}{4}(3 - 2n); \frac{1}{4}(7 - 2n); \cos^2(c + dx)\right) (b \sec(c + dx))^n \sin(c + dx)}{d(3 - 2n) \sec^{\frac{3}{2}}(c + dx) \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.117239, size = 81, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) (b \sec(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}\left(n - \frac{1}{2}\right), \frac{1}{2}\left(n + \frac{3}{2}\right), \sec^2(c + dx)\right)}{d\left(n - \frac{1}{2}\right) \sec^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^n/Sqrt[Sec[c + d*x]], x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (-1/2 + n)/2, (3/2 + n)/2, Sec[c + d*x]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-1/2 + n)*Sec[c + d*x]^(3/2))
```

**Maple [F]** time = 0.128, size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n \frac{1}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x)
```

```
[Out] int((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx + c))^n}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec(dx + c))^n}{\sqrt{\sec(dx + c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(c + dx))^n}{\sqrt{\sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n/sqrt(sec(c + d*x)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx + c))^n}{\sqrt{\sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^n/sec(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^n/sqrt(sec(d\*x + c)), x)

$$3.221 \quad \int \frac{(b \sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=80

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(5-2n), \frac{1}{4}(9-2n), \cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)} \sec^{\frac{5}{2}}(c+dx)}$$

[Out] (-2\*Hypergeometric2F1[1/2, (5 - 2\*n)/4, (9 - 2\*n)/4, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^n\*Sin[c + d\*x])/(d\*(5 - 2\*n)\*Sec[c + d\*x]^(5/2)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.0398434, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {20, 3772, 2643}

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(5-2n); \frac{1}{4}(9-2n); \cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)} \sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^n/Sec[c + d\*x]^(3/2), x]

[Out] (-2\*Hypergeometric2F1[1/2, (5 - 2\*n)/4, (9 - 2\*n)/4, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^n\*Sin[c + d\*x])/(d\*(5 - 2\*n)\*Sec[c + d\*x]^(5/2)\*Sqrt[Sin[c + d\*x]^2])

### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_.)\*((b\_.)\*(v\_))^(n\_.), x\_Symbol] := Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_.), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n-1)\*((Sin[c + d\*x]/b)^(n-1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; Fr

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \frac{(b \sec(c + dx))^n}{\sec^{\frac{3}{2}}(c + dx)} dx &= (\sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{-\frac{3}{2}+n}(c + dx) dx \\ &= \left( \cos^{\frac{1}{2}+n}(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n \right) \int \cos^{\frac{3}{2}-n}(c + dx) dx \\ &= -\frac{{}_2F_1\left(\frac{1}{2}, \frac{1}{4}(5 - 2n); \frac{1}{4}(9 - 2n); \cos^2(c + dx)\right) (b \sec(c + dx))^n \sin(c + dx)}{d(5 - 2n) \sec^{\frac{5}{2}}(c + dx) \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.139254, size = 81, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) (b \sec(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}\left(n - \frac{3}{2}\right), \frac{1}{2}\left(n + \frac{1}{2}\right), \sec^2(c + dx)\right)}{d\left(n - \frac{3}{2}\right) \sec^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Sec[c + d\*x])^n/Sec[c + d\*x]^(3/2), x]

[Out] (Csc[c + d\*x]\*Hypergeometric2F1[1/2, (-3/2 + n)/2, (1/2 + n)/2, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^n\*Sqrt[-Tan[c + d\*x]^2])/(d\*(-3/2 + n)\*Sec[c + d\*x]^(5/2))

**Maple [F]** time = 0.12, size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n (\sec(dx + c))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x)`

[Out] `int((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx + c))^n}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec(dx + c))^n}{\sec(dx + c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x, algorithm="fricas")`

[Out] `integral((b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(c + dx))^n}{\sec^{\frac{3}{2}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))**n/sec(d*x+c)**(3/2),x)`



[Out] Integral((b\*sec(c + d\*x))\*\*n/sec(c + d\*x)\*\*(3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx + c))^n}{\sec(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^n/sec(d\*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^n/sec(d\*x + c)^(3/2), x)

$$3.222 \quad \int \frac{(b \sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)} dx$$

**Optimal.** Leaf size=80

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(7-2n), \frac{1}{4}(11-2n), \cos^2(c+dx)\right)}{d(7-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{7}{2}}(c+dx)}$$

[Out] (-2\*Hypergeometric2F1[1/2, (7 - 2\*n)/4, (11 - 2\*n)/4, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^n\*Sin[c + d\*x])/(d\*(7 - 2\*n)\*Sec[c + d\*x]^(7/2)\*Sqrt[Sin[c + d\*x]^2])

**Rubi [A]** time = 0.038628, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {20, 3772, 2643}

$$\frac{2 \sin(c+dx)(b \sec(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(7-2n); \frac{1}{4}(11-2n); \cos^2(c+dx)\right)}{d(7-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Sec[c + d\*x])^n/Sec[c + d\*x]^(5/2), x]

[Out] (-2\*Hypergeometric2F1[1/2, (7 - 2\*n)/4, (11 - 2\*n)/4, Cos[c + d\*x]^2]\*(b\*Sec[c + d\*x])^n\*Sin[c + d\*x])/(d\*(7 - 2\*n)\*Sec[c + d\*x]^(7/2)\*Sqrt[Sin[c + d\*x]^2])

### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] := Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(b\*Csc[c + d\*x])^(n-1)\*((Sin[c + d\*x]/b)^(n-1)\*Int[1/(Sin[c + d\*x]/b)^n, x]), x] /; Fr

eeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

### Rubi steps

$$\begin{aligned} \int \frac{(b \sec(c + dx))^n}{\sec^{\frac{5}{2}}(c + dx)} dx &= (\sec^{-n}(c + dx)(b \sec(c + dx))^n) \int \sec^{-\frac{5}{2}+n}(c + dx) dx \\ &= \left( \cos^{\frac{1}{2}+n}(c + dx) \sqrt{\sec(c + dx)} (b \sec(c + dx))^n \right) \int \cos^{\frac{5}{2}-n}(c + dx) dx \\ &= -\frac{{}_2F_1\left(\frac{1}{2}, \frac{1}{4}(7 - 2n); \frac{1}{4}(11 - 2n); \cos^2(c + dx)\right) (b \sec(c + dx))^n \sin(c + dx)}{d(7 - 2n) \sec^{\frac{7}{2}}(c + dx) \sqrt{\sin^2(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.180298, size = 81, normalized size = 1.01

$$\frac{\sqrt{-\tan^2(c + dx)} \csc(c + dx) (b \sec(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}\left(n - \frac{5}{2}\right), \frac{1}{2}\left(n - \frac{1}{2}\right), \sec^2(c + dx)\right)}{d\left(n - \frac{5}{2}\right) \sec^{\frac{7}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Sec[c + d\*x])^n/Sec[c + d\*x]^(5/2), x]

[Out] (Csc[c + d\*x]\*Hypergeometric2F1[1/2, (-5/2 + n)/2, (-1/2 + n)/2, Sec[c + d\*x]^2]\*(b\*Sec[c + d\*x])^n\*Sqrt[-Tan[c + d\*x]^2])/(d\*(-5/2 + n)\*Sec[c + d\*x]^(7/2))

**Maple [F]** time = 0.123, size = 0, normalized size = 0.

$$\int (b \sec(dx + c))^n (\sec(dx + c))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x)`

[Out] `int((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx + c))^n}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(d*x + c))^n/sec(d*x + c)^(5/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec(dx + c))^n}{\sec(dx + c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x, algorithm="fricas")`

[Out] `integral((b*sec(d*x + c))^n/sec(d*x + c)^(5/2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c))**n/sec(d*x+c)**(5/2),x)`

[Out] Timed out

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**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx + c))^n}{\sec(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*sec(d\*x+c))^n/sec(d\*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((b\*sec(d\*x + c))^n/sec(d\*x + c)^(5/2), x)

### 3.223 $\int (d \sec(a + bx))^{7/2} \sin(a + bx) dx$

Optimal. Leaf size=20

$$\frac{2d(d \sec(a + bx))^{5/2}}{5b}$$

[Out] (2\*d\*(d\*Sec[a + b\*x])^(5/2))/(5\*b)

**Rubi [A]** time = 0.0351707, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2622, 30}

$$\frac{2d(d \sec(a + bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] Int[(d\*Sec[a + b\*x])^(7/2)\*Sin[a + b\*x],x]

[Out] (2\*d\*(d\*Sec[a + b\*x])^(5/2))/(5\*b)

#### Rule 2622

Int[csc[(e\_.) + (f\_.)\*(x\_.)]^(n\_.)\*((a\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(m\_), x\_Symbol] :> Dist[1/(f\*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a\*Sec[e + f\*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int (d \sec(a + bx))^{7/2} \sin(a + bx) dx &= \frac{d \text{Subst}\left(\int x^{3/2} dx, x, d \sec(a + bx)\right)}{b} \\ &= \frac{2d(d \sec(a + bx))^{5/2}}{5b} \end{aligned}$$

**Mathematica [A]** time = 0.0545522, size = 20, normalized size = 1.

$$\frac{2d(d \sec(a + bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Sec[a + b\*x])^(7/2)\*Sin[a + b\*x],x]

[Out] (2\*d\*(d\*Sec[a + b\*x])^(5/2))/(5\*b)

**Maple [A]** time = 0.021, size = 17, normalized size = 0.9

$$\frac{2d}{5b} (d \sec(bx + a))^{5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*sec(b\*x+a))^(7/2)\*sin(b\*x+a),x)

[Out] 2/5\*d\*(d\*sec(b\*x+a))^(5/2)/b

**Maxima [A]** time = 1.1273, size = 31, normalized size = 1.55

$$\frac{2 \left( \frac{d}{\cos(bx+a)} \right)^{7/2} \cos(bx + a)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))^(7/2)\*sin(b\*x+a),x, algorithm="maxima")

[Out] 2/5\*(d/cos(b\*x + a))^(7/2)\*cos(b\*x + a)/b

**Fricas [A]** time = 1.63252, size = 66, normalized size = 3.3

$$\frac{2d^3 \sqrt{\frac{d}{\cos(bx+a)}}}{5b \cos(bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))^(7/2)\*sin(b\*x+a),x, algorithm="fricas")

[Out]  $2/5*d^3*\sqrt{d/\cos(b*x + a)}/(b*\cos(b*x + a)^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))\*\*(7/2)\*sin(b\*x+a),x)

[Out] Timed out

**Giac [B]** time = 1.27447, size = 45, normalized size = 2.25

$$\frac{2d^4\operatorname{sgn}(\cos(bx+a))}{5\sqrt{d\cos(bx+a)}b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))^(7/2)\*sin(b\*x+a),x, algorithm="giac")

[Out]  $2/5*d^4*\operatorname{sgn}(\cos(b*x + a))/(\sqrt{d*\cos(b*x + a)}*b*\cos(b*x + a)^2)$



### 3.224 $\int (d \sec(a + bx))^{5/2} \sin(a + bx) dx$

Optimal. Leaf size=20

$$\frac{2d(d \sec(a + bx))^{3/2}}{3b}$$

[Out] (2\*d\*(d\*Sec[a + b\*x])^(3/2))/(3\*b)

**Rubi [A]** time = 0.0359804, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2622, 30}

$$\frac{2d(d \sec(a + bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In] Int[(d\*Sec[a + b\*x])^(5/2)\*Sin[a + b\*x],x]

[Out] (2\*d\*(d\*Sec[a + b\*x])^(3/2))/(3\*b)

#### Rule 2622

Int[csc[(e\_.) + (f\_.)\*(x\_.)]^(n\_.)\*((a\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(m\_.), x\_Symbol] :> Dist[1/(f\*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a\*Sec[e + f\*x], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int (d \sec(a + bx))^{5/2} \sin(a + bx) dx &= \frac{d \text{Subst} \left( \int \sqrt{x} dx, x, d \sec(a + bx) \right)}{b} \\ &= \frac{2d(d \sec(a + bx))^{3/2}}{3b} \end{aligned}$$

**Mathematica [A]** time = 0.0412481, size = 20, normalized size = 1.

$$\frac{2d(d \sec(a + bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Sec[a + b\*x])^(5/2)\*Sin[a + b\*x],x]

[Out] (2\*d\*(d\*Sec[a + b\*x])^(3/2))/(3\*b)

**Maple [A]** time = 0.017, size = 17, normalized size = 0.9

$$\frac{2d}{3b} (d \sec(bx + a))^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*sec(b\*x+a))^(5/2)\*sin(b\*x+a),x)

[Out] 2/3\*d\*(d\*sec(b\*x+a))^(3/2)/b

**Maxima [A]** time = 1.14477, size = 31, normalized size = 1.55

$$\frac{2 \left( \frac{d}{\cos(bx+a)} \right)^{\frac{5}{2}} \cos(bx + a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))^(5/2)\*sin(b\*x+a),x, algorithm="maxima")

[Out] 2/3\*(d/cos(b\*x + a))^(5/2)\*cos(b\*x + a)/b

**Fricas [A]** time = 1.62174, size = 63, normalized size = 3.15

$$\frac{2d^2 \sqrt{\frac{d}{\cos(bx+a)}}}{3b \cos(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a),x, algorithm="fricas")`

[Out]  $2/3*d^2*\sqrt{d/\cos(b*x + a)}/(b*\cos(b*x + a))$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*sec(b*x+a))**(5/2)*sin(b*x+a),x)`

[Out] Timed out

**Giac [B]** time = 1.2794, size = 45, normalized size = 2.25

$$\frac{2d^3 \operatorname{sgn}(\cos(bx + a))}{3\sqrt{d} \cos(bx + a) b \cos(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a),x, algorithm="giac")`

[Out]  $2/3*d^3*\operatorname{sgn}(\cos(b*x + a))/(\sqrt{d*\cos(b*x + a)}*b*\cos(b*x + a))$

### 3.225 $\int (d \sec(a + bx))^{3/2} \sin(a + bx) dx$

**Optimal.** Leaf size=18

$$\frac{2d\sqrt{d \sec(a + bx)}}{b}$$

[Out] (2\*d\*Sqrt[d\*Sec[a + b\*x]])/b

**Rubi [A]** time = 0.0362843, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2622, 30}

$$\frac{2d\sqrt{d \sec(a + bx)}}{b}$$

Antiderivative was successfully verified.

[In] Int[(d\*Sec[a + b\*x])^(3/2)\*Sin[a + b\*x],x]

[Out] (2\*d\*Sqrt[d\*Sec[a + b\*x]])/b

#### Rule 2622

Int[csc[(e\_.) + (f\_.)\*(x\_)]^(n\_.)\*((a\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(m\_), x\_Symbol] :> Dist[1/(f\*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a\*Sec[e + f\*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int (d \sec(a + bx))^{3/2} \sin(a + bx) dx &= \frac{d \operatorname{Subst}\left(\int \frac{1}{\sqrt{x}} dx, x, d \sec(a + bx)\right)}{b} \\ &= \frac{2d\sqrt{d \sec(a + bx)}}{b} \end{aligned}$$

**Mathematica [A]** time = 0.032755, size = 18, normalized size = 1.

$$\frac{2d\sqrt{d\sec(a+bx)}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Sec[a + b\*x])^(3/2)\*Sin[a + b\*x],x]

[Out] (2\*d\*Sqrt[d\*Sec[a + b\*x]])/b

**Maple [A]** time = 0.018, size = 17, normalized size = 0.9

$$2 \frac{d\sqrt{d\sec(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*sec(b\*x+a))^(3/2)\*sin(b\*x+a),x)

[Out] 2\*d\*(d\*sec(b\*x+a))^(1/2)/b

**Maxima [A]** time = 1.1578, size = 31, normalized size = 1.72

$$\frac{2 \left( \frac{d}{\cos(bx+a)} \right)^{\frac{3}{2}} \cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))^(3/2)\*sin(b\*x+a),x, algorithm="maxima")

[Out] 2\*(d/cos(b\*x + a))^(3/2)\*cos(b\*x + a)/b

**Fricas [A]** time = 1.66254, size = 38, normalized size = 2.11

$$\frac{2d\sqrt{\frac{d}{\cos(bx+a)}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))^(3/2)\*sin(b\*x+a),x, algorithm="fricas")

[Out] 2\*d\*sqrt(d/cos(b\*x + a))/b

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))\*\*(3/2)\*sin(b\*x+a),x)

[Out] Timed out

**Giac [A]** time = 1.33896, size = 34, normalized size = 1.89

$$\frac{2d^2 \operatorname{sgn}(\cos(bx + a))}{\sqrt{d \cos(bx + a)}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))^(3/2)\*sin(b\*x+a),x, algorithm="giac")

[Out] 2\*d^2\*sgn(cos(b\*x + a))/(sqrt(d\*cos(b\*x + a))\*b)

### 3.226 $\int \sqrt{d \sec(a + bx)} \sin(a + bx) dx$

Optimal. Leaf size=18

$$-\frac{2d}{b\sqrt{d \sec(a + bx)}}$$

[Out]  $(-2*d)/(b*\text{Sqrt}[d*\text{Sec}[a + b*x]])$

**Rubi [A]** time = 0.032692, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2622, 30}

$$-\frac{2d}{b\sqrt{d \sec(a + bx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[d*\text{Sec}[a + b*x]]*\text{Sin}[a + b*x], x]$

[Out]  $(-2*d)/(b*\text{Sqrt}[d*\text{Sec}[a + b*x]])$

#### Rule 2622

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]^{(n_.)}*((a_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/(f*a^n), \text{Subst}[\text{Int}[x^{(m+n-1)} / (-1 + x^2/a^2)^{((n+1)/2)}, x], x, a*\text{Sec}[e + f*x]], x] /;$   $\text{FreeQ}\{a, e, f, m\}, x \ \&\& \ \text{IntegerQ}[(n+1)/2] \ \&\& \ \text{IntegerQ}[(m+1)/2] \ \&\& \ \text{LtQ}[0, m, n]$

#### Rule 30

$\text{Int}[(x_)^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[x^{(m+1)} / (m+1), x] /;$   $\text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned} \int \sqrt{d \sec(a + bx)} \sin(a + bx) dx &= \frac{d \text{Subst}\left(\int \frac{1}{x^{3/2}} dx, x, d \sec(a + bx)\right)}{b} \\ &= -\frac{2d}{b\sqrt{d \sec(a + bx)}} \end{aligned}$$

**Mathematica [A]** time = 0.0376551, size = 18, normalized size = 1.

$$-\frac{2d}{b\sqrt{d\sec(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*Sec[a + b\*x]]\*Sin[a + b\*x],x]

[Out] (-2\*d)/(b\*Sqrt[d\*Sec[a + b\*x]])

**Maple [A]** time = 0.025, size = 17, normalized size = 0.9

$$-2\frac{d}{b\sqrt{d\sec(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*sec(b\*x+a))^(1/2)\*sin(b\*x+a),x)

[Out] -2\*d/b/(d\*sec(b\*x+a))^(1/2)

**Maxima [A]** time = 1.05814, size = 31, normalized size = 1.72

$$-\frac{2\sqrt{\frac{d}{\cos(bx+a)}}\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))^(1/2)\*sin(b\*x+a),x, algorithm="maxima")

[Out] -2\*sqrt(d/cos(b\*x + a))\*cos(b\*x + a)/b

**Fricas [A]** time = 1.6582, size = 54, normalized size = 3.

$$-\frac{2\sqrt{\frac{d}{\cos(bx+a)}}\cos(bx+a)}{b}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*sec(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="fricas")`

[Out] `-2*sqrt(d/cos(b*x + a))*cos(b*x + a)/b`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \sec(a + bx)} \sin(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*sec(b*x+a))**(1/2)*sin(b*x+a),x)`

[Out] `Integral(sqrt(d*sec(a + b*x))*sin(a + b*x), x)`

**Giac [A]** time = 1.32468, size = 30, normalized size = 1.67

$$-\frac{2\sqrt{d \cos(bx + a)} \operatorname{sgn}(\cos(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*sec(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="giac")`

[Out] `-2*sqrt(d*cos(b*x + a))*sgn(cos(b*x + a))/b`

$$3.227 \quad \int \frac{\sin(a+bx)}{\sqrt{d \sec(a+bx)}} dx$$

**Optimal.** Leaf size=20

$$-\frac{2d}{3b(d \sec(a+bx))^{3/2}}$$

[Out]  $(-2*d)/(3*b*(d*\text{Sec}[a + b*x])^{(3/2)})$

**Rubi [A]** time = 0.0326505, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2622, 30}

$$-\frac{2d}{3b(d \sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sin}[a + b*x]/\text{Sqrt}[d*\text{Sec}[a + b*x]], x]$

[Out]  $(-2*d)/(3*b*(d*\text{Sec}[a + b*x])^{(3/2)})$

#### Rule 2622

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]^{(n_.)}*((a_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/(f*a^n), \text{Subst}[\text{Int}[x^{(m+n-1)}/(-1+x^2/a^2)^{((n+1)/2)}], x], x, a*\text{Sec}[e+f*x], x] /; \text{FreeQ}\{a, e, f, m, x\} \ \&\& \ \text{IntegerQ}[(n+1)/2] \ \&\& \ !(\text{IntegerQ}[(m+1)/2] \ \&\& \ \text{LtQ}[0, m, n])$

#### Rule 30

$\text{Int}[(x_)^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[x^{(m+1)}/(m+1), x] /; \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

#### Rubi steps

$$\int \frac{\sin(a + bx)}{\sqrt{d \sec(a + bx)}} dx = \frac{d \operatorname{Subst} \left( \int \frac{1}{x^{5/2}} dx, x, d \sec(a + bx) \right)}{b}$$

$$= -\frac{2d}{3b(d \sec(a + bx))^{3/2}}$$

**Mathematica [A]** time = 0.0529927, size = 20, normalized size = 1.

$$-\frac{2d}{3b(d \sec(a + bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*x]/Sqrt[d\*Sec[a + b\*x]], x]

[Out] (-2\*d)/(3\*b\*(d\*Sec[a + b\*x])^(3/2))

**Maple [A]** time = 0.025, size = 17, normalized size = 0.9

$$-\frac{2d}{3b} (d \sec(bx + a))^{-3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(b\*x+a)/(d\*sec(b\*x+a))^(1/2), x)

[Out] -2/3\*d/b/(d\*sec(b\*x+a))^(3/2)

**Maxima [A]** time = 1.18467, size = 31, normalized size = 1.55

$$-\frac{2 \cos(bx + a)}{3b \sqrt{\frac{d}{\cos(bx+a)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b\*x+a)/(d\*sec(b\*x+a))^(1/2), x, algorithm="maxima")

[Out]  $-2/3*\cos(b*x + a)/(b*\sqrt{d/\cos(b*x + a)})$

**Fricas [A]** time = 1.70856, size = 65, normalized size = 3.25

$$-\frac{2\sqrt{\frac{d}{\cos(bx+a)}}\cos(bx+a)^2}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(b*x+a)/(d*sec(b*x+a))^(1/2),x, algorithm="fricas")`

[Out]  $-2/3*\sqrt{d/\cos(b*x + a)}*\cos(b*x + a)^2/(b*d)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a + bx)}{\sqrt{d \sec(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(b*x+a)/(d*sec(b*x+a))**(1/2),x)`

[Out] `Integral(sin(a + b*x)/sqrt(d*sec(a + b*x)), x)`

**Giac [B]** time = 3.62161, size = 47, normalized size = 2.35

$$-\frac{2\sqrt{d\cos(bx+a)}|b|\cos(bx+a)\operatorname{sgn}(b)\operatorname{sgn}(\cos(bx+a))}{3b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(b*x+a)/(d*sec(b*x+a))^(1/2),x, algorithm="giac")`

[Out]  $-2/3*\sqrt{d*\cos(b*x + a)}*abs(b)*\cos(b*x + a)*sgn(b)*sgn(\cos(b*x + a))/(b^2*d)$

### 3.228 $\int (d \sec(a + bx))^{5/2} \sin^3(a + bx) dx$

Optimal. Leaf size=41

$$\frac{2d^3}{b\sqrt{d \sec(a + bx)}} + \frac{2d(d \sec(a + bx))^{3/2}}{3b}$$

[Out]  $(2*d^3)/(b*\text{Sqrt}[d*\text{Sec}[a + b*x]]) + (2*d*(d*\text{Sec}[a + b*x])^{(3/2)})/(3*b)$

**Rubi [A]** time = 0.048539, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2622, 14}

$$\frac{2d^3}{b\sqrt{d \sec(a + bx)}} + \frac{2d(d \sec(a + bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*\text{Sec}[a + b*x])^{(5/2)}*\text{Sin}[a + b*x]^3, x]$

[Out]  $(2*d^3)/(b*\text{Sqrt}[d*\text{Sec}[a + b*x]]) + (2*d*(d*\text{Sec}[a + b*x])^{(3/2)})/(3*b)$

#### Rule 2622

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]^{(n_.)}*((a_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/(f*a^n), \text{Subst}[\text{Int}[x^{(m+n-1)}/(-1+x^2/a^2)^{((n+1)/2)}, x], x, a*\text{Sec}[e+f*x]], x] /;$  FreeQ[{a, e, f, m}, x] && IntegerQ[(n+1)/2] && !(IntegerQ[(m+1)/2] && LtQ[0, m, n])

#### Rule 14

$\text{Int}[(u_)*((c_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /;$  FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_.)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

#### Rubi steps

$$\begin{aligned}
\int (d \sec(a + bx))^{5/2} \sin^3(a + bx) dx &= \frac{d^3 \operatorname{Subst} \left( \int \frac{-1 + \frac{x^2}{d^2}}{x^{3/2}} dx, x, d \sec(a + bx) \right)}{b} \\
&= \frac{d^3 \operatorname{Subst} \left( \int \left( -\frac{1}{x^{3/2}} + \frac{\sqrt{x}}{d^2} \right) dx, x, d \sec(a + bx) \right)}{b} \\
&= \frac{2d^3}{b\sqrt{d \sec(a + bx)}} + \frac{2d(d \sec(a + bx))^{3/2}}{3b}
\end{aligned}$$

**Mathematica [A]** time = 0.219513, size = 32, normalized size = 0.78

$$\frac{d(3 \cos(2(a + bx)) + 5)(d \sec(a + bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Sec[a + b\*x])^(5/2)\*Sin[a + b\*x]^3,x]

[Out] (d\*(5 + 3\*Cos[2\*(a + b\*x)])\*(d\*Sec[a + b\*x])^(3/2))/(3\*b)

**Maple [B]** time = 0.199, size = 357, normalized size = 8.7

$$-\frac{(-1 + \cos(bx + a)) \cos(bx + a)}{6b(\sin(bx + a))^2} \left( 12 \sqrt{\frac{\cos(bx + a)}{(\cos(bx + a) + 1)^2}} (\cos(bx + a))^3 + 12 \sqrt{\frac{\cos(bx + a)}{(\cos(bx + a) + 1)^2}} (\cos(bx + a))^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*sec(b\*x+a))^(5/2)\*sin(b\*x+a)^3,x)

[Out]  $-1/6/b*(-1+\cos(b*x+a))*(12*(-\cos(b*x+a)/(\cos(b*x+a)+1)^2)^{(1/2)}*\cos(b*x+a)^3+12*(-\cos(b*x+a)/(\cos(b*x+a)+1)^2)^{(1/2)}*\cos(b*x+a)^2-3*\cos(b*x+a)^2*\ln(-2*(-\cos(b*x+a)/(\cos(b*x+a)+1)^2)^{(1/2)}*\cos(b*x+a)^2-\cos(b*x+a)^2-2*(-\cos(b*x+a)/(\cos(b*x+a)+1)^2)^{(1/2)}+2*\cos(b*x+a)-1)/\sin(b*x+a)^2)+3*\cos(b*x+a)^2*\ln(-2*(2*(-\cos(b*x+a)/(\cos(b*x+a)+1)^2)^{(1/2)}*\cos(b*x+a)^2-\cos(b*x+a)^2-2*(-\cos(b*x+a)/(\cos(b*x+a)+1)^2)^{(1/2)}+2*\cos(b*x+a)-1)/\sin(b*x+a)^2)+4*\cos(b*x+a)*(-\cos(b*x+a)/(\cos(b*x+a)+1)^2)^{(1/2)}+4*(-\cos(b*x+a)/(\cos(b*x+a)+1)^2)^{(1/2)}$

$/2)) * \cos(b*x+a) * (d/\cos(b*x+a))^{5/2} / (-\cos(b*x+a) / (\cos(b*x+a)+1)^2)^{1/2} / \sin(b*x+a)^2$

**Maxima [A]** time = 1.09725, size = 49, normalized size = 1.2

$$\frac{2 \left( \frac{3d^2}{\sqrt{\frac{d}{\cos(bx+a)}}} + \left( \frac{d}{\cos(bx+a)} \right)^{\frac{3}{2}} \right) d}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))^(5/2)\*sin(b\*x+a)^3,x, algorithm="maxima")

[Out] 2/3\*(3\*d^2/sqrt(d/cos(b\*x + a)) + (d/cos(b\*x + a))^(3/2))\*d/b

**Fricas [A]** time = 1.65811, size = 97, normalized size = 2.37

$$\frac{2 \left( 3d^2 \cos(bx+a)^2 + d^2 \right) \sqrt{\frac{d}{\cos(bx+a)}}}{3b \cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))^(5/2)\*sin(b\*x+a)^3,x, algorithm="fricas")

[Out] 2/3\*(3\*d^2\*cos(b\*x + a)^2 + d^2)\*sqrt(d/cos(b\*x + a))/(b\*cos(b\*x + a))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))\*\*(5/2)\*sin(b\*x+a)\*\*3,x)

[Out] Timed out

---

**Giac [A]** time = 1.18904, size = 66, normalized size = 1.61

$$\frac{2 \left( 3 \sqrt{d \cos(bx + a)} d + \frac{d^2}{\sqrt{d \cos(bx + a)} \cos(bx + a)} \right) d \operatorname{sgn}(\cos(bx + a))}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))^(5/2)\*sin(b\*x+a)^3,x, algorithm="giac")

[Out] 2/3\*(3\*sqrt(d\*cos(b\*x + a))\*d + d^2/(sqrt(d\*cos(b\*x + a))\*cos(b\*x + a)))\*d\*sgn(cos(b\*x + a))/b



### 3.229 $\int (d \sec(a + bx))^{9/2} \sin^3(a + bx) dx$

**Optimal.** Leaf size=43

$$\frac{2d(d \sec(a + bx))^{7/2}}{7b} - \frac{2d^3(d \sec(a + bx))^{3/2}}{3b}$$

[Out]  $(-2*d^3*(d*\text{Sec}[a + b*x])^{(3/2)})/(3*b) + (2*d*(d*\text{Sec}[a + b*x])^{(7/2)})/(7*b)$

**Rubi [A]** time = 0.0485292, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2622, 14}

$$\frac{2d(d \sec(a + bx))^{7/2}}{7b} - \frac{2d^3(d \sec(a + bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*\text{Sec}[a + b*x])^{(9/2)}*\text{Sin}[a + b*x]^3, x]$

[Out]  $(-2*d^3*(d*\text{Sec}[a + b*x])^{(3/2)})/(3*b) + (2*d*(d*\text{Sec}[a + b*x])^{(7/2)})/(7*b)$

#### Rule 2622

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]^{(n_.)}*((a_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/(f*a^n), \text{Subst}[\text{Int}[x^{(m+n-1)} / (-1 + x^2/a^2)^{((n+1)/2)}, x], x, a*\text{Sec}[e + f*x]], x] /;$  FreeQ[{a, e, f, m}, x] && IntegerQ[(n+1)/2] && !(IntegerQ[(m+1)/2] && LtQ[0, m, n])

#### Rule 14

$\text{Int}[(u_)*((c_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /;$  FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_.)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

#### Rubi steps

$$\begin{aligned} \int (d \sec(a + bx))^{9/2} \sin^3(a + bx) dx &= \frac{d^3 \text{Subst} \left( \int \sqrt{x} \left( -1 + \frac{x^2}{d^2} \right) dx, x, d \sec(a + bx) \right)}{b} \\ &= \frac{d^3 \text{Subst} \left( \int \left( -\sqrt{x} + \frac{x^{5/2}}{d^2} \right) dx, x, d \sec(a + bx) \right)}{b} \\ &= -\frac{2d^3 (d \sec(a + bx))^{3/2}}{3b} + \frac{2d (d \sec(a + bx))^{7/2}}{7b} \end{aligned}$$

**Mathematica [A]** time = 0.102751, size = 42, normalized size = 0.98

$$-\frac{d^4 (7 \cos(2(a + bx)) + 1) \sec^3(a + bx) \sqrt{d \sec(a + bx)}}{21b}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Sec[a + b\*x])^(9/2)\*Sin[a + b\*x]^3,x]

[Out] -(d^4\*(1 + 7\*Cos[2\*(a + b\*x)])\*Sec[a + b\*x]^3\*Sqrt[d\*Sec[a + b\*x]])/(21\*b)

**Maple [A]** time = 0.149, size = 36, normalized size = 0.8

$$-\frac{(14 (\cos(bx + a))^2 - 6) \cos(bx + a) \left( \frac{d}{\cos(bx + a)} \right)^{\frac{9}{2}}}{21b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*sec(b\*x+a))^(9/2)\*sin(b\*x+a)^3,x)

[Out] -2/21/b\*(7\*cos(b\*x+a)^2-3)\*(d/cos(b\*x+a))^(9/2)\*cos(b\*x+a)

**Maxima [A]** time = 1.2052, size = 51, normalized size = 1.19

$$-\frac{2 \left( 7 d^2 \left( \frac{d}{\cos(bx+a)} \right)^{\frac{3}{2}} - 3 \left( \frac{d}{\cos(bx+a)} \right)^{\frac{7}{2}} \right) d}{21b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))^(9/2)\*sin(b\*x+a)^3,x, algorithm="maxima")

[Out] -2/21\*(7\*d^2\*(d/cos(b\*x + a))^(3/2) - 3\*(d/cos(b\*x + a))^(7/2))\*d/b

**Fricas [A]** time = 1.69515, size = 105, normalized size = 2.44

$$-\frac{2(7d^4 \cos(bx+a)^2 - 3d^4) \sqrt{\frac{d}{\cos(bx+a)}}}{21 b \cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))^(9/2)\*sin(b\*x+a)^3,x, algorithm="fricas")

[Out] -2/21\*(7\*d^4\*cos(b\*x + a)^2 - 3\*d^4)\*sqrt(d/cos(b\*x + a))/(b\*cos(b\*x + a)^3)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*sec(b\*x+a))\*\*(9/2)\*sin(b\*x+a)\*\*3,x)

[Out] Timed out

**Giac [A]** time = 1.3429, size = 66, normalized size = 1.53

$$-\frac{2(7d^5 \cos(bx+a)^2 - 3d^5) \operatorname{sgn}(\cos(bx+a))}{21 \sqrt{d \cos(bx+a)} b \cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(9/2)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2/21*(7*d^5*cos(b*x + a)^2 - 3*d^5)*sgn(cos(b*x + a))/(sqrt(d*cos(b*x + a))  
)*b*cos(b*x + a)^3
```

### 3.230 $\int (d \csc(a + bx))^{9/2} \sqrt{c \sec(a + bx)} dx$

**Optimal.** Leaf size=128

$$\frac{4d^4 \sqrt{\sin(2a + 2bx)} \operatorname{EllipticF}\left(a + bx - \frac{\pi}{4}, 2\right) \sqrt{c \sec(a + bx)} \sqrt{d \csc(a + bx)}}{7b} - \frac{4cd^3 (d \csc(a + bx))^{3/2}}{7b \sqrt{c \sec(a + bx)}} - \frac{2cd (d \csc(a + bx))^{7/2}}{7b \sqrt{c \sec(a + bx)}}$$

[Out]  $(-4*c*d^3*(d*\operatorname{Csc}[a + b*x])^{(3/2)})/(7*b*\operatorname{Sqrt}[c*\operatorname{Sec}[a + b*x]]) - (2*c*d*(d*\operatorname{Csc}[a + b*x])^{(7/2)})/(7*b*\operatorname{Sqrt}[c*\operatorname{Sec}[a + b*x]]) + (4*d^4*\operatorname{Sqrt}[d*\operatorname{Csc}[a + b*x]]*\operatorname{EllipticF}[a - \operatorname{Pi}/4 + b*x, 2]*\operatorname{Sqrt}[c*\operatorname{Sec}[a + b*x]]*\operatorname{Sqrt}[\operatorname{Sin}[2*a + 2*b*x]])/(7*b)$

**Rubi [A]** time = 0.203931, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2625, 2630, 2573, 2641}

$$\frac{4cd^3 (d \csc(a + bx))^{3/2}}{7b \sqrt{c \sec(a + bx)}} + \frac{4d^4 \sqrt{\sin(2a + 2bx)} F\left(a + bx - \frac{\pi}{4} \middle| 2\right) \sqrt{c \sec(a + bx)} \sqrt{d \csc(a + bx)}}{7b} - \frac{2cd (d \csc(a + bx))^{7/2}}{7b \sqrt{c \sec(a + bx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d*\operatorname{Csc}[a + b*x])^{(9/2)}*\operatorname{Sqrt}[c*\operatorname{Sec}[a + b*x]], x]$

[Out]  $(-4*c*d^3*(d*\operatorname{Csc}[a + b*x])^{(3/2)})/(7*b*\operatorname{Sqrt}[c*\operatorname{Sec}[a + b*x]]) - (2*c*d*(d*\operatorname{Csc}[a + b*x])^{(7/2)})/(7*b*\operatorname{Sqrt}[c*\operatorname{Sec}[a + b*x]]) + (4*d^4*\operatorname{Sqrt}[d*\operatorname{Csc}[a + b*x]]*\operatorname{EllipticF}[a - \operatorname{Pi}/4 + b*x, 2]*\operatorname{Sqrt}[c*\operatorname{Sec}[a + b*x]]*\operatorname{Sqrt}[\operatorname{Sin}[2*a + 2*b*x]])/(7*b)$

#### Rule 2625

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_)]*(a_.)^{(m_)}*((b_.)*\operatorname{sec}[(e_.) + (f_.)*(x_)]))^{(n_.)}, x\_Symbol] :> -\operatorname{Simp}[(a*b*(a*\operatorname{Csc}[e + f*x])^{(m - 1)}*(b*\operatorname{Sec}[e + f*x])^{(n - 1)})/(f*(m - 1)), x] + \operatorname{Dist}[(a^2*(m + n - 2))/(m - 1), \operatorname{Int}[(a*\operatorname{Csc}[e + f*x])^{(m - 2)}*(b*\operatorname{Sec}[e + f*x])^n, x], x] /;$  FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && IntegersQ[2\*m, 2\*n] && !GtQ[n, m]

#### Rule 2630

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_)]*(a_.)^{(m_)}*((b_.)*\operatorname{sec}[(e_.) + (f_.)*(x_)]))^{(n_.)}, x\_Symbol] :> \operatorname{Dist}[(a*\operatorname{Csc}[e + f*x])^m*(b*\operatorname{Sec}[e + f*x])^n*(a*\operatorname{Sin}[e + f*x])^m*(b*\operatorname{Cos}[e + f*x])^n, \operatorname{Int}[1/((a*\operatorname{Sin}[e + f*x])^m*(b*\operatorname{Cos}[e + f*x])^n), x],$

x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]

### Rule 2573

Int[1/(Sqrt[cos[(e\_.) + (f\_.)\*(x\_.)]\*(b\_.)]\*Sqrt[(a\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]]), x\_Symbol] := Dist[Sqrt[Sin[2\*e + 2\*f\*x]]/(Sqrt[a\*Sin[e + f\*x]]\*Sqrt[b\*Cos[e + f\*x]]), Int[1/Sqrt[Sin[2\*e + 2\*f\*x]], x], x] /; FreeQ[{a, b, e, f}, x]

### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] := Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned}
 \int (d \csc(a + bx))^{9/2} \sqrt{c \sec(a + bx)} dx &= -\frac{2cd(d \csc(a + bx))^{7/2}}{7b\sqrt{c \sec(a + bx)}} + \frac{1}{7} (6d^2) \int (d \csc(a + bx))^{5/2} \sqrt{c \sec(a + bx)} dx \\
 &= -\frac{4cd^3(d \csc(a + bx))^{3/2}}{7b\sqrt{c \sec(a + bx)}} - \frac{2cd(d \csc(a + bx))^{7/2}}{7b\sqrt{c \sec(a + bx)}} + \frac{1}{7} (4d^4) \int \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} dx \\
 &= -\frac{4cd^3(d \csc(a + bx))^{3/2}}{7b\sqrt{c \sec(a + bx)}} - \frac{2cd(d \csc(a + bx))^{7/2}}{7b\sqrt{c \sec(a + bx)}} + \frac{1}{7} (4d^4 \sqrt{c \cos(a + bx)} \sqrt{d \csc(a + bx)}) \\
 &= -\frac{4cd^3(d \csc(a + bx))^{3/2}}{7b\sqrt{c \sec(a + bx)}} - \frac{2cd(d \csc(a + bx))^{7/2}}{7b\sqrt{c \sec(a + bx)}} + \frac{1}{7} (4d^4 \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)}) \\
 &= -\frac{4cd^3(d \csc(a + bx))^{3/2}}{7b\sqrt{c \sec(a + bx)}} - \frac{2cd(d \csc(a + bx))^{7/2}}{7b\sqrt{c \sec(a + bx)}} + \frac{4d^4 \sqrt{d \csc(a + bx)} F\left(a - \frac{\pi}{4} + bx, 2\right)}{7b\sqrt{c \sec(a + bx)}}
 \end{aligned}$$

**Mathematica [C]** time = 1.48662, size = 122, normalized size = 0.95

$$\frac{2d^4 \cos(2(a + bx)) \cot(a + bx) \sqrt{c \sec(a + bx)} \sqrt{d \csc(a + bx)} \left( (\cos(2(a + bx)) - 2) \csc^4(a + bx) - 2(-\cot^2(a + bx))^{3/4} \sec(a + bx) \right)}{7b(c \sec^2(a + bx) - 2)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(9/2)\*Sqrt[c\*Sec[a + b\*x]],x]

[Out] (2\*d^4\*Cos[2\*(a + b\*x)]\*Cot[a + b\*x]\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]])\*((-2 + Cos[2\*(a + b\*x)])\*Csc[a + b\*x]^4 - 2\*(-Cot[a + b\*x]^2)^(3/4)\*Hyp

ergeometric2F1[1/2, 3/4, 3/2, Csc[a + b\*x]^2]\*Sec[a + b\*x]^2))/(7\*b\*(-2 + Csc[a + b\*x]^2))

**Maple [B]** time = 0.253, size = 542, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(9/2)\*(c\*sec(b\*x+a))^(1/2), x)

[Out] 
$$\begin{aligned} & -1/7/b*2^{(1/2)}*(4*\cos(b*x+a)^3*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}*EllipticF((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}, 1/2*2^{(1/2)}+4*\cos(b*x+a)^2*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}*EllipticF((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}, 1/2*2^{(1/2)})-4*\cos(b*x+a)*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}*EllipticF((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}, 1/2*2^{(1/2)})-4*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}*EllipticF((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}, 1/2*2^{(1/2)})-2*\cos(b*x+a)^3*2^{(1/2)}+3*\cos(b*x+a)*2^{(1/2)})*(d/\sin(b*x+a))^{(9/2)}*(c/\cos(b*x+a))^{(1/2)}*\sin(b*x+a) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc(bx + a))^{\frac{9}{2}} \sqrt{c \sec(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(9/2)\*(c\*sec(b\*x+a))^(1/2), x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(9/2)\*sqrt(c\*sec(b\*x + a)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} d^4 \csc(bx + a)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(9/2)\*(c\*sec(b\*x+a))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a))\*d^4\*csc(b\*x + a)^4, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(9/2)\*(c\*sec(b\*x+a))\*\*(1/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc(bx + a))^{\frac{9}{2}} \sqrt{c \sec(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(9/2)\*(c\*sec(b\*x+a))^(1/2),x, algorithm="giac")

[Out] integrate((d\*csc(b\*x + a))^(9/2)\*sqrt(c\*sec(b\*x + a)), x)



### 3.231 $\int (d \csc(a + bx))^{7/2} \sqrt{c \sec(a + bx)} dx$

**Optimal.** Leaf size=69

$$-\frac{8cd^3\sqrt{d \csc(a + bx)}}{5b\sqrt{c \sec(a + bx)}} - \frac{2cd(d \csc(a + bx))^{5/2}}{5b\sqrt{c \sec(a + bx)}}$$

[Out]  $(-8*c*d^3*\text{Sqrt}[d*\text{Csc}[a + b*x]])/(5*b*\text{Sqrt}[c*\text{Sec}[a + b*x]]) - (2*c*d*(d*\text{Csc}[a + b*x])^{5/2})/(5*b*\text{Sqrt}[c*\text{Sec}[a + b*x]])$

**Rubi [A]** time = 0.0970203, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {2625, 2619}

$$-\frac{8cd^3\sqrt{d \csc(a + bx)}}{5b\sqrt{c \sec(a + bx)}} - \frac{2cd(d \csc(a + bx))^{5/2}}{5b\sqrt{c \sec(a + bx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*\text{Csc}[a + b*x])^{7/2}*\text{Sqrt}[c*\text{Sec}[a + b*x]], x]$

[Out]  $(-8*c*d^3*\text{Sqrt}[d*\text{Csc}[a + b*x]])/(5*b*\text{Sqrt}[c*\text{Sec}[a + b*x]]) - (2*c*d*(d*\text{Csc}[a + b*x])^{5/2})/(5*b*\text{Sqrt}[c*\text{Sec}[a + b*x]])$

#### Rule 2625

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] :> -\text{Simp}[(a*b*(a*\text{Csc}[e + f*x])^{(m - 1)}*(b*\text{Sec}[e + f*x])^{(n - 1)})/(f*(m - 1)), x] + \text{Dist}[(a^2*(m + n - 2))/(m - 1), \text{Int}[(a*\text{Csc}[e + f*x])^{(m - 2)}*(b*\text{Sec}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, e, f, n\}, x] \&\& \text{GtQ}[m, 1] \&\& \text{IntegersQ}[2*m, 2*n] \&\& !\text{GtQ}[n, m]$

#### Rule 2619

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] :> \text{Simp}[(a*b*(a*\text{Csc}[e + f*x])^{(m - 1)}*(b*\text{Sec}[e + f*x])^{(n - 1)})/(f*(n - 1)), x] /; \text{FreeQ}\{a, b, e, f, m, n\}, x] \&\& \text{EqQ}[m + n - 2, 0] \&\& \text{NeQ}[n, 1]$

#### Rubi steps

$$\int (d \csc(a + bx))^{7/2} \sqrt{c \sec(a + bx)} dx = -\frac{2cd(d \csc(a + bx))^{5/2}}{5b\sqrt{c \sec(a + bx)}} + \frac{1}{5} (4d^2) \int (d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)} dx$$

$$= -\frac{8cd^3 \sqrt{d \csc(a + bx)}}{5b\sqrt{c \sec(a + bx)}} - \frac{2cd(d \csc(a + bx))^{5/2}}{5b\sqrt{c \sec(a + bx)}}$$

**Mathematica [A]** time = 0.116126, size = 56, normalized size = 0.81

$$-\frac{2d^3 \sqrt{c \sec(a + bx)} \sqrt{d \csc(a + bx)} (4 \cos(a + bx) + \cot(a + bx) \csc(a + bx))}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(7/2)\*Sqrt[c\*Sec[a + b\*x]], x]

[Out] (-2\*d^3\*Sqrt[d\*Csc[a + b\*x]]\*(4\*Cos[a + b\*x] + Cot[a + b\*x]\*Csc[a + b\*x])\*Sqrt[c\*Sec[a + b\*x]])/(5\*b)

**Maple [A]** time = 0.191, size = 54, normalized size = 0.8

$$\frac{(8 (\cos(bx + a))^2 - 10) \cos(bx + a) \sin(bx + a)}{5b} \left( \frac{d}{\sin(bx + a)} \right)^{7/2} \sqrt{\frac{c}{\cos(bx + a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(7/2)\*(c\*sec(b\*x+a))^(1/2), x)

[Out] 2/5/b\*(4\*cos(b\*x+a)^2-5)\*cos(b\*x+a)\*(d/sin(b\*x+a))^(7/2)\*(c/cos(b\*x+a))^(1/2)\*sin(b\*x+a)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc(bx + a))^{7/2} \sqrt{c \sec(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(7/2)\*(c\*sec(b\*x+a))^(1/2),x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(7/2)\*sqrt(c\*sec(b\*x + a)), x)

**Fricas [A]** time = 1.82622, size = 155, normalized size = 2.25

$$\frac{2 \left( 4 d^3 \cos(bx + a)^3 - 5 d^3 \cos(bx + a) \right) \sqrt{\frac{c}{\cos(bx+a)}} \sqrt{\frac{d}{\sin(bx+a)}}}{5 \left( b \cos(bx + a)^2 - b \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(7/2)\*(c\*sec(b\*x+a))^(1/2),x, algorithm="fricas")

[Out]  $-2/5*(4*d^3*\cos(b*x + a)^3 - 5*d^3*\cos(b*x + a))*\sqrt{c/\cos(b*x + a)}*\sqrt{d/\sin(b*x + a)}/(b*\cos(b*x + a)^2 - b)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(7/2)\*(c\*sec(b\*x+a))\*\*(1/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc(bx + a))^{\frac{7}{2}} \sqrt{c \sec(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(7/2)\*(c\*sec(b\*x+a))^(1/2),x, algorithm="giac")

[Out] integrate((d\*csc(b\*x + a))^(7/2)\*sqrt(c\*sec(b\*x + a)), x)

### 3.232 $\int (d \csc(a + bx))^{5/2} \sqrt{c \sec(a + bx)} dx$

**Optimal.** Leaf size=93

$$\frac{2d^2 \sqrt{\sin(2a + 2bx)} \operatorname{EllipticF}\left(a + bx - \frac{\pi}{4}, 2\right) \sqrt{c \sec(a + bx)} \sqrt{d \csc(a + bx)}}{3b} - \frac{2cd(d \csc(a + bx))^{3/2}}{3b \sqrt{c \sec(a + bx)}}$$

[Out]  $(-2*c*d*(d*\operatorname{Csc}[a + b*x])^{3/2})/(3*b*\operatorname{Sqrt}[c*\operatorname{Sec}[a + b*x]]) + (2*d^2*\operatorname{Sqrt}[d*\operatorname{Csc}[a + b*x]]*\operatorname{EllipticF}[a - \operatorname{Pi}/4 + b*x, 2]*\operatorname{Sqrt}[c*\operatorname{Sec}[a + b*x]]*\operatorname{Sqrt}[\operatorname{Sin}[2*a + 2*b*x]])/(3*b)$

**Rubi [A]** time = 0.143491, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2625, 2630, 2573, 2641}

$$\frac{2d^2 \sqrt{\sin(2a + 2bx)} F\left(a + bx - \frac{\pi}{4} \middle| 2\right) \sqrt{c \sec(a + bx)} \sqrt{d \csc(a + bx)}}{3b} - \frac{2cd(d \csc(a + bx))^{3/2}}{3b \sqrt{c \sec(a + bx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d*\operatorname{Csc}[a + b*x])^{5/2}*\operatorname{Sqrt}[c*\operatorname{Sec}[a + b*x]],x]$

[Out]  $(-2*c*d*(d*\operatorname{Csc}[a + b*x])^{3/2})/(3*b*\operatorname{Sqrt}[c*\operatorname{Sec}[a + b*x]]) + (2*d^2*\operatorname{Sqrt}[d*\operatorname{Csc}[a + b*x]]*\operatorname{EllipticF}[a - \operatorname{Pi}/4 + b*x, 2]*\operatorname{Sqrt}[c*\operatorname{Sec}[a + b*x]]*\operatorname{Sqrt}[\operatorname{Sin}[2*a + 2*b*x]])/(3*b)$

#### Rule 2625

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_)]*(a_.)^{(m_)}*((b_.)*\operatorname{sec}[(e_.) + (f_.)*(x_)]])^{(n_.)}, x\_Symbol] :> -\operatorname{Simp}[(a*b*(a*\operatorname{Csc}[e + f*x])^{(m - 1)}*(b*\operatorname{Sec}[e + f*x])^{(n - 1)})/(f*(m - 1)), x] + \operatorname{Dist}[(a^{2*(m + n - 2)})/(m - 1), \operatorname{Int}[(a*\operatorname{Csc}[e + f*x])^{(m - 2)}*(b*\operatorname{Sec}[e + f*x])^n, x], x] /; \operatorname{FreeQ}\{a, b, e, f, n\}, x] \&\& \operatorname{GtQ}[m, 1] \&\& \operatorname{IntegersQ}[2*m, 2*n] \&\& !\operatorname{GtQ}[n, m]$

#### Rule 2630

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_)]*(a_.)^{(m_)}*((b_.)*\operatorname{sec}[(e_.) + (f_.)*(x_)]])^{(n_.)}, x\_Symbol] :> \operatorname{Dist}[(a*\operatorname{Csc}[e + f*x])^m*(b*\operatorname{Sec}[e + f*x])^n*(a*\operatorname{Sin}[e + f*x])^m*(b*\operatorname{Cos}[e + f*x])^n, \operatorname{Int}[1/((a*\operatorname{Sin}[e + f*x])^m*(b*\operatorname{Cos}[e + f*x])^n), x], x] /; \operatorname{FreeQ}\{a, b, e, f, m, n\}, x] \&\& \operatorname{IntegerQ}[m - 1/2] \&\& \operatorname{IntegerQ}[n - 1/2]$

Rule 2573

Int[1/(Sqrt[cos[(e\_.) + (f\_.)\*(x\_.)]\*(b\_.)]\*Sqrt[(a\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]]), x\_Symbol] :> Dist[Sqrt[Sin[2\*e + 2\*f\*x]]/(Sqrt[a\*Sin[e + f\*x]]\*Sqrt[b\*Cos[e + f\*x]]), Int[1/Sqrt[Sin[2\*e + 2\*f\*x]], x], x] /; FreeQ[{a, b, e, f}, x]

Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] :> Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int (d \csc(a + bx))^{5/2} \sqrt{c \sec(a + bx)} dx &= -\frac{2cd(d \csc(a + bx))^{3/2}}{3b\sqrt{c \sec(a + bx)}} + \frac{1}{3} (2d^2) \int \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} dx \\
 &= -\frac{2cd(d \csc(a + bx))^{3/2}}{3b\sqrt{c \sec(a + bx)}} + \frac{1}{3} (2d^2 \sqrt{c \cos(a + bx)} \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)}) \\
 &= -\frac{2cd(d \csc(a + bx))^{3/2}}{3b\sqrt{c \sec(a + bx)}} + \frac{1}{3} (2d^2 \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} \sqrt{\sin(2a + 2bx)}) \\
 &= -\frac{2cd(d \csc(a + bx))^{3/2}}{3b\sqrt{c \sec(a + bx)}} + \frac{2d^2 \sqrt{d \csc(a + bx)} F\left(a - \frac{\pi}{4} + bx \middle| 2\right) \sqrt{c \sec(a + bx)} \sqrt{\sin(2a + 2bx)}}{3b}
 \end{aligned}$$

**Mathematica [C]** time = 0.85784, size = 109, normalized size = 1.17

$$\frac{d(\cos(a + bx) + \cos(3(a + bx))) \sec^2(a + bx) \sqrt{c \sec(a + bx)} (d \csc(a + bx))^{3/2} \left(-\cot^2(a + bx)\right)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \text{Csc}[a + bx]^2\right]}{3b(c \sec^2(a + bx) - 2)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(5/2)\*Sqrt[c\*Sec[a + b\*x]],x]

[Out] -(d\*(Cos[a + b\*x] + Cos[3\*(a + b\*x)])\*(d\*Csc[a + b\*x])^(3/2)\*(Cot[a + b\*x]^2 + (-Cot[a + b\*x]^2)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b\*x]^2])\*Sec[a + b\*x]^2\*Sqrt[c\*Sec[a + b\*x]])/(3\*b\*(-2 + Csc[a + b\*x]^2))

**Maple [B]** time = 0.2, size = 285, normalized size = 3.1

$$\frac{\sqrt{2} \sin (bx+a)}{3 b} \left( 2 \cos (bx+a) \sin (bx+a) \sqrt{\frac{-1+\cos (bx+a)-\sin (bx+a)}{\sin (bx+a)}} \sqrt{\frac{-1+\cos (bx+a)+\sin (bx+a)}{\sin (bx+a)}} \sqrt{\frac{-1-\cos (bx+a)+\sin (bx+a)}{\sin (bx+a)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(5/2)\*(c\*sec(b\*x+a))^(1/2),x)

[Out] 1/3/b\*2^(1/2)\*(2\*cos(b\*x+a)\*sin(b\*x+a)\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1/2\*2^(1/2))+2\*sin(b\*x+a)\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1/2\*2^(1/2))-cos(b\*x+a)\*2^(1/2)\*(d/sin(b\*x+a))^(5/2)\*(c/cos(b\*x+a))^(1/2)\*sin(b\*x+a)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc (bx+a))^{\frac{5}{2}} \sqrt{c \sec (bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(5/2)\*(c\*sec(b\*x+a))^(1/2),x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(5/2)\*sqrt(c\*sec(b\*x + a)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \sqrt{d \csc (bx+a)} \sqrt{c \sec (bx+a)} d^2 \csc (bx+a)^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(5/2)\*(c\*sec(b\*x+a))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a))\*d^2\*csc(b\*x + a)^2, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(5/2)\*(c\*sec(b\*x+a))\*\*(1/2), x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc (bx + a))^{\frac{5}{2}} \sqrt{c \sec (bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(5/2)\*(c\*sec(b\*x+a))^(1/2), x, algorithm="giac")

[Out] integrate((d\*csc(b\*x + a))^(5/2)\*sqrt(c\*sec(b\*x + a)), x)

$$3.233 \quad \int (d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)} dx$$

**Optimal.** Leaf size=31

$$-\frac{2cd\sqrt{d \csc(a + bx)}}{b\sqrt{c \sec(a + bx)}}$$

[Out]  $(-2*c*d*Sqrt[d*Csc[a + b*x]])/(b*Sqrt[c*Sec[a + b*x]])$

**Rubi [A]** time = 0.0486029, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {2619}

$$-\frac{2cd\sqrt{d \csc(a + bx)}}{b\sqrt{c \sec(a + bx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*Csc[a + b*x])^{(3/2)}*Sqrt[c*Sec[a + b*x]],x]$

[Out]  $(-2*c*d*Sqrt[d*Csc[a + b*x]])/(b*Sqrt[c*Sec[a + b*x]])$

**Rule 2619**

$\text{Int}[(\csc[e_.] + (f_.)*(x_))* (a_.)^{(m_)} * ((b_.) * \sec[e_.] + (f_.)*(x_))^{(n_)}], x\_Symbol] \rightarrow \text{Simp}[(a*b*(a*Csc[e + f*x])^{(m - 1)} * (b*Sec[e + f*x])^{(n - 1)}) / (f*(n - 1)), x] /;$  FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] && NeQ[n, 1]

**Rubi steps**

$$\int (d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)} dx = -\frac{2cd\sqrt{d \csc(a + bx)}}{b\sqrt{c \sec(a + bx)}}$$

**Mathematica [A]** time = 0.0633391, size = 31, normalized size = 1.

$$-\frac{2cd\sqrt{d \csc(a + bx)}}{b\sqrt{c \sec(a + bx)}}$$



Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(3/2)\*Sqrt[c\*Sec[a + b\*x]],x]

[Out] (-2\*c\*d\*Sqrt[d\*Csc[a + b\*x]])/(b\*Sqrt[c\*Sec[a + b\*x]])

**Maple [A]** time = 0.164, size = 42, normalized size = 1.4

$$-2 \frac{\cos(bx+a)\sin(bx+a)}{b} \left( \frac{d}{\sin(bx+a)} \right)^{3/2} \sqrt{\frac{c}{\cos(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(3/2)\*(c\*sec(b\*x+a))^(1/2),x)

[Out] -2/b\*cos(b\*x+a)\*sin(b\*x+a)\*(d/sin(b\*x+a))^(3/2)\*(c/cos(b\*x+a))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc(bx+a))^{\frac{3}{2}} \sqrt{c \sec(bx+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(3/2)\*(c\*sec(b\*x+a))^(1/2),x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(3/2)\*sqrt(c\*sec(b\*x + a)), x)

**Fricas [A]** time = 1.70862, size = 85, normalized size = 2.74

$$\frac{2d \sqrt{\frac{c}{\cos(bx+a)}} \sqrt{\frac{d}{\sin(bx+a)}} \cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(3/2)\*(c\*sec(b\*x+a))^(1/2),x, algorithm="fricas")

[Out]  $-2*d*\sqrt{c/\cos(b*x + a)}*\sqrt{d/\sin(b*x + a)}*\cos(b*x + a)/b$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))**(3/2)*(c*sec(b*x+a))**(1/2), x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc(bx + a))^{\frac{3}{2}} \sqrt{c \sec(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2), x, algorithm="giac")`

[Out] `integrate((d*csc(b*x + a))^(3/2)*sqrt(c*sec(b*x + a)), x)`

### 3.234 $\int \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} dx$

**Optimal.** Leaf size=53

$$\frac{\sqrt{\sin(2a + 2bx)} \operatorname{EllipticF}\left(a + bx - \frac{\pi}{4}, 2\right) \sqrt{c \sec(a + bx)} \sqrt{d \csc(a + bx)}}{b}$$

[Out] (Sqrt[d\*Csc[a + b\*x]]\*EllipticF[a - Pi/4 + b\*x, 2]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])/b

**Rubi [A]** time = 0.0927454, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$ , Rules used = {2630, 2573, 2641}

$$\frac{\sqrt{\sin(2a + 2bx)} F\left(a + bx - \frac{\pi}{4} \middle| 2\right) \sqrt{c \sec(a + bx)} \sqrt{d \csc(a + bx)}}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]],x]

[Out] (Sqrt[d\*Csc[a + b\*x]]\*EllipticF[a - Pi/4 + b\*x, 2]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])/b

#### Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]
```

#### Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

#### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \sqrt{d \csc(a+bx)} \sqrt{c \sec(a+bx)} dx &= \left( \sqrt{c \cos(a+bx)} \sqrt{d \csc(a+bx)} \sqrt{c \sec(a+bx)} \sqrt{d \sin(a+bx)} \right) \int \frac{1}{\sqrt{c \cos(a+bx)} \sqrt{d \sin(a+bx)}} dx \\ &= \left( \sqrt{d \csc(a+bx)} \sqrt{c \sec(a+bx)} \sqrt{\sin(2a+2bx)} \right) \int \frac{1}{\sqrt{\sin(2a+2bx)}} dx \\ &= \frac{\sqrt{d \csc(a+bx)} F\left(a - \frac{\pi}{4} + bx \mid 2\right) \sqrt{c \sec(a+bx)} \sqrt{\sin(2a+2bx)}}{b} \end{aligned}$$

**Mathematica [C]** time = 0.670832, size = 68, normalized size = 1.28

$$\frac{\tan^3(a+bx) \left(-\cot^2(a+bx)\right)^{7/4} \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \csc^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]], x]
```

```
[Out] ((-Cot[a + b*x]^2)^(7/4)*Sqrt[d*Csc[a + b*x]]*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2]*Sqrt[c*Sec[a + b*x]]*Tan[a + b*x]^3)/b
```

**Maple [B]** time = 0.196, size = 157, normalized size = 3.

$$-\frac{\sqrt{2}(\sin(bx+a))^2}{b(-1+\cos(bx+a))} \sqrt{\frac{c}{\cos(bx+a)}} \sqrt{\frac{d}{\sin(bx+a)}} \sqrt{\frac{-1+\cos(bx+a)-\sin(bx+a)}{\sin(bx+a)}} \sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2), x)
```

```
[Out] -1/b*2^(1/2)*(c/cos(b*x+a))^(1/2)*(d/sin(b*x+a))^(1/2)*sin(b*x+a)^2*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a)), 2)
```

$n(b*x+a)/\sin(b*x+a)^{(1/2)}, 1/2*2^{(1/2))/(-1+\cos(b*x+a))$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(1/2)\*(c\*sec(b\*x+a))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(1/2)\*(c\*sec(b\*x+a))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \sec(a + bx)} \sqrt{d \csc(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(1/2)\*(c\*sec(b\*x+a))\*\*(1/2),x)

[Out] Integral(sqrt(c\*sec(a + b\*x))\*sqrt(d\*csc(a + b\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a)), x)
```

$$3.235 \quad \int \frac{\sqrt{c \sec(a+bx)}}{\sqrt{d \csc(a+bx)}} dx$$

**Optimal.** Leaf size=270

$$\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{c \sec(a+bx)}}{\sqrt{2b}\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}} + \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{c \sec(a+bx)}}{\sqrt{2b}\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}} + \frac{\sqrt{c \sec(a+bx)} \log(\tan(a+bx))}{2\sqrt{2b}\sqrt{\tan(a+bx)}}$$

```
[Out] -((ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]])*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*
Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a
+ b*x]])*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a +
b*x]]) + (Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a
+ b*x]])/(2*Sqrt[2]*b*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (Log[1 + S
qrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(2*Sqrt[2]*
b*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

**Rubi [A]** time = 0.140481, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.36$ , Rules used = {2629, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{c \sec(a+bx)}}{\sqrt{2b}\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}} + \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{c \sec(a+bx)}}{\sqrt{2b}\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}} + \frac{\sqrt{c \sec(a+bx)} \log(\tan(a+bx))}{2\sqrt{2b}\sqrt{\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sec[a + b*x]]/Sqrt[d*Csc[a + b*x]], x]
```

```
[Out] -((ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]])*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*
Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a
+ b*x]])*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a +
b*x]]) + (Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a
+ b*x]])/(2*Sqrt[2]*b*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (Log[1 + S
qrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(2*Sqrt[2]*
b*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

### Rule 2629

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_.)])^(n
_), x_Symbol] :> Dist[((a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n)/Tan[e + f*x]^
n, Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3476

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c\_.)\*(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n))/c^n)^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x],



$x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

### Rule 628

$\text{Int}[(d_.) + (e_.)*(x_.)]/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x\_Symbol] \text{:} \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{c \sec(a+bx)}}{\sqrt{d \csc(a+bx)}} dx &= \frac{\sqrt{c \sec(a+bx)} \int \sqrt{\tan(a+bx)} dx}{\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\ &= \frac{\sqrt{c \sec(a+bx)} \text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2} dx, x, \tan(a+bx)\right)}{b\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\ &= \frac{(2\sqrt{c \sec(a+bx)}) \text{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{\tan(a+bx)}\right)}{b\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\ &= -\frac{\sqrt{c \sec(a+bx)} \text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(a+bx)}\right)}{b\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} + \frac{\sqrt{c \sec(a+bx)} \text{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\tan(a+bx)}\right)}{b\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\ &= \frac{\sqrt{c \sec(a+bx)} \text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\tan(a+bx)}\right)}{2b\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} + \frac{\sqrt{c \sec(a+bx)} \text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2} dx, x, \sqrt{\tan(a+bx)}\right)}{2b\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\ &= \frac{\log\left(1 - \sqrt{2}\sqrt{\tan(a+bx)} + \tan(a+bx)\right) \sqrt{c \sec(a+bx)}}{2\sqrt{2}b\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} - \frac{\log\left(1 + \sqrt{2}\sqrt{\tan(a+bx)} + \tan(a+bx)\right) \sqrt{c \sec(a+bx)}}{2\sqrt{2}b\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\ &= -\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{c \sec(a+bx)}}{\sqrt{2}b\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} + \frac{\tan^{-1}\left(1 + \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{c \sec(a+bx)}}{\sqrt{2}b\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \end{aligned}$$

**Mathematica [A]** time = 1.32641, size = 171, normalized size = 0.63

$$\frac{\cot(a+bx)\sqrt{c \sec(a+bx)} \left( \log\left(\sqrt{\cot^2(a+bx)} - \sqrt{2}\sqrt[4]{\cot^2(a+bx)} + 1\right) - \log\left(\sqrt{\cot^2(a+bx)} + \sqrt{2}\sqrt[4]{\cot^2(a+bx)} + 1\right) \right)}{2\sqrt{2}b\sqrt[4]{\cot^2(a+bx)}\sqrt{d \csc(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c\*Sec[a + b\*x]]/Sqrt[d\*Csc[a + b\*x]], x]

```
[Out] (Cot[a + b*x]*(2*ArcTan[1 - Sqrt[2]*(Cot[a + b*x]^2)^(1/4)] - 2*ArcTan[1 + Sqrt[2]*(Cot[a + b*x]^2)^(1/4)] + Log[1 - Sqrt[2]*(Cot[a + b*x]^2)^(1/4) + Sqrt[Cot[a + b*x]^2]] - Log[1 + Sqrt[2]*(Cot[a + b*x]^2)^(1/4) + Sqrt[Cot[a + b*x]^2]])*Sqrt[c*Sec[a + b*x]])/(2*Sqrt[2]*b*(Cot[a + b*x]^2)^(1/4)*Sqrt[d*Csc[a + b*x]])
```

**Maple [C]** time = 0.148, size = 276, normalized size = 1.

$$\frac{\sqrt{2} \sin(bx + a)}{2b(-1 + \cos(bx + a))} \sqrt{\frac{c}{\cos(bx + a)}} \sqrt{\frac{-1 + \cos(bx + a) - \sin(bx + a)}{\sin(bx + a)}} \sqrt{\frac{-1 + \cos(bx + a) + \sin(bx + a)}{\sin(bx + a)}} \sqrt{\frac{-1 + \cos(bx + a)}{\sin(bx + a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2),x)
```

```
[Out] 1/2/b*2^(1/2)*(c/cos(b*x+a))^(1/2)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*(I*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2)))*sin(b*x+a)/(d/sin(b*x+a))^(1/2)/(-1+cos(b*x+a))
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c \sec(bx + a)}}{\sqrt{d \csc(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sec(b*x + a))/sqrt(d*csc(b*x + a)), x)
```

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(1/2)/(d\*csc(b\*x+a))^(1/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c \sec(a + bx)}}{\sqrt{d \csc(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))\*\*(1/2)/(d\*csc(b\*x+a))\*\*(1/2),x)

[Out] Integral(sqrt(c\*sec(a + b\*x))/sqrt(d\*csc(a + b\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c \sec(bx + a)}}{\sqrt{d \csc(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(1/2)/(d\*csc(b\*x+a))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(c\*sec(b\*x + a))/sqrt(d\*csc(b\*x + a)), x)

$$3.236 \quad \int \frac{\sqrt{c \sec(a+bx)}}{(d \csc(a+bx))^{3/2}} dx$$

**Optimal.** Leaf size=93

$$\frac{\sqrt{\sin(2a+2bx)} \operatorname{EllipticF}\left(a+bx-\frac{\pi}{4}, 2\right) \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}{2bd^2} - \frac{c}{bd \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}$$

[Out]  $-(c/(b*d*\operatorname{Sqrt}[d*\operatorname{Csc}[a+b*x]]*\operatorname{Sqrt}[c*\operatorname{Sec}[a+b*x]])) + (\operatorname{Sqrt}[d*\operatorname{Csc}[a+b*x]]*\operatorname{EllipticF}[a-\operatorname{Pi}/4+b*x, 2]*\operatorname{Sqrt}[c*\operatorname{Sec}[a+b*x]]*\operatorname{Sqrt}[\operatorname{Sin}[2*a+2*b*x]])/(2*b*d^2)$

**Rubi [A]** time = 0.147486, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2627, 2630, 2573, 2641}

$$\frac{\sqrt{\sin(2a+2bx)} F\left(a+bx-\frac{\pi}{4} \middle| 2\right) \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}{2bd^2} - \frac{c}{bd \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[c*\operatorname{Sec}[a+b*x]]/(d*\operatorname{Csc}[a+b*x])^{3/2}, x]$

[Out]  $-(c/(b*d*\operatorname{Sqrt}[d*\operatorname{Csc}[a+b*x]]*\operatorname{Sqrt}[c*\operatorname{Sec}[a+b*x]])) + (\operatorname{Sqrt}[d*\operatorname{Csc}[a+b*x]]*\operatorname{EllipticF}[a-\operatorname{Pi}/4+b*x, 2]*\operatorname{Sqrt}[c*\operatorname{Sec}[a+b*x]]*\operatorname{Sqrt}[\operatorname{Sin}[2*a+2*b*x]])/(2*b*d^2)$

### Rule 2627

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_)]*(a_.))^{(m_)}*((b_.)*\operatorname{sec}[(e_.) + (f_.)*(x_)])^{(n_)}], x\_Symbol] \rightarrow \operatorname{Simp}[(b*(a*\operatorname{Csc}[e+f*x])^{(m+1)}*(b*\operatorname{Sec}[e+f*x])^{(n-1)})/(a*f*(m+n)), x] + \operatorname{Dist}[(m+1)/(a^2*(m+n)), \operatorname{Int}[(a*\operatorname{Csc}[e+f*x])^{(m+2)}*(b*\operatorname{Sec}[e+f*x])^n, x], x] /;$  FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] && NeQ[m+n, 0] && IntegersQ[2\*m, 2\*n]

### Rule 2630

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_)]*(a_.))^{(m_)}*((b_.)*\operatorname{sec}[(e_.) + (f_.)*(x_)])^{(n_)}], x\_Symbol] \rightarrow \operatorname{Dist}[(a*\operatorname{Csc}[e+f*x])^m*(b*\operatorname{Sec}[e+f*x])^n*(a*\operatorname{Sin}[e+f*x])^m*(b*\operatorname{Cos}[e+f*x])^n, \operatorname{Int}[1/((a*\operatorname{Sin}[e+f*x])^m*(b*\operatorname{Cos}[e+f*x])^n), x], x] /;$  FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m-1/2] && IntegerQ[n-1/2]

]

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_.)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_.)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*SIN[e + f*x]]*Sqrt[b*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{c \sec(a+bx)}}{(d \csc(a+bx))^{3/2}} dx &= -\frac{c}{bd\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}} + \frac{\int \sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)} dx}{2d^2} \\ &= -\frac{c}{bd\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}} + \frac{(\sqrt{c \cos(a+bx)}\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}\sqrt{d \sin(a+bx)})}{2d^2} \\ &= -\frac{c}{bd\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}} + \frac{(\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}\sqrt{\sin(2a+2bx)})}{2d^2} \int \frac{1}{\sqrt{\sin(2a+2bx)}} dx \\ &= -\frac{c}{bd\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}} + \frac{\sqrt{d \csc(a+bx)}F\left(a - \frac{\pi}{4} + bx \middle| 2\right) \sqrt{c \sec(a+bx)}\sqrt{\sin(2a+2bx)}}{2bd^2} \end{aligned}$$

**Mathematica [C]** time = 0.672873, size = 80, normalized size = 0.86

$$\frac{(c \sec(a+bx))^{3/2} \left( (-\cot^2(a+bx))^{3/4} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \csc^2(a+bx)\right) + \cos(2(a+bx)) + 1 \right)}{2bcd\sqrt{d \csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sec[a + b*x]]/(d*Csc[a + b*x])^(3/2), x]
```

```
[Out] -((1 + Cos[2*(a + b*x)] + (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2])*(c*Sec[a + b*x])^(3/2))/(2*b*c*d*Sqrt[d*Csc[a + b*x]])
```

---

**Maple [A]** time = 0.186, size = 188, normalized size = 2.

$$-\frac{\sqrt{2}}{2b(-1 + \cos(bx + a))\sin(bx + a)} \left( \sin(bx + a) \sqrt{-\frac{-1 + \cos(bx + a) - \sin(bx + a)}{\sin(bx + a)}} \sqrt{\frac{-1 + \cos(bx + a) + \sin(bx + a)}{\sin(bx + a)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*sec(b\*x+a))^(1/2)/(d\*csc(b\*x+a))^(3/2), x)

[Out]  $-1/2/b*2^{(1/2)}*(\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*(-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a)^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}*\text{EllipticF}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}, 1/2*2^{(1/2)})+\cos(b*x+a)^2*2^{(1/2)}-\cos(b*x+a)*2^{(1/2)}*(c/\cos(b*x+a))^{(1/2)/(-1+\cos(b*x+a))}/(d/\sin(b*x+a))^{(3/2)}/\sin(b*x+a)$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c \sec(bx + a)}}{(d \csc(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(1/2)/(d\*csc(b\*x+a))^(3/2), x, algorithm="maxima")

[Out] integrate(sqrt(c\*sec(b\*x + a))/(d\*csc(b\*x + a))^(3/2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d \csc(bx + a)}\sqrt{c \sec(bx + a)}}{d^2 \csc(bx + a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(1/2)/(d\*csc(b\*x+a))^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a))/(d^2\*csc(b\*x + a)^2), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c \sec(a + bx)}}{(d \csc(a + bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))\*\*(1/2)/(d\*csc(b\*x+a))\*\*(3/2), x)

[Out] Integral(sqrt(c\*sec(a + b\*x))/(d\*csc(a + b\*x))\*\*(3/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c \sec(bx + a)}}{(d \csc(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(1/2)/(d\*csc(b\*x+a))^(3/2), x, algorithm="giac")

[Out] integrate(sqrt(c\*sec(b\*x + a))/(d\*csc(b\*x + a))^(3/2), x)

$$3.237 \quad \int \frac{\sqrt{c \sec(a+bx)}}{(d \csc(a+bx))^{5/2}} dx$$

**Optimal.** Leaf size=322

$$-\frac{3 \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{c \sec(a+bx)}}{4\sqrt{2}bd^2\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}} + \frac{3 \tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{c \sec(a+bx)}}{4\sqrt{2}bd^2\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}} + \frac{3\sqrt{c \sec(a+bx)} \log\left(\frac{\sqrt{2}\sqrt{\tan(a+bx)} + 1}{1 - \sqrt{2}\sqrt{\tan(a+bx)}}\right)}{8\sqrt{2}bd^2\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}}$$

[Out]  $-c/(2*b*d*(d*Csc[a + b*x])^{(3/2)*Sqrt[c*Sec[a + b*x]]) - (3*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(4*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(4*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (3*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(8*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (3*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(8*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])$

**Rubi [A]** time = 0.211808, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2627, 2629, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3 \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{c \sec(a+bx)}}{4\sqrt{2}bd^2\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}} + \frac{3 \tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{c \sec(a+bx)}}{4\sqrt{2}bd^2\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}} + \frac{3\sqrt{c \sec(a+bx)} \log\left(\frac{\sqrt{2}\sqrt{\tan(a+bx)} + 1}{1 - \sqrt{2}\sqrt{\tan(a+bx)}}\right)}{8\sqrt{2}bd^2\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c\*Sec[a + b\*x]]/(d\*Csc[a + b\*x])^(5/2), x]

[Out]  $-c/(2*b*d*(d*Csc[a + b*x])^{(3/2)*Sqrt[c*Sec[a + b*x]]) - (3*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(4*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(4*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (3*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(8*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (3*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(8*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])$

**Rule 2627**

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Simp[(b\*(a\*Csc[e + f\*x])^(m + 1)\*(b\*Sec[e + f\*x])^(n - 1)



)/(a\*f\*(m + n)), x] + Dist[(m + 1)/(a^2\*(m + n)), Int[(a\*Csc[e + f\*x])^(m + 2)\*(b\*Sec[e + f\*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] && NeQ[m + n, 0] && IntegersQ[2\*m, 2\*n]

### Rule 2629

Int[(csc[(e\_) + (f\_)\*(x\_)]\*(a\_))^(m\_)\*((b\_)\*sec[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :=> Dist[((a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n)/Tan[e + f\*x]^n, Int[Tan[e + f\*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && EqQ[m + n, 0]

### Rule 3476

Int[((b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] :=> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 329

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :=> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 297

Int[(x\_)^2/((a\_) + (b\_)\*(x\_)^4), x\_Symbol] :=> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] :=> With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] :=> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

#### Rule 204

$\text{Int}[\frac{(a_+) + (b_+)(x_+)^2}{(x_+)^{-1}}, x\_Symbol] \rightarrow -\text{Simp}[\frac{\text{ArcTan}[\frac{\text{Rt}[-b, 2]x}{\text{Rt}[-a, 2] - \text{Rt}[-a, 2] \text{Rt}[-b, 2]}]}{\text{Rt}[-a, 2] \text{Rt}[-b, 2]}, x] \ /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

#### Rule 1165

$\text{Int}[\frac{(d_+) + (e_+)(x_+)^2}{(a_+) + (c_+)(x_+)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2d/e, 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2d - ae^2, 0] \ \&\& \ \text{NegQ}[de]$

#### Rule 628

$\text{Int}[\frac{(d_+) + (e_+)(x_+)}{(a_+) + (b_+)(x_+) + (c_+)(x_+)^2}, x\_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c \sec(a+bx)}}{(d \csc(a+bx))^{5/2}} dx &= -\frac{c}{2bd(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} + \frac{3 \int \frac{\sqrt{c \sec(a+bx)}}{\sqrt{d \csc(a+bx)}} dx}{4d^2} \\
&= -\frac{c}{2bd(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} + \frac{(3\sqrt{c \sec(a+bx)}) \int \sqrt{\tan(a+bx)} dx}{4d^2 \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\
&= -\frac{c}{2bd(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} + \frac{(3\sqrt{c \sec(a+bx)}) \text{Subst} \left( \int \frac{\sqrt{x}}{1+x^2} dx, x, \tan(a+bx) \right)}{4bd^2 \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\
&= -\frac{c}{2bd(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} + \frac{(3\sqrt{c \sec(a+bx)}) \text{Subst} \left( \int \frac{x^2}{1+x^4} dx, x, \sqrt{\tan(a+bx)} \right)}{2bd^2 \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\
&= -\frac{c}{2bd(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} - \frac{(3\sqrt{c \sec(a+bx)}) \text{Subst} \left( \int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(a+bx)} \right)}{4bd^2 \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\
&= -\frac{c}{2bd(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} + \frac{(3\sqrt{c \sec(a+bx)}) \text{Subst} \left( \int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\tan(a+bx)} \right)}{8bd^2 \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\
&= -\frac{c}{2bd(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} + \frac{3 \log \left( 1 - \sqrt{2} \sqrt{\tan(a+bx)} + \tan(a+bx) \right) \sqrt{c \sec(a+bx)}}{8\sqrt{2}bd^2 \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\
&= -\frac{c}{2bd(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} - \frac{3 \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(a+bx)} \right) \sqrt{c \sec(a+bx)}}{4\sqrt{2}bd^2 \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} + \frac{3 \tan^{-1} \left( 1 + \sqrt{2} \sqrt{\tan(a+bx)} \right) \sqrt{c \sec(a+bx)}}{4\sqrt{2}bd^2 \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}}
\end{aligned}$$

**Mathematica [A]** time = 1.78913, size = 222, normalized size = 0.69

$$\frac{c\sqrt{d \csc(a+bx)} \left( -4\sqrt[4]{\cot^2(a+bx)} + 4 \cos(2(a+bx))\sqrt[4]{\cot^2(a+bx)} + 3\sqrt{2} \log \left( \sqrt{\cot^2(a+bx)} - \sqrt{2}\sqrt[4]{\cot^2(a+bx)} + \sqrt{2}\sqrt[4]{\cot^2(a+bx)} \right) \right)}{16bd^3 \sqrt[4]{\cot^2(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c\*Sec[a + b\*x]]/(d\*Csc[a + b\*x])^(5/2), x]

[Out] (c\*Sqrt[d\*Csc[a + b\*x]]\*(6\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4)] - 6\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4)] - 4\*(Cot[a + b\*x]^2)^(1/4) + 4\*Cos[2\*(a + b\*x)]\*(Cot[a + b\*x]^2)^(1/4) + 3\*Sqrt[2]\*Log[1 - Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4) + Sqrt[Cot[a + b\*x]^2]] - 3\*Sqrt[2]\*Log[1 + Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4) + Sqrt[Cot[a + b\*x]^2]]))/(16\*b\*d^3\*(Cot[a + b\*x]^2)^(1/4)\*Sqrt[c\*Sec[a + b\*x]])

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**Maple [C]** time = 0.225, size = 522, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((c \sec(bx+a))^{1/2} / (d \csc(bx+a))^{5/2}, x)$

[Out]  $-1/8/b^2^{1/2} * (3I * (-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1 + \cos(bx+a) + \sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1 + \cos(bx+a)) / \sin(bx+a))^{1/2} * \text{EllipticPi}((-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{1/2}, 1/2 - 1/2 * I, 1/2 * 2^{1/2}) - 3I * (-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1 + \cos(bx+a) + \sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1 + \cos(bx+a)) / \sin(bx+a))^{1/2} * \text{EllipticPi}((-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{1/2}, 1/2 + 1/2 * I, 1/2 * 2^{1/2}) - 3 * (-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1 + \cos(bx+a) + \sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1 + \cos(bx+a)) / \sin(bx+a))^{1/2} * \text{EllipticPi}((-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{1/2}, 1/2 - 1/2 * I, 1/2 * 2^{1/2}) - 3 * (-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1 + \cos(bx+a) + \sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1 + \cos(bx+a)) / \sin(bx+a))^{1/2} * \text{EllipticPi}((-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{1/2}, 1/2 + 1/2 * I, 1/2 * 2^{1/2}) + 2 * \cos(bx+a)^2 * 2^{1/2} - 2 * \cos(bx+a) * 2^{1/2}) * (c / \cos(bx+a))^{1/2} / (-1 + \cos(bx+a)) / (d / \sin(bx+a))^{5/2} / \sin(bx+a)$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c \sec(bx+a)}}{(d \csc(bx+a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c \sec(bx+a))^{1/2} / (d \csc(bx+a))^{5/2}, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}(\text{sqrt}(c \sec(bx+a)) / (d \csc(bx+a))^{5/2}, x)$

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(5/2),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*sec(b*x+a))**(1/2)/(d*csc(b*x+a))**(5/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c \sec(bx + a)}}{(d \csc(bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(5/2),x, algorithm="giac")`

[Out] `integrate(sqrt(c*sec(b*x + a))/(d*csc(b*x + a))^(5/2), x)`

### 3.238 $\int (d \csc(a + bx))^{9/2} (c \sec(a + bx))^{3/2} dx$

**Optimal.** Leaf size=104

$$\frac{64cd^5\sqrt{c\sec(a+bx)}}{21b\sqrt{d\csc(a+bx)}} - \frac{16cd^3\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}{21b} - \frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{7/2}}{7b}$$

[Out]  $(64*c*d^5*\text{Sqrt}[c*\text{Sec}[a + b*x]])/(21*b*\text{Sqrt}[d*\text{Csc}[a + b*x]]) - (16*c*d^3*(d*\text{Csc}[a + b*x])^{3/2}*\text{Sqrt}[c*\text{Sec}[a + b*x]])/(21*b) - (2*c*d*(d*\text{Csc}[a + b*x])^{7/2}*\text{Sqrt}[c*\text{Sec}[a + b*x]])/(7*b)$

**Rubi [A]** time = 0.162464, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {2625, 2619}

$$\frac{64cd^5\sqrt{c\sec(a+bx)}}{21b\sqrt{d\csc(a+bx)}} - \frac{16cd^3\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}{21b} - \frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*\text{Csc}[a + b*x])^{9/2}*(c*\text{Sec}[a + b*x])^{3/2}, x]$

[Out]  $(64*c*d^5*\text{Sqrt}[c*\text{Sec}[a + b*x]])/(21*b*\text{Sqrt}[d*\text{Csc}[a + b*x]]) - (16*c*d^3*(d*\text{Csc}[a + b*x])^{3/2}*\text{Sqrt}[c*\text{Sec}[a + b*x]])/(21*b) - (2*c*d*(d*\text{Csc}[a + b*x])^{7/2}*\text{Sqrt}[c*\text{Sec}[a + b*x]])/(7*b)$

#### Rule 2625

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] :> -\text{Simp}[(a*b*(a*\text{Csc}[e + f*x])^{(m-1)}*(b*\text{Sec}[e + f*x])^{(n-1)})/(f*(m-1)), x] + \text{Dist}[(a^2*(m+n-2))/(m-1), \text{Int}[(a*\text{Csc}[e + f*x])^{(m-2)}*(b*\text{Sec}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, e, f, n\}, x] \&\& \text{GtQ}[m, 1] \&\& \text{IntegersQ}[2*m, 2*n] \&\& !\text{GtQ}[n, m]$

#### Rule 2619

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] :> \text{Simp}[(a*b*(a*\text{Csc}[e + f*x])^{(m-1)}*(b*\text{Sec}[e + f*x])^{(n-1)})/(f*(n-1)), x] /; \text{FreeQ}\{a, b, e, f, m, n\}, x] \&\& \text{EqQ}[m + n - 2, 0] \&\& \text{NeQ}[n, 1]$

Rubi steps

$$\begin{aligned}
\int (d \csc(a + bx))^{9/2} (c \sec(a + bx))^{3/2} dx &= -\frac{2cd(d \csc(a + bx))^{7/2} \sqrt{c \sec(a + bx)}}{7b} + \frac{1}{7} (8d^2) \int (d \csc(a + bx))^{5/2} (c \sec(a + bx))^{3/2} dx \\
&= -\frac{16cd^3 (d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)}}{21b} - \frac{2cd(d \csc(a + bx))^{7/2} \sqrt{c \sec(a + bx)}}{7b} \\
&= \frac{64cd^5 \sqrt{c \sec(a + bx)}}{21b \sqrt{d \csc(a + bx)}} - \frac{16cd^3 (d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)}}{21b} - \frac{2cd(d \csc(a + bx))^{7/2} \sqrt{c \sec(a + bx)}}{7b}
\end{aligned}$$

**Mathematica [A]** time = 0.298261, size = 57, normalized size = 0.55

$$\frac{2cd^5 (3 \csc^4(a + bx) + 8 \csc^2(a + bx) - 32) \sqrt{c \sec(a + bx)}}{21b \sqrt{d \csc(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(9/2)\*(c\*Sec[a + b\*x])^(3/2), x]

[Out] (-2\*c\*d^5\*(-32 + 8\*Csc[a + b\*x]^2 + 3\*Csc[a + b\*x]^4)\*Sqrt[c\*Sec[a + b\*x]])/(21\*b\*Sqrt[d\*Csc[a + b\*x]])

**Maple [A]** time = 0.196, size = 64, normalized size = 0.6

$$\frac{(64 (\cos(bx + a))^4 - 112 (\cos(bx + a))^2 + 42) \cos(bx + a) \sin(bx + a)}{21b} \left( \frac{d}{\sin(bx + a)} \right)^{\frac{9}{2}} \left( \frac{c}{\cos(bx + a)} \right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(9/2)\*(c\*sec(b\*x+a))^(3/2), x)

[Out] 2/21/b\*(32\*cos(b\*x+a)^4-56\*cos(b\*x+a)^2+21)\*cos(b\*x+a)\*(d/sin(b\*x+a))^(9/2)\*(c/cos(b\*x+a))^(3/2)\*sin(b\*x+a)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc(bx + a))^{\frac{9}{2}} (c \sec(bx + a))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(9/2)\*(c\*sec(b\*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(9/2)\*(c\*sec(b\*x + a))^(3/2), x)

**Fricas [A]** time = 2.44703, size = 203, normalized size = 1.95

$$\frac{2 \left( 32 cd^4 \cos^4(bx + a) - 56 cd^4 \cos^2(bx + a) + 21 cd^4 \right) \sqrt{\frac{c}{\cos(bx+a)}} \sqrt{\frac{d}{\sin(bx+a)}}}{21 \left( b \cos^2(bx + a) - b \right) \sin(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(9/2)\*(c\*sec(b\*x+a))^(3/2),x, algorithm="fricas")

[Out] -2/21\*(32\*c\*d^4\*cos(b\*x + a)^4 - 56\*c\*d^4\*cos(b\*x + a)^2 + 21\*c\*d^4)\*sqrt(c/cos(b\*x + a))\*sqrt(d/sin(b\*x + a))/((b\*cos(b\*x + a)^2 - b)\*sin(b\*x + a))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(9/2)\*(c\*sec(b\*x+a))\*\*(3/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc(bx + a))^{\frac{9}{2}} (c \sec(bx + a))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(9/2)\*(c\*sec(b\*x+a))^(3/2),x, algorithm="giac")



```
[Out] integrate((d*csc(b*x + a))^(9/2)*(c*sec(b*x + a))^(3/2), x)
```

### 3.239 $\int (d \csc(a + bx))^{7/2} (c \sec(a + bx))^{3/2} dx$

**Optimal.** Leaf size=166

$$-\frac{24c^2d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}} + \frac{24cd^5\sqrt{c\sec(a+bx)}}{5b(d\csc(a+bx))^{3/2}} - \frac{12cd^3\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{5b} - \frac{2cd\sqrt{c\sec(a+bx)}}{5b}$$

[Out] (24\*c\*d^5\*Sqrt[c\*Sec[a + b\*x]])/(5\*b\*(d\*Csc[a + b\*x])^(3/2)) - (12\*c\*d^3\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]])/(5\*b) - (2\*c\*d\*(d\*Csc[a + b\*x])^(5/2)\*Sqrt[c\*Sec[a + b\*x]])/(5\*b) - (24\*c^2\*d^4\*EllipticE[a - Pi/4 + b\*x, 2])/(5\*b\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])

**Rubi [A]** time = 0.267906, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2625, 2626, 2630, 2572, 2639}

$$-\frac{24c^2d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}} + \frac{24cd^5\sqrt{c\sec(a+bx)}}{5b(d\csc(a+bx))^{3/2}} - \frac{12cd^3\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{5b} - \frac{2cd\sqrt{c\sec(a+bx)}}{5b}$$

Antiderivative was successfully verified.

[In] Int[(d\*Csc[a + b\*x])^(7/2)\*(c\*Sec[a + b\*x])^(3/2), x]

[Out] (24\*c\*d^5\*Sqrt[c\*Sec[a + b\*x]])/(5\*b\*(d\*Csc[a + b\*x])^(3/2)) - (12\*c\*d^3\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]])/(5\*b) - (2\*c\*d\*(d\*Csc[a + b\*x])^(5/2)\*Sqrt[c\*Sec[a + b\*x]])/(5\*b) - (24\*c^2\*d^4\*EllipticE[a - Pi/4 + b\*x, 2])/(5\*b\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])

#### Rule 2625

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] :> -Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*(m - 1)), x] + Dist[(a^2\*(m + n - 2))/(m - 1), Int[(a\*Csc[e + f\*x])^(m - 2)\*(b\*Sec[e + f\*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && IntegersQ[2\*m, 2\*n] && !GtQ[n, m]

#### Rule 2626

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] :> Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*(n - 1)), x] + Dist[(b^2\*(m + n - 2))/(n - 1), Int[(a\*Csc[e + f\*x])^m,

$m*(b*\text{Sec}[e + f*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{a, b, e, f, m\}, x\} \&\& \text{GtQ}\{n, 1\} \&\& \text{IntegersQ}\{2*m, 2*n\}$

### Rule 2630

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[(a*\text{Csc}[e + f*x])^m*(b*\text{Sec}[e + f*x])^n*(a*\text{Sin}[e + f*x])^m*(b*\text{Cos}[e + f*x])^n, \text{Int}[1/((a*\text{Sin}[e + f*x])^m*(b*\text{Cos}[e + f*x])^n), x], x] /; \text{FreeQ}\{a, b, e, f, m, n\}, x\} \&\& \text{IntegerQ}\{m - 1/2\} \&\& \text{IntegerQ}\{n - 1/2\}$

### Rule 2572

$\text{Int}[\text{Sqrt}[\cos[(e_.) + (f_.)*(x_.)]*(b_.)]*\text{Sqrt}[(a_.)*\sin[(e_.) + (f_.)*(x_.)]], x\_Symbol] \rightarrow \text{Dist}[(\text{Sqrt}[a*\text{Sin}[e + f*x]]*\text{Sqrt}[b*\text{Cos}[e + f*x]])/\text{Sqrt}[\text{Sin}[2*e + 2*f*x]], \text{Int}[\text{Sqrt}[\text{Sin}[2*e + 2*f*x]], x], x] /; \text{FreeQ}\{a, b, e, f\}, x\}$

### Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x\_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x\}$

### Rubi steps

$$\begin{aligned} \int (d \csc(a + bx))^{7/2} (c \sec(a + bx))^{3/2} dx &= -\frac{2cd(d \csc(a + bx))^{5/2} \sqrt{c \sec(a + bx)}}{5b} + \frac{1}{5} (6d^2) \int (d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2} dx \\ &= -\frac{12cd^3 \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)}}{5b} - \frac{2cd(d \csc(a + bx))^{5/2} \sqrt{c \sec(a + bx)}}{5b} \\ &= \frac{24cd^5 \sqrt{c \sec(a + bx)}}{5b(d \csc(a + bx))^{3/2}} - \frac{12cd^3 \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)}}{5b} - \frac{2cd(d \csc(a + bx))^{5/2} \sqrt{c \sec(a + bx)}}{5b} \\ &= \frac{24cd^5 \sqrt{c \sec(a + bx)}}{5b(d \csc(a + bx))^{3/2}} - \frac{12cd^3 \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)}}{5b} - \frac{2cd(d \csc(a + bx))^{5/2} \sqrt{c \sec(a + bx)}}{5b} \\ &= \frac{24cd^5 \sqrt{c \sec(a + bx)}}{5b(d \csc(a + bx))^{3/2}} - \frac{12cd^3 \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)}}{5b} - \frac{2cd(d \csc(a + bx))^{5/2} \sqrt{c \sec(a + bx)}}{5b} \\ &= \frac{24cd^5 \sqrt{c \sec(a + bx)}}{5b(d \csc(a + bx))^{3/2}} - \frac{12cd^3 \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)}}{5b} - \frac{2cd(d \csc(a + bx))^{5/2} \sqrt{c \sec(a + bx)}}{5b} \end{aligned}$$

**Mathematica [C]** time = 1.11952, size = 114, normalized size = 0.69

$$\frac{2cd^3 \tan^2(a + bx) \sqrt{c \sec(a + bx)} \sqrt{d \csc(a + bx)} \left( 12 \cos^2(a + bx) \sqrt{-\cot^2(a + bx)} \operatorname{Hypergeometric2F1} \left( -\frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \csc^2(a + bx) \right) \right)}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(7/2)\*(c\*Sec[a + b\*x])^(3/2), x]

[Out] (-2\*c\*d^3\*Sqrt[d\*Csc[a + b\*x]]\*(Cot[a + b\*x]^2\*(6\*Cos[2\*(a + b\*x)] + Csc[a + b\*x]^2) + 12\*Cos[a + b\*x]^2\*(-Cot[a + b\*x]^2)^(1/4)\*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b\*x]^2])\*Sqrt[c\*Sec[a + b\*x]]\*Tan[a + b\*x]^2)/(5\*b)

**Maple [B]** time = 0.207, size = 996, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(7/2)\*(c\*sec(b\*x+a))^(3/2), x)

[Out] -1/5/b\*2^(1/2)\*(24\*cos(b\*x+a)^3\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticE((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2), 1/2\*2^(1/2))-12\*cos(b\*x+a)^3\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2), 1/2\*2^(1/2))+24\*cos(b\*x+a)^2\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticE((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2), 1/2\*2^(1/2))-12\*cos(b\*x+a)^2\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2), 1/2\*2^(1/2))-24\*cos(b\*x+a)\*((-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticE((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2), 1/2\*2^(1/2))+12\*cos(b\*x+a)\*((-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2), 1/2\*2^(1/2))-12\*cos(b\*x+a)^3\*2^(1/2)-24\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticE((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2), 1/2\*2^(1/2))

)^(1/2), 1/2\*2^(1/2))+12\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2), 1/2\*2^(1/2))+6\*cos(b\*x+a)^2\*2^(1/2)+12\*cos(b\*x+a)\*2^(1/2)-5\*2^(1/2))\*cos(b\*x+a)\*(d/sin(b\*x+a))^(7/2)\*(c/cos(b\*x+a))^(3/2)\*sin(b\*x+a)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc(bx + a))^{7/2} (c \sec(bx + a))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(7/2)\*(c\*sec(b\*x+a))^(3/2), x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(7/2)\*(c\*sec(b\*x + a))^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} c d^3 \csc(bx + a)^3 \sec(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(7/2)\*(c\*sec(b\*x+a))^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a))\*c\*d^3\*csc(b\*x + a)^3\*sec(b\*x + a), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(7/2)\*(c\*sec(b\*x+a))\*\*(3/2), x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc (bx + a))^{\frac{7}{2}} (c \sec (bx + a))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(3/2), x)
```

### 3.240 $\int (d \csc(a + bx))^{5/2} (c \sec(a + bx))^{3/2} dx$

**Optimal.** Leaf size=69

$$\frac{8cd^3\sqrt{c\sec(a+bx)}}{3b\sqrt{d\csc(a+bx)}} - \frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}{3b}$$

[Out]  $(8*c*d^3*\text{Sqrt}[c*\text{Sec}[a + b*x]])/(3*b*\text{Sqrt}[d*\text{Csc}[a + b*x]]) - (2*c*d*(d*\text{Csc}[a + b*x])^{3/2}*\text{Sqrt}[c*\text{Sec}[a + b*x]])/(3*b)$

**Rubi [A]** time = 0.102838, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {2625, 2619}

$$\frac{8cd^3\sqrt{c\sec(a+bx)}}{3b\sqrt{d\csc(a+bx)}} - \frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*\text{Csc}[a + b*x])^{5/2}*(c*\text{Sec}[a + b*x])^{3/2}, x]$

[Out]  $(8*c*d^3*\text{Sqrt}[c*\text{Sec}[a + b*x]])/(3*b*\text{Sqrt}[d*\text{Csc}[a + b*x]]) - (2*c*d*(d*\text{Csc}[a + b*x])^{3/2}*\text{Sqrt}[c*\text{Sec}[a + b*x]])/(3*b)$

#### Rule 2625

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] :> -\text{Simp}[(a*b*(a*\text{Csc}[e + f*x])^{(m-1)}*(b*\text{Sec}[e + f*x])^{(n-1)})/(f*(m-1)), x] + \text{Dist}[(a^2*(m+n-2))/(m-1), \text{Int}[(a*\text{Csc}[e + f*x])^{(m-2)}*(b*\text{Sec}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, e, f, n\}, x] \&\& \text{GtQ}[m, 1] \&\& \text{IntegersQ}[2*m, 2*n] \&\& !\text{GtQ}[n, m]$

#### Rule 2619

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] :> \text{Simp}[(a*b*(a*\text{Csc}[e + f*x])^{(m-1)}*(b*\text{Sec}[e + f*x])^{(n-1)})/(f*(n-1)), x] /; \text{FreeQ}\{a, b, e, f, m, n\}, x] \&\& \text{EqQ}[m+n-2, 0] \&\& \text{NeQ}[n, 1]$

#### Rubi steps

$$\int (d \csc(a + bx))^{5/2} (c \sec(a + bx))^{3/2} dx = -\frac{2cd(d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)}}{3b} + \frac{1}{3} (4d^2) \int \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{3/2} dx$$

$$= \frac{8cd^3 \sqrt{c \sec(a + bx)}}{3b \sqrt{d \csc(a + bx)}} - \frac{2cd(d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)}}{3b}$$

**Mathematica [A]** time = 0.135484, size = 45, normalized size = 0.65

$$-\frac{2cd^3 (\csc^2(a + bx) - 4) \sqrt{c \sec(a + bx)}}{3b \sqrt{d \csc(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(5/2)\*(c\*Sec[a + b\*x])^(3/2),x]

[Out] (-2\*c\*d^3\*(-4 + Csc[a + b\*x]^2)\*Sqrt[c\*Sec[a + b\*x]])/(3\*b\*Sqrt[d\*Csc[a + b\*x]])

**Maple [A]** time = 0.162, size = 54, normalized size = 0.8

$$-\frac{(8 (\cos(bx + a))^2 - 6) \cos(bx + a) \sin(bx + a)}{3b} \left( \frac{d}{\sin(bx + a)} \right)^{5/2} \left( \frac{c}{\cos(bx + a)} \right)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(5/2)\*(c\*sec(b\*x+a))^(3/2),x)

[Out] -2/3/b\*(4\*cos(b\*x+a)^2-3)\*cos(b\*x+a)\*(d/sin(b\*x+a))^(5/2)\*(c/cos(b\*x+a))^(3/2)\*sin(b\*x+a)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc(bx + a))^{5/2} (c \sec(bx + a))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((d\*csc(b\*x+a))^(5/2)\*(c\*sec(b\*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(5/2)\*(c\*sec(b\*x + a))^(3/2), x)

**Fricas [A]** time = 2.06235, size = 135, normalized size = 1.96

$$\frac{2 \left( 4 c d^2 \cos (b x + a)^2 - 3 c d^2 \right) \sqrt{\frac{c}{\cos (b x + a)}} \sqrt{\frac{d}{\sin (b x + a)}}}{3 b \sin (b x + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(5/2)\*(c\*sec(b\*x+a))^(3/2),x, algorithm="fricas")

[Out] -2/3\*(4\*c\*d^2\*cos(b\*x + a)^2 - 3\*c\*d^2)\*sqrt(c/cos(b\*x + a))\*sqrt(d/sin(b\*x + a))/(b\*sin(b\*x + a))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(5/2)\*(c\*sec(b\*x+a))\*\*(3/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc (b x + a))^{\frac{5}{2}} (c \sec (b x + a))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(5/2)\*(c\*sec(b\*x+a))^(3/2),x, algorithm="giac")

[Out] integrate((d\*csc(b\*x + a))^(5/2)\*(c\*sec(b\*x + a))^(3/2), x)

### 3.241 $\int (d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2} dx$

**Optimal.** Leaf size=125

$$-\frac{4c^2d^2E\left(a + bx - \frac{\pi}{4} \middle| 2\right)}{b\sqrt{\sin(2a + 2bx)}\sqrt{c \sec(a + bx)}\sqrt{d \csc(a + bx)}} + \frac{4cd^3\sqrt{c \sec(a + bx)}}{b(d \csc(a + bx))^{3/2}} - \frac{2cd\sqrt{c \sec(a + bx)}\sqrt{d \csc(a + bx)}}{b}$$

[Out] (4\*c\*d^3\*Sqrt[c\*Sec[a + b\*x]])/(b\*(d\*Csc[a + b\*x])^(3/2)) - (2\*c\*d\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]])/b - (4\*c^2\*d^2\*EllipticE[a - Pi/4 + b\*x, 2])/(b\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])

**Rubi [A]** time = 0.204733, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2625, 2626, 2630, 2572, 2639}

$$-\frac{4c^2d^2E\left(a + bx - \frac{\pi}{4} \middle| 2\right)}{b\sqrt{\sin(2a + 2bx)}\sqrt{c \sec(a + bx)}\sqrt{d \csc(a + bx)}} + \frac{4cd^3\sqrt{c \sec(a + bx)}}{b(d \csc(a + bx))^{3/2}} - \frac{2cd\sqrt{c \sec(a + bx)}\sqrt{d \csc(a + bx)}}{b}$$

Antiderivative was successfully verified.

[In] Int[(d\*Csc[a + b\*x])^(3/2)\*(c\*Sec[a + b\*x])^(3/2),x]

[Out] (4\*c\*d^3\*Sqrt[c\*Sec[a + b\*x]])/(b\*(d\*Csc[a + b\*x])^(3/2)) - (2\*c\*d\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]])/b - (4\*c^2\*d^2\*EllipticE[a - Pi/4 + b\*x, 2])/(b\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])

#### Rule 2625

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> -Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*(m - 1)), x] + Dist[(a^2\*(m + n - 2))/(m - 1), Int[(a\*Csc[e + f\*x])^(m - 2)\*(b\*Sec[e + f\*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && IntegersQ[2\*m, 2\*n] && !GtQ[n, m]

#### Rule 2626

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*(n - 1)), x] + Dist[(b^2\*(m + n - 2))/(n - 1), Int[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1]

] && IntegersQ[2\*m, 2\*n]

### Rule 2630

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Dist[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n\*(a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]

### Rule 2572

Int[Sqrt[cos[(e\_.) + (f\_.)\*(x\_.)]\*(b\_.)]\*Sqrt[(a\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]], x\_Symbol] :> Dist[(Sqrt[a\*Sin[e + f\*x]]\*Sqrt[b\*Cos[e + f\*x]])/Sqrt[Sin[2\*e + 2\*f\*x]], Int[Sqrt[Sin[2\*e + 2\*f\*x]], x], x] /; FreeQ[{a, b, e, f}, x]

### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] :> Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned}
 \int (d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2} dx &= -\frac{2cd\sqrt{d} \csc(a + bx)\sqrt{c \sec(a + bx)}}{b} + (2d^2) \int \frac{(c \sec(a + bx))^{3/2}}{\sqrt{d} \csc(a + bx)} dx \\
 &= \frac{4cd^3\sqrt{c \sec(a + bx)}}{b(d \csc(a + bx))^{3/2}} - \frac{2cd\sqrt{d} \csc(a + bx)\sqrt{c \sec(a + bx)}}{b} - (4c^2d^2) \int \frac{\sqrt{d} \csc(a + bx)}{\sqrt{d} \csc(a + bx)} dx \\
 &= \frac{4cd^3\sqrt{c \sec(a + bx)}}{b(d \csc(a + bx))^{3/2}} - \frac{2cd\sqrt{d} \csc(a + bx)\sqrt{c \sec(a + bx)}}{b} - \frac{(4c^2d^2) \int \sqrt{d} \csc(a + bx)}{\sqrt{c \cos(a + bx)}\sqrt{d}} dx \\
 &= \frac{4cd^3\sqrt{c \sec(a + bx)}}{b(d \csc(a + bx))^{3/2}} - \frac{2cd\sqrt{d} \csc(a + bx)\sqrt{c \sec(a + bx)}}{b} - \frac{(4c^2d^2) \int \sqrt{d} \csc(a + bx)}{\sqrt{d} \csc(a + bx)\sqrt{c}} dx \\
 &= \frac{4cd^3\sqrt{c \sec(a + bx)}}{b(d \csc(a + bx))^{3/2}} - \frac{2cd\sqrt{d} \csc(a + bx)\sqrt{c \sec(a + bx)}}{b} - \frac{4c^2d^2}{b\sqrt{d} \csc(a + bx)\sqrt{c}}
 \end{aligned}$$

**Mathematica [C]** time = 0.511562, size = 99, normalized size = 0.79

$$\frac{2cd \tan^2(a + bx)\sqrt{c \sec(a + bx)}\sqrt{d \csc(a + bx)} \left( 2 \cos^2(a + bx) \sqrt{-\cot^2(a + bx)} \operatorname{Hypergeometric2F1} \left( -\frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \csc^2(a + bx) \right) \right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (-2*c*d*Sqrt[d*Csc[a + b*x]]*(Cos[2*(a + b*x)]*Cot[a + b*x]^2 + 2*Cos[a + b*x]^2*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec[a + b*x]]*Tan[a + b*x]^2)/b
```

**Maple [B]** time = 0.187, size = 503, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2),x)
```

```
[Out] 1/b*2^(1/2)*(4*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+4*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a)*2^(1/2)+2^(1/2))*cos(b*x+a)*(d/sin(b*x+a))^(3/2)*(c/cos(b*x+a))^(3/2)*sin(b*x+a)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc (bx + a))^{\frac{3}{2}} (c \sec (bx + a))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

[Out] integrate((d\*csc(b\*x + a))^(3/2)\*(c\*sec(b\*x + a))^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d \csc(bx + a)}\sqrt{c \sec(bx + a)}cd \csc(bx + a) \sec(bx + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(3/2)\*(c\*sec(b\*x+a))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a))\*c\*d\*csc(b\*x + a)\*sec(b\*x + a), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(3/2)\*(c\*sec(b\*x+a))\*\*(3/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc(bx + a))^{\frac{3}{2}} (c \sec(bx + a))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(3/2)\*(c\*sec(b\*x+a))^(3/2),x, algorithm="giac")

[Out] integrate((d\*csc(b\*x + a))^(3/2)\*(c\*sec(b\*x + a))^(3/2), x)

$$3.242 \quad \int \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{3/2} dx$$

**Optimal.** Leaf size=31

$$\frac{2cd\sqrt{c \sec(a + bx)}}{b\sqrt{d \csc(a + bx)}}$$

[Out] (2\*c\*d\*Sqrt[c\*Sec[a + b\*x]])/(b\*Sqrt[d\*Csc[a + b\*x]])

**Rubi [A]** time = 0.0476698, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {2619}

$$\frac{2cd\sqrt{c \sec(a + bx)}}{b\sqrt{d \csc(a + bx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*Csc[a + b\*x]]\*(c\*Sec[a + b\*x])^(3/2),x]

[Out] (2\*c\*d\*Sqrt[c\*Sec[a + b\*x]])/(b\*Sqrt[d\*Csc[a + b\*x]])

**Rule 2619**

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] := Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] && NeQ[n, 1]

**Rubi steps**

$$\int \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{3/2} dx = \frac{2cd\sqrt{c \sec(a + bx)}}{b\sqrt{d \csc(a + bx)}}$$

**Mathematica [A]** time = 0.0613883, size = 31, normalized size = 1.

$$\frac{2cd\sqrt{c \sec(a + bx)}}{b\sqrt{d \csc(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*Csc[a + b\*x]]\*(c\*Sec[a + b\*x])^(3/2),x]

[Out] (2\*c\*d\*Sqrt[c\*Sec[a + b\*x]])/(b\*Sqrt[d\*Csc[a + b\*x]])

**Maple [A]** time = 0.169, size = 42, normalized size = 1.4

$$2 \frac{\cos(bx + a) \sin(bx + a)}{b} \sqrt{\frac{d}{\sin(bx + a)}} \left( \frac{c}{\cos(bx + a)} \right)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(1/2)\*(c\*sec(b\*x+a))^(3/2),x)

[Out] 2/b\*cos(b\*x+a)\*sin(b\*x+a)\*(d/sin(b\*x+a))^(1/2)\*(c/cos(b\*x+a))^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \csc(bx + a)} (c \sec(bx + a))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(1/2)\*(c\*sec(b\*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(d\*csc(b\*x + a))\*(c\*sec(b\*x + a))^(3/2), x)

**Fricas [A]** time = 1.65792, size = 84, normalized size = 2.71

$$\frac{2c \sqrt{\frac{c}{\cos(bx+a)}} \sqrt{\frac{d}{\sin(bx+a)}} \sin(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(1/2)\*(c\*sec(b\*x+a))^(3/2),x, algorithm="fricas")

[Out]  $2*c*\sqrt{c/\cos(b*x + a)}*\sqrt{d/\sin(b*x + a)}*\sin(b*x + a)/b$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))**(1/2)*(c*sec(b*x+a))**(3/2), x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \csc(bx + a)} (c \sec(bx + a))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(3/2), x, algorithm="giac")`

[Out] `integrate(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(3/2), x)`



$$3.243 \quad \int \frac{(c \sec(a+bx))^{3/2}}{\sqrt{d} \csc(a+bx)} dx$$

**Optimal.** Leaf size=89

$$\frac{2cd\sqrt{c \sec(a+bx)}}{b(d \csc(a+bx))^{3/2}} - \frac{2c^2 E\left(a+bx - \frac{\pi}{4} \middle| 2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c \sec(a+bx)}\sqrt{d \csc(a+bx)}}$$

[Out] (2\*c\*d\*Sqrt[c\*Sec[a + b\*x]])/(b\*(d\*Csc[a + b\*x])^(3/2)) - (2\*c^2\*EllipticE[a - Pi/4 + b\*x, 2])/(b\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])

**Rubi [A]** time = 0.143064, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2626, 2630, 2572, 2639}

$$\frac{2cd\sqrt{c \sec(a+bx)}}{b(d \csc(a+bx))^{3/2}} - \frac{2c^2 E\left(a+bx - \frac{\pi}{4} \middle| 2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c \sec(a+bx)}\sqrt{d \csc(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[(c\*Sec[a + b\*x])^(3/2)/Sqrt[d\*Csc[a + b\*x]], x]

[Out] (2\*c\*d\*Sqrt[c\*Sec[a + b\*x]])/(b\*(d\*Csc[a + b\*x])^(3/2)) - (2\*c^2\*EllipticE[a - Pi/4 + b\*x, 2])/(b\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])

### Rule 2626

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] :> Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*(n - 1)), x] + Dist[(b^2\*(m + n - 2))/(n - 1), Int[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && IntegersQ[2\*m, 2\*n]

### Rule 2630

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] :> Dist[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n\*(a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]

]

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] := Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]] , x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c \sec(a + bx))^{3/2}}{\sqrt{d} \csc(a + bx)} dx &= \frac{2cd\sqrt{c \sec(a + bx)}}{b(d \csc(a + bx))^{3/2}} - (2c^2) \int \frac{1}{\sqrt{d} \csc(a + bx)\sqrt{c \sec(a + bx)}} dx \\
&= \frac{2cd\sqrt{c \sec(a + bx)}}{b(d \csc(a + bx))^{3/2}} - \frac{(2c^2) \int \sqrt{c \cos(a + bx)}\sqrt{d \sin(a + bx)} dx}{\sqrt{c \cos(a + bx)}\sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}\sqrt{d \sin(a + bx)}} \\
&= \frac{2cd\sqrt{c \sec(a + bx)}}{b(d \csc(a + bx))^{3/2}} - \frac{(2c^2) \int \sqrt{\sin(2a + 2bx)} dx}{\sqrt{d} \csc(a + bx)\sqrt{c \sec(a + bx)}\sqrt{\sin(2a + 2bx)}} \\
&= \frac{2cd\sqrt{c \sec(a + bx)}}{b(d \csc(a + bx))^{3/2}} - \frac{2c^2 E\left(a - \frac{\pi}{4} + bx \mid 2\right)}{b\sqrt{d} \csc(a + bx)\sqrt{c \sec(a + bx)}\sqrt{\sin(2a + 2bx)}}
\end{aligned}$$

**Mathematica [C]** time = 0.366759, size = 66, normalized size = 0.74

$$\frac{2cd\sqrt{c \sec(a + bx)} \left( \sqrt[4]{-\cot^2(a + bx)} \operatorname{Hypergeometric2F1} \left( -\frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \csc^2(a + bx) \right) - 1 \right)}{b(d \csc(a + bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(3/2)/Sqrt[d*Csc[a + b*x]],x]
```

```
[Out] (-2*c*d*(-1 + (-Cot[a + b*x]^2)^(1/4))*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc
[a + b*x]^2])*Sqrt[c*Sec[a + b*x]]/(b*(d*Csc[a + b*x])^(3/2))
```

**Maple [B]** time = 0.191, size = 505, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x)`

[Out] 
$$\frac{1}{b^2} 2^{1/2} (2 \cos(bx+a) (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} \text{EllipticE}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2) 2^{1/2} - \cos(bx+a) (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} \text{EllipticF}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2) 2^{1/2} + 2 (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} \text{EllipticE}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2) 2^{1/2} - (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} \text{EllipticF}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2) 2^{1/2} - \cos(bx+a) 2^{1/2} + 2^{1/2} \cos(bx+a) (c/\cos(bx+a))^{3/2} / (d/\sin(bx+a))^{1/2} / \sin(bx+a)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sec(bx + a))^{\frac{3}{2}}}{\sqrt{d} \csc(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x, algorithm="maxima")`

[Out] `integrate((c*sec(b*x + a))^(3/2)/sqrt(d*csc(b*x + a)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\sqrt{d} \csc(bx + a) \sqrt{c \sec(bx + a)} c \sec(bx + a)}{d \csc(bx + a)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*c*sec(b*x + a)/(d*csc(b*x + a)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(3/2)/(d*csc(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sec(bx + a))^{\frac{3}{2}}}{\sqrt{d} \csc(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2)/sqrt(d*csc(b*x + a)), x)
```

$$3.244 \quad \int \frac{(c \sec(a+bx))^{3/2}}{(d \csc(a+bx))^{3/2}} dx$$

**Optimal.** Leaf size=327

$$\frac{c^2 \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{\sqrt{2}bd^2\sqrt{c \sec(a+bx)}} - \frac{c^2 \tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{\sqrt{2}bd^2\sqrt{c \sec(a+bx)}}$$

```
[Out] (2*c*Sqrt[c*Sec[a + b*x]])/(b*d*Sqrt[d*Csc[a + b*x]]) + (c^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) - (c^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) + (c^2*Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) - (c^2*Sqrt[d*Csc[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]])
```

**Rubi [A]** time = 0.21939, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2624, 2629, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{c^2 \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{\sqrt{2}bd^2\sqrt{c \sec(a+bx)}} - \frac{c^2 \tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{\sqrt{2}bd^2\sqrt{c \sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(3/2)/(d*Csc[a + b*x])^(3/2), x]
```

```
[Out] (2*c*Sqrt[c*Sec[a + b*x]])/(b*d*Sqrt[d*Csc[a + b*x]]) + (c^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) - (c^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) + (c^2*Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) - (c^2*Sqrt[d*Csc[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]])
```

#### Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
```

$$\frac{1}{(f*a*(n - 1)), x] + \text{Dist}[(b^2*(m + 1))/(a^2*(n - 1)), \text{Int}[(a*\text{Csc}[e + f*x])^{m+2}*(b*\text{Sec}[e + f*x])^{n-2}], x], x] /; \text{FreeQ}[\{a, b, e, f\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntegersQ}[2*m, 2*n]$$

### Rule 2629

$$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[(a*\text{Csc}[e + f*x])^m*(b*\text{Sec}[e + f*x])^n/\text{Tan}[e + f*x]^n, \text{Int}[\text{Tan}[e + f*x]^n, x], x] /; \text{FreeQ}[\{a, b, e, f, m, n\}, x] \ \&\& \ !\text{IntegerQ}[n] \ \&\& \ \text{EqQ}[m + n, 0]$$

### Rule 3476

$$\text{Int}[(b_.)*\tan[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[b/d, \text{Subst}[\text{Int}[x^n/(b^2 + x^2), x], x, b*\text{Tan}[c + d*x]], x] /; \text{FreeQ}[\{b, c, d, n\}, x] \ \&\& \ !\text{IntegerQ}[n]$$

### Rule 329

$$\text{Int}[(c_.)*(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{k*(m+1)-1}*(a + (b*x^{k*n}))/c^n]^p, x], x, (c*x)^{1/k}], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{FractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$$

### Rule 211

$$\text{Int}[(a_.) + (b_.)*(x_.)^4]^{(-1)}, x\_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1/(2*r), \text{Int}[(r - s*x^2)/(a + b*x^4), x], x] + \text{Dist}[1/(2*r), \text{Int}[(r + s*x^2)/(a + b*x^4), x], x]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ (\text{GtQ}[a/b, 0] \ || \ (\text{PosQ}[a/b] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, b]]))$$

### Rule 1165

$$\text{Int}[(d_.) + (e_.)*(x_.)^2]/((a_.) + (c_.)*(x_.)^4), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(-2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$$

### Rule 628

$$\text{Int}[(d_.) + (e_.)*(x_.)]/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x\_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$$

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c \sec(a + bx))^{3/2}}{(d \csc(a + bx))^{3/2}} dx &= \frac{2c\sqrt{c \sec(a + bx)}}{bd\sqrt{d \csc(a + bx)}} - \frac{c^2 \int \frac{\sqrt{d \csc(a+bx)}}{\sqrt{c \sec(a+bx)}} dx}{d^2} \\
&= \frac{2c\sqrt{c \sec(a + bx)}}{bd\sqrt{d \csc(a + bx)}} - \frac{(c^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}) \int \frac{1}{\sqrt{\tan(a+bx)}} dx}{d^2 \sqrt{c \sec(a + bx)}} \\
&= \frac{2c\sqrt{c \sec(a + bx)}}{bd\sqrt{d \csc(a + bx)}} - \frac{(c^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}) \text{Subst} \left( \int \frac{1}{\sqrt{x(1+x^2)}} dx, x, \tan(a + bx) \right)}{bd^2 \sqrt{c \sec(a + bx)}} \\
&= \frac{2c\sqrt{c \sec(a + bx)}}{bd\sqrt{d \csc(a + bx)}} - \frac{(2c^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}) \text{Subst} \left( \int \frac{1}{1+x^4} dx, x, \sqrt{\tan(a + bx)} \right)}{bd^2 \sqrt{c \sec(a + bx)}} \\
&= \frac{2c\sqrt{c \sec(a + bx)}}{bd\sqrt{d \csc(a + bx)}} - \frac{(c^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}) \text{Subst} \left( \int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(a + bx)} \right)}{bd^2 \sqrt{c \sec(a + bx)}} - \frac{c^2 \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(a + bx)} + \tan(a + bx) \right) \sqrt{\tan(a + bx)}}{2\sqrt{2}bd^2 \sqrt{c \sec(a + bx)}} \\
&= \frac{2c\sqrt{c \sec(a + bx)}}{bd\sqrt{d \csc(a + bx)}} + \frac{c^2 \sqrt{d \csc(a + bx)} \log \left( 1 - \sqrt{2} \sqrt{\tan(a + bx)} + \tan(a + bx) \right) \sqrt{\tan(a + bx)}}{2\sqrt{2}bd^2 \sqrt{c \sec(a + bx)}} \\
&= \frac{2c\sqrt{c \sec(a + bx)}}{bd\sqrt{d \csc(a + bx)}} + \frac{c^2 \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(a + bx)} \right) \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}}{\sqrt{2}bd^2 \sqrt{c \sec(a + bx)}} - \frac{c^2 \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(a + bx)} + \tan(a + bx) \right) \sqrt{\tan(a + bx)}}{2\sqrt{2}bd^2 \sqrt{c \sec(a + bx)}}
\end{aligned}$$

**Mathematica [C]** time = 0.242222, size = 64, normalized size = 0.2

$$\frac{2c\sqrt{c \sec(a + bx)} \left( \cot^2(a + bx) \text{Hypergeometric2F1} \left( \frac{3}{4}, 1, \frac{7}{4}, -\cot^2(a + bx) \right) + 3 \right)}{3bd\sqrt{d \csc(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*Sec[a + b\*x])^(3/2)/(d\*Csc[a + b\*x])^(3/2), x]

[Out] (2\*c\*(3 + Cot[a + b\*x]^2\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[a + b\*x]^2])\*Sqrt[c\*Sec[a + b\*x]])/(3\*b\*d\*Sqrt[d\*Csc[a + b\*x]])

**Maple [C]** time = 0.171, size = 656, normalized size = 2.

result too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x)`

[Out]  $\frac{1}{2}b^{1/2}*(I*\sin(b*x+a)*\text{EllipticPi}(\frac{-(-1+\cos(b*x+a)-\sin(b*x+a))}{\sin(b*x+a)})^{1/2},1/2-1/2*I,1/2*2^{1/2})*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}-I*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2})*\text{EllipticPi}(\frac{-(-1+\cos(b*x+a)-\sin(b*x+a))}{\sin(b*x+a)})^{1/2},1/2+1/2*I,1/2*2^{1/2})+\sin(b*x+a)*\text{EllipticPi}(\frac{-(-1+\cos(b*x+a)-\sin(b*x+a))}{\sin(b*x+a)})^{1/2},1/2-1/2*I,1/2*2^{1/2})*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}+\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2})*\text{EllipticPi}(\frac{-(-1+\cos(b*x+a)-\sin(b*x+a))}{\sin(b*x+a)})^{1/2},1/2+1/2*I,1/2*2^{1/2})-2*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2})*\text{EllipticF}(\frac{-(-1+\cos(b*x+a)-\sin(b*x+a))}{\sin(b*x+a)})^{1/2},1/2*2^{1/2})+2*\cos(b*x+a)*2^{1/2}-2*2^{1/2})*\cos(b*x+a)*(c/\cos(b*x+a))^{3/2}/(-1+\cos(b*x+a))/(d/\sin(b*x+a))^{3/2}/\sin(b*x+a)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sec(bx + a))^{\frac{3}{2}}}{(d \csc(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x, algorithm="maxima")`

[Out] `integrate((c*sec(b*x + a))^(3/2)/(d*csc(b*x + a))^(3/2), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(3/2)/(d*csc(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sec(bx + a))^{\frac{3}{2}}}{(d \csc(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2)/(d*csc(b*x + a))^(3/2), x)
```

$$3.245 \quad \int \frac{(c \sec(a+bx))^{3/2}}{(d \csc(a+bx))^{5/2}} dx$$

**Optimal.** Leaf size=94

$$\frac{2c\sqrt{c \sec(a+bx)}}{bd(d \csc(a+bx))^{3/2}} - \frac{3c^2 E\left(a+bx - \frac{\pi}{4} \middle| 2\right)}{bd^2 \sqrt{\sin(2a+2bx)} \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}$$

[Out] (2\*c\*Sqrt[c\*Sec[a + b\*x]])/(b\*d\*(d\*Csc[a + b\*x])^(3/2)) - (3\*c^2\*EllipticE[a - Pi/4 + b\*x, 2])/(b\*d^2\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])

**Rubi [A]** time = 0.152684, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2624, 2630, 2572, 2639}

$$\frac{2c\sqrt{c \sec(a+bx)}}{bd(d \csc(a+bx))^{3/2}} - \frac{3c^2 E\left(a+bx - \frac{\pi}{4} \middle| 2\right)}{bd^2 \sqrt{\sin(2a+2bx)} \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[(c\*Sec[a + b\*x])^(3/2)/(d\*Csc[a + b\*x])^(5/2), x]

[Out] (2\*c\*Sqrt[c\*Sec[a + b\*x]])/(b\*d\*(d\*Csc[a + b\*x])^(3/2)) - (3\*c^2\*EllipticE[a - Pi/4 + b\*x, 2])/(b\*d^2\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])

#### Rule 2624

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(b\*(a\*Csc[e + f\*x])^(m + 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*a\*(n - 1)), x] + Dist[(b^2\*(m + 1))/(a^2\*(n - 1)), Int[(a\*Csc[e + f\*x])^(m + 2)\*(b\*Sec[e + f\*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && LtQ[m, -1] && IntegersQ[2\*m, 2\*n]

#### Rule 2630

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Dist[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n\*(a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]

]

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] := Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]] , x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(c \sec(a + bx))^{3/2}}{(d \csc(a + bx))^{5/2}} dx &= \frac{2c\sqrt{c \sec(a + bx)}}{bd(d \csc(a + bx))^{3/2}} - \frac{(3c^2) \int \frac{1}{\sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}} dx}{d^2} \\ &= \frac{2c\sqrt{c \sec(a + bx)}}{bd(d \csc(a + bx))^{3/2}} - \frac{(3c^2) \int \sqrt{c \cos(a + bx)}\sqrt{d \sin(a + bx)} dx}{d^2 \sqrt{c \cos(a + bx)}\sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}\sqrt{d \sin(a + bx)}} \\ &= \frac{2c\sqrt{c \sec(a + bx)}}{bd(d \csc(a + bx))^{3/2}} - \frac{(3c^2) \int \sqrt{\sin(2a + 2bx)} dx}{d^2 \sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}\sqrt{\sin(2a + 2bx)}} \\ &= \frac{2c\sqrt{c \sec(a + bx)}}{bd(d \csc(a + bx))^{3/2}} - \frac{3c^2 E\left(a - \frac{\pi}{4} + bx \middle| 2\right)}{bd^2 \sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}\sqrt{\sin(2a + 2bx)}} \end{aligned}$$

**Mathematica [C]** time = 0.406607, size = 69, normalized size = 0.73

$$\frac{c\sqrt{c \sec(a + bx)} \left( 3\sqrt[4]{-\cot^2(a + bx)} \operatorname{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \csc^2(a + bx)\right) - 2 \right)}{bd(d \csc(a + bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(3/2)/(d*Csc[a + b*x])^(5/2), x]
```

```
[Out] -((c*(-2 + 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[
a + b*x]^2]))*Sqrt[c*Sec[a + b*x]])/(b*d*(d*Csc[a + b*x])^(3/2))
```

**Maple [B]** time = 0.17, size = 520, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((c \sec(bx+a))^{3/2}/(d \csc(bx+a))^{5/2}, x)$

[Out]  $\frac{1}{2} b^2 \sqrt{2} (6 \cos(bx+a) (-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1 + \cos(bx+a) + \sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1 + \cos(bx+a)) / \sin(bx+a))^{1/2} * \text{EllipticE}((-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{1/2}, 1/2 * 2^{1/2}) - 3 \cos(bx+a) * (-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1 + \cos(bx+a) + \sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1 + \cos(bx+a)) / \sin(bx+a))^{1/2} * \text{EllipticF}((-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{1/2}, 1/2 * 2^{1/2}) + 6 * (-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1 + \cos(bx+a) + \sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1 + \cos(bx+a)) / \sin(bx+a))^{1/2} * \text{EllipticE}((-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{1/2}, 1/2 * 2^{1/2}) - 3 * (-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1 + \cos(bx+a) + \sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1 + \cos(bx+a)) / \sin(bx+a))^{1/2} * \text{EllipticF}((-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{1/2}, 1/2 * 2^{1/2}) + \cos(bx+a)^2 * 2^{1/2} - 3 \cos(bx+a) * 2^{1/2} + 2 * 2^{1/2}) * \cos(bx+a) * (c / \cos(bx+a))^{3/2} / \sin(bx+a)^3 / (d / \sin(bx+a))^{5/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sec(bx + a))^{\frac{3}{2}}}{(d \csc(bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c \sec(bx+a))^{3/2}/(d \csc(bx+a))^{5/2}, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}((c \sec(bx + a))^{3/2}/(d \csc(bx + a))^{5/2}, x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\sqrt{d} \csc(bx + a) \sqrt{c \sec(bx + a)} c \sec(bx + a)}{d^3 \csc(bx + a)^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*c*sec(b*x + a)/(d^3*csc(b*x + a)^3), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(3/2)/(d*csc(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sec (bx + a))^{\frac{3}{2}}}{(d \csc (bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2)/(d*csc(b*x + a))^(5/2), x)
```

### 3.246 $\int (d \csc(a + bx))^{9/2} (c \sec(a + bx))^{5/2} dx$

**Optimal.** Leaf size=166

$$\frac{40c^2d^4\sqrt{\sin(2a+2bx)}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{21b} + \frac{40cd^5(c\sec(a+bx))^{3/2}}{21b\sqrt{d\csc(a+bx)}} - \frac{20cd^3(c\sec(a+bx))^{5/2}}{21b}$$

```
[Out] (40*c*d^5*(c*Sec[a + b*x])^(3/2))/(21*b*Sqrt[d*Csc[a + b*x]]) - (20*c*d^3*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2))/(21*b) - (2*c*d*(d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(3/2))/(7*b) + (40*c^2*d^4*Sqrt[d*Csc[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/(21*b)
```

**Rubi [A]** time = 0.264753, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2625, 2626, 2630, 2573, 2641}

$$\frac{40c^2d^4\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{21b} + \frac{40cd^5(c\sec(a+bx))^{3/2}}{21b\sqrt{d\csc(a+bx)}} - \frac{20cd^3(c\sec(a+bx))^{5/2}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(9/2)*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (40*c*d^5*(c*Sec[a + b*x])^(3/2))/(21*b*Sqrt[d*Csc[a + b*x]]) - (20*c*d^3*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2))/(21*b) - (2*c*d*(d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(3/2))/(7*b) + (40*c^2*d^4*Sqrt[d*Csc[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/(21*b)
```

#### Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && IntegersQ[2*m, 2*n] && !GtQ[n, m]
```

#### Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && IntegersQ[2*m, 2*n] && !GtQ[n, m]
```

1))/(f\*(n - 1)), x] + Dist[(b^2\*(m + n - 2))/(n - 1), Int[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && IntegersQ[2\*m, 2\*n]

### Rule 2630

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^m\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^n), x\_Symbol] := Dist[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n\*(a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]

### Rule 2573

Int[1/(Sqrt[cos[(e\_.) + (f\_.)\*(x\_.)]\*(b\_.)]\*Sqrt[(a\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]]), x\_Symbol] := Dist[Sqrt[Sin[2\*e + 2\*f\*x]]/(Sqrt[a\*Sin[e + f\*x]]\*Sqrt[b\*Cos[e + f\*x]]), Int[1/Sqrt[Sin[2\*e + 2\*f\*x]], x], x] /; FreeQ[{a, b, e, f}, x]

### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] := Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned}
 \int (d \csc(a + bx))^{9/2} (c \sec(a + bx))^{5/2} dx &= -\frac{2cd(d \csc(a + bx))^{7/2} (c \sec(a + bx))^{3/2}}{7b} + \frac{1}{7} (10d^2) \int (d \csc(a + bx))^{5/2} (c \sec(a + bx))^{3/2} dx \\
 &= -\frac{20cd^3(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2}}{21b} - \frac{2cd(d \csc(a + bx))^{7/2} (c \sec(a + bx))^{3/2}}{7b} \\
 &= \frac{40cd^5(c \sec(a + bx))^{3/2}}{21b\sqrt{d} \csc(a + bx)} - \frac{20cd^3(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2}}{21b} - \frac{2cd(d \csc(a + bx))^{7/2} (c \sec(a + bx))^{3/2}}{7b} \\
 &= \frac{40cd^5(c \sec(a + bx))^{3/2}}{21b\sqrt{d} \csc(a + bx)} - \frac{20cd^3(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2}}{21b} - \frac{2cd(d \csc(a + bx))^{7/2} (c \sec(a + bx))^{3/2}}{7b} \\
 &= \frac{40cd^5(c \sec(a + bx))^{3/2}}{21b\sqrt{d} \csc(a + bx)} - \frac{20cd^3(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2}}{21b} - \frac{2cd(d \csc(a + bx))^{7/2} (c \sec(a + bx))^{3/2}}{7b} \\
 &= \frac{40cd^5(c \sec(a + bx))^{3/2}}{21b\sqrt{d} \csc(a + bx)} - \frac{20cd^3(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2}}{21b} - \frac{2cd(d \csc(a + bx))^{7/2} (c \sec(a + bx))^{3/2}}{7b}
 \end{aligned}$$



**Mathematica [C]** time = 1.42326, size = 92, normalized size = 0.55

$$\frac{2cd^5(c \sec(a + bx))^{3/2} \left( 20 (-\cot^2(a + bx))^{3/4} \text{Hypergeometric2F1} \left( \frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \csc^2(a + bx) \right) + \cot^2(a + bx) (3 \csc^2(a + bx) + 2) \right)}{21b\sqrt{d \csc(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(9/2)\*(c\*Sec[a + b\*x])^(5/2),x]

[Out] (-2\*c\*d^5\*(-7 + Cot[a + b\*x]^2\*(13 + 3\*Csc[a + b\*x]^2) + 20\*(-Cot[a + b\*x]^2)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b\*x]^2])\*(c\*Sec[a + b\*x])^(3/2))/(21\*b\*Sqrt[d\*Csc[a + b\*x]])

**Maple [B]** time = 0.232, size = 563, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(9/2)\*(c\*sec(b\*x+a))^(5/2),x)

[Out] -1/21/b\*2^(1/2)\*(40\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1/2\*2^(1/2))\*cos(b\*x+a)^4\*sin(b\*x+a)+40\*cos(b\*x+a)^3\*sin(b\*x+a)\*((-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1/2\*2^(1/2))-40\*cos(b\*x+a)^2\*sin(b\*x+a)\*((-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1/2\*2^(1/2))-40\*cos(b\*x+a)\*sin(b\*x+a)\*((-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1/2\*2^(1/2))-20\*2^(1/2)\*cos(b\*x+a)^4+30\*cos(b\*x+a)^2\*2^(1/2)-7\*2^(1/2))\*cos(b\*x+a)\*(d/sin(b\*x+a))^(9/2)\*(c/cos(b\*x+a))^(5/2)\*sin(b\*x+a)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc (bx + a))^{\frac{9}{2}} (c \sec (bx + a))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(9/2)\*(c\*sec(b\*x+a))^(5/2),x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(9/2)\*(c\*sec(b\*x + a))^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d \csc(bx + a)}\sqrt{c \sec(bx + a)}c^2d^4 \csc(bx + a)^4 \sec(bx + a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(9/2)\*(c\*sec(b\*x+a))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a))\*c^2\*d^4\*csc(b\*x + a)^4\*sec(b\*x + a)^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(9/2)\*(c\*sec(b\*x+a))\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc(bx + a))^{\frac{9}{2}} (c \sec(bx + a))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(9/2)\*(c\*sec(b\*x+a))^(5/2),x, algorithm="giac")

[Out] integrate((d\*csc(b\*x + a))^(9/2)\*(c\*sec(b\*x + a))^(5/2), x)

### 3.247 $\int (d \csc(a + bx))^{7/2} (c \sec(a + bx))^{5/2} dx$

**Optimal.** Leaf size=106

$$-\frac{64c^3d^3\sqrt{d \csc(a + bx)}}{15b\sqrt{c \sec(a + bx)}} + \frac{16cd^3(c \sec(a + bx))^{3/2}\sqrt{d \csc(a + bx)}}{15b} - \frac{2cd(c \sec(a + bx))^{3/2}(d \csc(a + bx))^{5/2}}{5b}$$

[Out]  $(-64*c^3*d^3*\text{Sqrt}[d*\text{Csc}[a + b*x]])/(15*b*\text{Sqrt}[c*\text{Sec}[a + b*x]]) + (16*c*d^3*\text{Sqrt}[d*\text{Csc}[a + b*x]]*(c*\text{Sec}[a + b*x])^{(3/2)})/(15*b) - (2*c*d*(d*\text{Csc}[a + b*x])^{(5/2)}*(c*\text{Sec}[a + b*x])^{(3/2)})/(5*b)$

**Rubi [A]** time = 0.162101, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$ , Rules used = {2625, 2626, 2619}

$$-\frac{64c^3d^3\sqrt{d \csc(a + bx)}}{15b\sqrt{c \sec(a + bx)}} + \frac{16cd^3(c \sec(a + bx))^{3/2}\sqrt{d \csc(a + bx)}}{15b} - \frac{2cd(c \sec(a + bx))^{3/2}(d \csc(a + bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*\text{Csc}[a + b*x])^{(7/2)}*(c*\text{Sec}[a + b*x])^{(5/2)}, x]$

[Out]  $(-64*c^3*d^3*\text{Sqrt}[d*\text{Csc}[a + b*x]])/(15*b*\text{Sqrt}[c*\text{Sec}[a + b*x]]) + (16*c*d^3*\text{Sqrt}[d*\text{Csc}[a + b*x]]*(c*\text{Sec}[a + b*x])^{(3/2)})/(15*b) - (2*c*d*(d*\text{Csc}[a + b*x])^{(5/2)}*(c*\text{Sec}[a + b*x])^{(3/2)})/(5*b)$

#### Rule 2625

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(a_.)^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_)])^{(n_.)}, x\_Symbol] :> -\text{Simp}[(a*b*(a*\text{Csc}[e + f*x])^{(m - 1)}*(b*\text{Sec}[e + f*x])^{(n - 1)})/(f*(m - 1)), x] + \text{Dist}[(a^2*(m + n - 2))/(m - 1), \text{Int}[(a*\text{Csc}[e + f*x])^{(m - 2)}*(b*\text{Sec}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, e, f, n\}, x] \&\& \text{GtQ}[m, 1] \&\& \text{IntegersQ}[2*m, 2*n] \&\& !\text{GtQ}[n, m]$

#### Rule 2626

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(a_.)^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_)])^{(n_.)}, x\_Symbol] :> \text{Simp}[(a*b*(a*\text{Csc}[e + f*x])^{(m - 1)}*(b*\text{Sec}[e + f*x])^{(n - 1)})/(f*(n - 1)), x] + \text{Dist}[(b^2*(m + n - 2))/(n - 1), \text{Int}[(a*\text{Csc}[e + f*x])^m*(b*\text{Sec}[e + f*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{a, b, e, f, m\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegersQ}[2*m, 2*n]$

**Rule 2619**

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1))/(f*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] && NeQ[n, 1]
```

**Rubi steps**

$$\begin{aligned} \int (d \csc(a + bx))^{7/2} (c \sec(a + bx))^{5/2} dx &= -\frac{2cd(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{3/2}}{5b} + \frac{1}{5} (8d^2) \int (d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2} dx \\ &= \frac{16cd^3 \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{3/2}}{15b} - \frac{2cd(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{3/2}}{5b} \\ &= -\frac{64c^3 d^3 \sqrt{d \csc(a + bx)}}{15b \sqrt{c \sec(a + bx)}} + \frac{16cd^3 \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{3/2}}{15b} - \frac{2cd(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{3/2}}{5b} \end{aligned}$$

**Mathematica [A]** time = 0.193343, size = 57, normalized size = 0.54

$$-\frac{2cd^3 (c \sec(a + bx))^{3/2} \sqrt{d \csc(a + bx)} (32 \cos^2(a + bx) + 3 \cot^2(a + bx) - 5)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(5/2), x]
```

```
[Out] (-2*c*d^3*(-5 + 32*Cos[a + b*x]^2 + 3*Cot[a + b*x]^2)*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2))/(15*b)
```

**Maple [A]** time = 0.165, size = 64, normalized size = 0.6

$$\frac{(64 (\cos(bx + a))^4 - 80 (\cos(bx + a))^2 + 10) \cos(bx + a) \sin(bx + a)}{15b} \left( \frac{d}{\sin(bx + a)} \right)^{7/2} \left( \frac{c}{\cos(bx + a)} \right)^{5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2), x)
```

[Out]  $2/15/b*(32*\cos(b*x+a)^4-40*\cos(b*x+a)^2+5)*\cos(b*x+a)*(d/\sin(b*x+a))^{(7/2)}*(c/\cos(b*x+a))^{(5/2)}*\sin(b*x+a)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc (bx + a))^{\frac{7}{2}} (c \sec (bx + a))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")`

[Out] `integrate((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(5/2), x)`

**Fricas [A]** time = 1.87092, size = 207, normalized size = 1.95

$$\frac{2 \left( 32 c^2 d^3 \cos (bx + a)^4 - 40 c^2 d^3 \cos (bx + a)^2 + 5 c^2 d^3 \right) \sqrt{\frac{c}{\cos (bx + a)}} \sqrt{\frac{d}{\sin (bx + a)}}}{15 \left( b \cos (bx + a)^3 - b \cos (bx + a) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")`

[Out] `-2/15*(32*c^2*d^3*cos(b*x + a)^4 - 40*c^2*d^3*cos(b*x + a)^2 + 5*c^2*d^3)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))/(b*cos(b*x + a)^3 - b*cos(b*x + a))`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))**(7/2)*(c*sec(b*x+a))**(5/2),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc (bx + a))^{\frac{7}{2}} (c \sec (bx + a))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(5/2), x)
```

### 3.248 $\int (d \csc(a + bx))^{5/2} (c \sec(a + bx))^{5/2} dx$

**Optimal.** Leaf size=131

$$\frac{4c^2d^2\sqrt{\sin(2a+2bx)}\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3b} + \frac{4cd^3(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}} - \frac{2cd(c\sec(a+bx))^{3/2}}{3b}$$

[Out] (4\*c\*d^3\*(c\*Sec[a + b\*x])^(3/2))/(3\*b\*Sqrt[d\*Csc[a + b\*x]]) - (2\*c\*d\*(d\*Csc[a + b\*x])^(3/2)\*(c\*Sec[a + b\*x])^(3/2))/(3\*b) + (4\*c^2\*d^2\*Sqrt[d\*Csc[a + b\*x]]\*EllipticF[a - Pi/4 + b\*x, 2]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])/(3\*b)

**Rubi [A]** time = 0.2044, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2625, 2626, 2630, 2573, 2641}

$$\frac{4c^2d^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3b} + \frac{4cd^3(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}} - \frac{2cd(c\sec(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In] Int[(d\*Csc[a + b\*x])^(5/2)\*(c\*Sec[a + b\*x])^(5/2),x]

[Out] (4\*c\*d^3\*(c\*Sec[a + b\*x])^(3/2))/(3\*b\*Sqrt[d\*Csc[a + b\*x]]) - (2\*c\*d\*(d\*Csc[a + b\*x])^(3/2)\*(c\*Sec[a + b\*x])^(3/2))/(3\*b) + (4\*c^2\*d^2\*Sqrt[d\*Csc[a + b\*x]]\*EllipticF[a - Pi/4 + b\*x, 2]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])/(3\*b)

#### Rule 2625

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> -Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*(m - 1)), x] + Dist[(a^2\*(m + n - 2))/(m - 1), Int[(a\*Csc[e + f\*x])^(m - 2)\*(b\*Sec[e + f\*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && IntegersQ[2\*m, 2\*n] && !GtQ[n, m]

#### Rule 2626

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*(n - 1)), x] + Dist[(b^2\*(m + n - 2))/(n - 1), Int[(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[n, m]

$m*(b*\text{Sec}[e + f*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{a, b, e, f, m\}, x\} \&\& \text{GtQ}[n, 1]$   
 $] \&\& \text{IntegersQ}[2*m, 2*n]$

### Rule 2630

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] := \text{Dist}[(a*\text{Csc}[e + f*x])^{m*(b*\text{Sec}[e + f*x])^n*(a*\text{Sin}[e + f*x])^m*(b*\text{Cos}[e + f*x])^n}, \text{Int}[1/((a*\text{Sin}[e + f*x])^m*(b*\text{Cos}[e + f*x])^n), x], x] /; \text{FreeQ}\{a, b, e, f, m, n\}, x\} \&\& \text{IntegerQ}[m - 1/2] \&\& \text{IntegerQ}[n - 1/2]$

### Rule 2573

$\text{Int}[1/(\text{Sqrt}[\text{cos}[(e_.) + (f_.)*(x_.)]*(b_.)]*\text{Sqrt}[(a_.)*\text{sin}[(e_.) + (f_.)*(x_.)]]), x\_Symbol] := \text{Dist}[\text{Sqrt}[\text{Sin}[2*e + 2*f*x]]/(\text{Sqrt}[a*\text{Sin}[e + f*x]]*\text{Sqrt}[b*\text{Cos}[e + f*x]]), \text{Int}[1/\text{Sqrt}[\text{Sin}[2*e + 2*f*x]], x], x] /; \text{FreeQ}\{a, b, e, f\}, x]$

### Rule 2641

$\text{Int}[1/\text{Sqrt}[\text{sin}[(c_.) + (d_.)*(x_.)]], x\_Symbol] := \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

### Rubi steps

$$\begin{aligned} \int (d \csc(a + bx))^{5/2} (c \sec(a + bx))^{5/2} dx &= -\frac{2cd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2}}{3b} + (2d^2) \int \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{3/2} dx \\ &= \frac{4cd^3 (c \sec(a + bx))^{3/2}}{3b\sqrt{d \csc(a + bx)}} - \frac{2cd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2}}{3b} + \frac{1}{3} (4c^2 d^2) \int \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{3/2} dx \\ &= \frac{4cd^3 (c \sec(a + bx))^{3/2}}{3b\sqrt{d \csc(a + bx)}} - \frac{2cd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2}}{3b} + \frac{1}{3} (4c^2 d^2 \sqrt{cd}) \int \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{3/2} dx \\ &= \frac{4cd^3 (c \sec(a + bx))^{3/2}}{3b\sqrt{d \csc(a + bx)}} - \frac{2cd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2}}{3b} + \frac{1}{3} (4c^2 d^2 \sqrt{cd}) \int \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{3/2} dx \\ &= \frac{4cd^3 (c \sec(a + bx))^{3/2}}{3b\sqrt{d \csc(a + bx)}} - \frac{2cd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2}}{3b} + \frac{4c^2 d^2 \sqrt{cd} \int \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{3/2} dx}{3} \end{aligned}$$

**Mathematica [C]** time = 0.65213, size = 87, normalized size = 0.66

$$\frac{2c^3 d \tan^2(a + bx) (d \csc(a + bx))^{3/2} \left( 2 \left( -\cot^2(a + bx) \right)^{3/4} \text{Hypergeometric2F1} \left( \frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \csc^2(a + bx) \right) + \cot^2(a + bx) \right)}{3b\sqrt{c \sec(a + bx)}}$$



Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(5/2)\*(c\*Sec[a + b\*x])^(5/2),x]

[Out]  $(-2*c^3*d*(d*Csc[a + b*x])^{3/2}*(-1 + Cot[a + b*x]^2 + 2*(-Cot[a + b*x]^2)^{3/4}*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2])*Tan[a + b*x]^2)/(3*b*Sqrt[c*Sec[a + b*x]])$

**Maple [B]** time = 0.195, size = 304, normalized size = 2.3

$$\frac{\cos(bx + a) \sqrt{2} \sin(bx + a)}{3b} \left( 4 (\cos(bx + a))^2 \sin(bx + a) \sqrt{\frac{-1 + \cos(bx + a) - \sin(bx + a)}{\sin(bx + a)}} \sqrt{\frac{-1 + \cos(bx + a) + \sin(bx + a)}{\sin(bx + a)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(5/2)\*(c\*sec(b\*x+a))^(5/2),x)

[Out]  $\frac{1}{3} \frac{1}{b^2} \sqrt{2} \cos(bx+a)^2 \sin(bx+a) \left( \frac{-1 + \cos(bx+a) - \sin(bx+a)}{\sin(bx+a)} \right)^{1/2} \left( \frac{-1 + \cos(bx+a) + \sin(bx+a)}{\sin(bx+a)} \right)^{1/2} \left( \frac{-1 + \cos(bx+a)}{\sin(bx+a)} \right)^{1/2} \text{EllipticF}\left(\frac{-1 + \cos(bx+a) - \sin(bx+a)}{\sin(bx+a)}\right)^{1/2}, \frac{1}{2} \sqrt{2} \cos(bx+a) \sin(bx+a) \left( \frac{-1 + \cos(bx+a) - \sin(bx+a)}{\sin(bx+a)} \right)^{1/2} \left( \frac{-1 + \cos(bx+a) + \sin(bx+a)}{\sin(bx+a)} \right)^{1/2} \left( \frac{-1 + \cos(bx+a)}{\sin(bx+a)} \right)^{1/2} \text{EllipticF}\left(\frac{-1 + \cos(bx+a) - \sin(bx+a)}{\sin(bx+a)}\right)^{1/2}, \frac{1}{2} \sqrt{2} \cos(bx+a)^2 \sqrt{2} \cos(bx+a) \left( \frac{d}{\sin(bx+a)} \right)^{5/2} \left( \frac{c}{\cos(bx+a)} \right)^{5/2} \sin(bx+a)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc(bx + a))^{\frac{5}{2}} (c \sec(bx + a))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(5/2)\*(c\*sec(b\*x+a))^(5/2),x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(5/2)\*(c\*sec(b\*x + a))^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d \csc(bx+a)}\sqrt{c \sec(bx+a)}c^2d^2 \csc(bx+a)^2 \sec(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(5/2)\*(c\*sec(b\*x+a))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a))\*c^2\*d^2\*csc(b\*x + a)^2\*sec(b\*x + a)^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(5/2)\*(c\*sec(b\*x+a))\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc(bx+a))^{\frac{5}{2}} (c \sec(bx+a))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(5/2)\*(c\*sec(b\*x+a))^(5/2),x, algorithm="giac")

[Out] integrate((d\*csc(b\*x + a))^(5/2)\*(c\*sec(b\*x + a))^(5/2), x)

### 3.249 $\int (d \csc(a + bx))^{3/2} (c \sec(a + bx))^{5/2} dx$

**Optimal.** Leaf size=69

$$\frac{2cd(c \sec(a + bx))^{3/2} \sqrt{d \csc(a + bx)}}{3b} - \frac{8c^3 d \sqrt{d \csc(a + bx)}}{3b \sqrt{c \sec(a + bx)}}$$

[Out]  $(-8*c^3*d*\text{Sqrt}[d*\text{Csc}[a + b*x]])/(3*b*\text{Sqrt}[c*\text{Sec}[a + b*x]]) + (2*c*d*\text{Sqrt}[d*\text{Csc}[a + b*x]]*(c*\text{Sec}[a + b*x])^{(3/2)})/(3*b)$

**Rubi [A]** time = 0.102423, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {2626, 2619}

$$\frac{2cd(c \sec(a + bx))^{3/2} \sqrt{d \csc(a + bx)}}{3b} - \frac{8c^3 d \sqrt{d \csc(a + bx)}}{3b \sqrt{c \sec(a + bx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*\text{Csc}[a + b*x])^{(3/2)}*(c*\text{Sec}[a + b*x])^{(5/2)}, x]$

[Out]  $(-8*c^3*d*\text{Sqrt}[d*\text{Csc}[a + b*x]])/(3*b*\text{Sqrt}[c*\text{Sec}[a + b*x]]) + (2*c*d*\text{Sqrt}[d*\text{Csc}[a + b*x]]*(c*\text{Sec}[a + b*x])^{(3/2)})/(3*b)$

#### Rule 2626

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] :> \text{Simp}[(a*b*(a*\text{Csc}[e + f*x])^{(m - 1)}*(b*\text{Sec}[e + f*x])^{(n - 1)})/(f*(n - 1)), x] + \text{Dist}[(b^2*(m + n - 2))/(n - 1), \text{Int}[(a*\text{Csc}[e + f*x])^{(m)}*(b*\text{Sec}[e + f*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{a, b, e, f, m\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegersQ}[2*m, 2*n]$

#### Rule 2619

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] :> \text{Simp}[(a*b*(a*\text{Csc}[e + f*x])^{(m - 1)}*(b*\text{Sec}[e + f*x])^{(n - 1)})/(f*(n - 1)), x] /; \text{FreeQ}\{a, b, e, f, m, n\}, x] \&\& \text{EqQ}[m + n - 2, 0] \&\& \text{NeQ}[n, 1]$

#### Rubi steps

$$\int (d \csc(a + bx))^{3/2} (c \sec(a + bx))^{5/2} dx = \frac{2cd\sqrt{d \csc(a + bx)}(c \sec(a + bx))^{3/2}}{3b} + \frac{1}{3} (4c^2) \int (d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)} dx$$

$$= -\frac{8c^3 d \sqrt{d \csc(a + bx)}}{3b \sqrt{c \sec(a + bx)}} + \frac{2cd\sqrt{d \csc(a + bx)}(c \sec(a + bx))^{3/2}}{3b}$$

**Mathematica [A]** time = 0.207171, size = 45, normalized size = 0.65

$$\frac{2cd(2 \cos(2(a + bx)) + 1)(c \sec(a + bx))^{3/2} \sqrt{d \csc(a + bx)}}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(3/2)\*(c\*Sec[a + b\*x])^(5/2), x]

[Out] (-2\*c\*d\*(1 + 2\*Cos[2\*(a + b\*x)])\*Sqrt[d\*Csc[a + b\*x]]\*(c\*Sec[a + b\*x])^(3/2))/(3\*b)

**Maple [A]** time = 0.161, size = 54, normalized size = 0.8

$$-\frac{(8 (\cos (bx + a))^2 - 2) \cos (bx + a) \sin (bx + a)}{3b} \left( \frac{d}{\sin (bx + a)} \right)^{\frac{3}{2}} \left( \frac{c}{\cos (bx + a)} \right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(3/2)\*(c\*sec(b\*x+a))^(5/2), x)

[Out] -2/3/b\*(4\*cos(b\*x+a)^2-1)\*cos(b\*x+a)\*(d/sin(b\*x+a))^(3/2)\*(c/cos(b\*x+a))^(5/2)\*sin(b\*x+a)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc (bx + a))^{\frac{3}{2}} (c \sec (bx + a))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(3/2)\*(c\*sec(b\*x+a))^(5/2),x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(3/2)\*(c\*sec(b\*x + a))^(5/2), x)

**Fricas [A]** time = 1.54961, size = 132, normalized size = 1.91

$$\frac{2 \left( 4c^2d \cos(bx + a)^2 - c^2d \right) \sqrt{\frac{c}{\cos(bx+a)}} \sqrt{\frac{d}{\sin(bx+a)}}}{3b \cos(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(3/2)\*(c\*sec(b\*x+a))^(5/2),x, algorithm="fricas")

[Out] -2/3\*(4\*c^2\*d\*cos(b\*x + a)^2 - c^2\*d)\*sqrt(c/cos(b\*x + a))\*sqrt(d/sin(b\*x + a))/(b\*cos(b\*x + a))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(3/2)\*(c\*sec(b\*x+a))\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (d \csc(bx + a))^{\frac{3}{2}} (c \sec(bx + a))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(3/2)\*(c\*sec(b\*x+a))^(5/2),x, algorithm="giac")

[Out] integrate((d\*csc(b\*x + a))^(3/2)\*(c\*sec(b\*x + a))^(5/2), x)

### 3.250 $\int \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{5/2} dx$

**Optimal.** Leaf size=93

$$\frac{2c^2 \sqrt{\sin(2a + 2bx)} \operatorname{EllipticF}\left(a + bx - \frac{\pi}{4}, 2\right) \sqrt{c \sec(a + bx)} \sqrt{d \csc(a + bx)}}{3b} + \frac{2cd(c \sec(a + bx))^{3/2}}{3b \sqrt{d \csc(a + bx)}}$$

[Out] (2\*c\*d\*(c\*Sec[a + b\*x])^(3/2))/(3\*b\*Sqrt[d\*Csc[a + b\*x]]) + (2\*c^2\*Sqrt[d\*Csc[a + b\*x]]\*EllipticF[a - Pi/4 + b\*x, 2]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])/(3\*b)

**Rubi [A]** time = 0.140581, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2626, 2630, 2573, 2641}

$$\frac{2c^2 \sqrt{\sin(2a + 2bx)} F\left(a + bx - \frac{\pi}{4} \mid 2\right) \sqrt{c \sec(a + bx)} \sqrt{d \csc(a + bx)}}{3b} + \frac{2cd(c \sec(a + bx))^{3/2}}{3b \sqrt{d \csc(a + bx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*Csc[a + b\*x]]\*(c\*Sec[a + b\*x])^(5/2),x]

[Out] (2\*c\*d\*(c\*Sec[a + b\*x])^(3/2))/(3\*b\*Sqrt[d\*Csc[a + b\*x]]) + (2\*c^2\*Sqrt[d\*Csc[a + b\*x]]\*EllipticF[a - Pi/4 + b\*x, 2]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])/(3\*b)

#### Rule 2626

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*(n - 1)), x] + Dist[(b^2\*(m + n - 2))/(n - 1), Int[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && IntegersQ[2\*m, 2\*n]

#### Rule 2630

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n\*(a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]

Rule 2573

Int[1/(Sqrt[cos[(e\_.) + (f\_.)\*(x\_.)]\*(b\_.)]\*Sqrt[(a\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]]), x\_Symbol] :> Dist[Sqrt[Sin[2\*e + 2\*f\*x]]/(Sqrt[a\*Sin[e + f\*x]]\*Sqrt[b\*Cos[e + f\*x]]), Int[1/Sqrt[Sin[2\*e + 2\*f\*x]], x], x] /; FreeQ[{a, b, e, f}, x]

Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] :> Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{5/2} dx &= \frac{2cd(c \sec(a + bx))^{3/2}}{3b\sqrt{d} \csc(a + bx)} + \frac{1}{3} (2c^2) \int \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} dx \\ &= \frac{2cd(c \sec(a + bx))^{3/2}}{3b\sqrt{d} \csc(a + bx)} + \frac{1}{3} (2c^2 \sqrt{c \cos(a + bx)} \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} \sqrt{d} dx) \\ &= \frac{2cd(c \sec(a + bx))^{3/2}}{3b\sqrt{d} \csc(a + bx)} + \frac{1}{3} (2c^2 \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} \sqrt{\sin(2a + 2bx)}) \int dx \\ &= \frac{2cd(c \sec(a + bx))^{3/2}}{3b\sqrt{d} \csc(a + bx)} + \frac{2c^2 \sqrt{d \csc(a + bx)} F\left(a - \frac{\pi}{4} + bx \middle| 2\right) \sqrt{c \sec(a + bx)} \sqrt{\sin(2a + 2bx)}}{3b} \end{aligned}$$

**Mathematica [C]** time = 0.57948, size = 68, normalized size = 0.73

$$\frac{2cd(c \sec(a + bx))^{3/2} \left( (-\cot^2(a + bx))^{3/4} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \csc^2(a + bx)\right) - 1 \right)}{3b\sqrt{d} \csc(a + bx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*Csc[a + b\*x]]\*(c\*Sec[a + b\*x])^(5/2),x]

[Out] (-2\*c\*d\*(-1 + (-Cot[a + b\*x]^2)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b\*x]^2])\*(c\*Sec[a + b\*x])^(3/2))/(3\*b\*Sqrt[d\*Csc[a + b\*x]])

**Maple [A]** time = 0.206, size = 190, normalized size = 2.

$$\frac{\cos(bx + a) \sqrt{2} \sin(bx + a)}{3b(-1 + \cos(bx + a))} \left( 2 \cos(bx + a) \sin(bx + a) \sqrt{\frac{-1 + \cos(bx + a) - \sin(bx + a)}{\sin(bx + a)}} \sqrt{\frac{-1 + \cos(bx + a) + \sin(bx + a)}{\sin(bx + a)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x)`

[Out] 
$$-1/3/b*2^{(1/2)}*(2*\cos(b*x+a)*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}*\text{EllipticF}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}, 1/2*2^{(1/2)})-\cos(b*x+a)*2^{(1/2)}+2^{(1/2)}*\cos(b*x+a)*(d/\sin(b*x+a))^{(1/2)}*(c/\cos(b*x+a))^{(5/2)}*\sin(b*x+a)/(-1+\cos(b*x+a))$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \csc(bx + a)} (c \sec(bx + a))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(5/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} c^2 \sec(bx + a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*c^2*sec(b*x + a)^2, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((d\*csc(b\*x+a))\*\*(1/2)\*(c\*sec(b\*x+a))\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \csc(bx + a)} (c \sec(bx + a))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(1/2)\*(c\*sec(b\*x+a))^(5/2),x, algorithm="giac")

[Out] integrate(sqrt(d\*csc(b\*x + a))\*(c\*sec(b\*x + a))^(5/2), x)

$$3.251 \quad \int \frac{(c \sec(a+bx))^{5/2}}{\sqrt{d} \csc(a+bx)} dx$$

**Optimal.** Leaf size=33

$$\frac{2cd(c \sec(a+bx))^{3/2}}{3b(d \csc(a+bx))^{3/2}}$$

[Out] (2\*c\*d\*(c\*Sec[a + b\*x])^(3/2))/(3\*b\*(d\*Csc[a + b\*x])^(3/2))

**Rubi [A]** time = 0.0494305, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {2619}

$$\frac{2cd(c \sec(a+bx))^{3/2}}{3b(d \csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*Sec[a + b\*x])^(5/2)/Sqrt[d\*Csc[a + b\*x]],x]

[Out] (2\*c\*d\*(c\*Sec[a + b\*x])^(3/2))/(3\*b\*(d\*Csc[a + b\*x])^(3/2))

### Rule 2619

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] && NeQ[n, 1]

### Rubi steps

$$\int \frac{(c \sec(a+bx))^{5/2}}{\sqrt{d} \csc(a+bx)} dx = \frac{2cd(c \sec(a+bx))^{3/2}}{3b(d \csc(a+bx))^{3/2}}$$

**Mathematica [A]** time = 0.111973, size = 33, normalized size = 1.

$$\frac{2cd(c \sec(a+bx))^{3/2}}{3b(d \csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*Sec[a + b\*x])^(5/2)/Sqrt[d\*Csc[a + b\*x]], x]

[Out] (2\*c\*d\*(c\*Sec[a + b\*x])^(3/2))/(3\*b\*(d\*Csc[a + b\*x])^(3/2))

**Maple [A]** time = 0.149, size = 42, normalized size = 1.3

$$\frac{2 \cos(bx + a) \sin(bx + a)}{3b} \left( \frac{c}{\cos(bx + a)} \right)^{\frac{5}{2}} \frac{1}{\sqrt{\frac{d}{\sin(bx+a)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*sec(b\*x+a))^(5/2)/(d\*csc(b\*x+a))^(1/2), x)

[Out] 2/3/b\*cos(b\*x+a)\*sin(b\*x+a)\*(c/cos(b\*x+a))^(5/2)/(d/sin(b\*x+a))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sec(bx + a))^{\frac{5}{2}}}{\sqrt{d \csc(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(5/2)/(d\*csc(b\*x+a))^(1/2), x, algorithm="maxima")

[Out] integrate((c\*sec(b\*x + a))^(5/2)/sqrt(d\*csc(b\*x + a)), x)

**Fricas [B]** time = 1.36281, size = 127, normalized size = 3.85

$$\frac{2 \left( c^2 \cos(bx + a)^2 - c^2 \right) \sqrt{\frac{c}{\cos(bx+a)}} \sqrt{\frac{d}{\sin(bx+a)}}}{3bd \cos(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(5/2)/(d\*csc(b\*x+a))^(1/2),x, algorithm="fricas")

[Out] -2/3\*(c^2\*cos(b\*x + a)^2 - c^2)\*sqrt(c/cos(b\*x + a))\*sqrt(d/sin(b\*x + a))/(b\*d\*cos(b\*x + a))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))\*\*(5/2)/(d\*csc(b\*x+a))\*\*(1/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sec(bx + a))^{\frac{5}{2}}}{\sqrt{d} \csc(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(5/2)/(d\*csc(b\*x+a))^(1/2),x, algorithm="giac")

[Out] integrate((c\*sec(b\*x + a))^(5/2)/sqrt(d\*csc(b\*x + a)), x)

$$3.252 \quad \int \frac{(c \sec(a+bx))^{5/2}}{(d \csc(a+bx))^{3/2}} dx$$

**Optimal.** Leaf size=98

$$\frac{2c(c \sec(a+bx))^{3/2}}{3bd\sqrt{d \csc(a+bx)}} - \frac{c^2 \sqrt{\sin(2a+2bx)} \text{EllipticF}\left(a+bx - \frac{\pi}{4}, 2\right) \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}{3bd^2}$$

[Out] (2\*c\*(c\*Sec[a + b\*x])^(3/2))/(3\*b\*d\*Sqrt[d\*Csc[a + b\*x]]) - (c^2\*Sqrt[d\*Csc[a + b\*x]]\*EllipticF[a - Pi/4 + b\*x, 2]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])/(3\*b\*d^2)

**Rubi [A]** time = 0.149594, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2624, 2630, 2573, 2641}

$$\frac{2c(c \sec(a+bx))^{3/2}}{3bd\sqrt{d \csc(a+bx)}} - \frac{c^2 \sqrt{\sin(2a+2bx)} F\left(a+bx - \frac{\pi}{4} \middle| 2\right) \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}{3bd^2}$$

Antiderivative was successfully verified.

[In] Int[(c\*Sec[a + b\*x])^(5/2)/(d\*Csc[a + b\*x])^(3/2), x]

[Out] (2\*c\*(c\*Sec[a + b\*x])^(3/2))/(3\*b\*d\*Sqrt[d\*Csc[a + b\*x]]) - (c^2\*Sqrt[d\*Csc[a + b\*x]]\*EllipticF[a - Pi/4 + b\*x, 2]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])/(3\*b\*d^2)

#### Rule 2624

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(b\*(a\*Csc[e + f\*x])^(m + 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*a\*(n - 1)), x] + Dist[(b^2\*(m + 1))/(a^2\*(n - 1)), Int[(a\*Csc[e + f\*x])^(m + 2)\*(b\*Sec[e + f\*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && LtQ[m, -1] && IntegersQ[2\*m, 2\*n]

#### Rule 2630

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Dist[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n\*(a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]

]

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_.)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_.)]]), x_Symbol] := Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*SIN[e + f*x]]*Sqrt[b*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(c \sec(a + bx))^{5/2}}{(d \csc(a + bx))^{3/2}} dx &= \frac{2c(c \sec(a + bx))^{3/2}}{3bd\sqrt{d} \csc(a + bx)} - \frac{c^2 \int \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} dx}{3d^2} \\ &= \frac{2c(c \sec(a + bx))^{3/2}}{3bd\sqrt{d} \csc(a + bx)} - \frac{(c^2 \sqrt{c \cos(a + bx)} \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} \sqrt{d \sin(a + bx)}) \int \frac{1}{\sqrt{c \cos(a + bx)}} dx}{3d^2} \\ &= \frac{2c(c \sec(a + bx))^{3/2}}{3bd\sqrt{d} \csc(a + bx)} - \frac{(c^2 \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} \sqrt{\sin(2a + 2bx)}) \int \frac{1}{\sqrt{\sin(2a + 2bx)}} dx}{3d^2} \\ &= \frac{2c(c \sec(a + bx))^{3/2}}{3bd\sqrt{d} \csc(a + bx)} - \frac{c^2 \sqrt{d \csc(a + bx)} F\left(a - \frac{\pi}{4} + bx \mid 2\right) \sqrt{c \sec(a + bx)} \sqrt{\sin(2a + 2bx)}}{3bd^2} \end{aligned}$$

**Mathematica [C]** time = 0.527651, size = 70, normalized size = 0.71

$$\frac{c(c \sec(a + bx))^{3/2} \left( (-\cot^2(a + bx))^{3/4} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \csc^2(a + bx)\right) + 2 \right)}{3bd\sqrt{d} \csc(a + bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(5/2)/(d*Csc[a + b*x])^(3/2), x]
```

```
[Out] (c*(2 + (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2]))*(c*Sec[a + b*x])^(3/2)/(3*b*d*Sqrt[d*Csc[a + b*x]])
```

**Maple [A]** time = 0.176, size = 192, normalized size = 2.

$$\frac{\cos(bx+a)\sqrt{2}}{3b(-1+\cos(bx+a))\sin(bx+a)} \left( \cos(bx+a)\sin(bx+a) \sqrt{\frac{-1+\cos(bx+a)-\sin(bx+a)}{\sin(bx+a)}} \sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*sec(b\*x+a))^(5/2)/(d\*csc(b\*x+a))^(3/2), x)

[Out] 1/3/b\*2^(1/2)\*(cos(b\*x+a)\*sin(b\*x+a)\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a)))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2), 1/2\*2^(1/2))+cos(b\*x+a)\*2^(1/2)-2^(1/2))\*cos(b\*x+a)\*(c/cos(b\*x+a))^(5/2)/(-1+cos(b\*x+a))/(d/sin(b\*x+a))^(3/2)/sin(b\*x+a)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sec(bx+a))^{\frac{5}{2}}}{(d \csc(bx+a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(5/2)/(d\*csc(b\*x+a))^(3/2), x, algorithm="maxima")

[Out] integrate((c\*sec(b\*x + a))^(5/2)/(d\*csc(b\*x + a))^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d \csc(bx+a)}\sqrt{c \sec(bx+a)}c^2 \sec(bx+a)^2}{d^2 \csc(bx+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(5/2)/(d\*csc(b\*x+a))^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a))\*c^2\*sec(b\*x + a)^2/(d^2\*csc(b\*x + a)^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))\*\*(5/2)/(d\*csc(b\*x+a))\*\*(3/2), x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sec (bx + a))^{\frac{5}{2}}}{(d \csc (bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(5/2)/(d\*csc(b\*x+a))^(3/2), x, algorithm="giac")

[Out] integrate((c\*sec(b\*x + a))^(5/2)/(d\*csc(b\*x + a))^(3/2), x)



$$3.253 \quad \int \frac{(c \sec(a+bx))^{5/2}}{(d \csc(a+bx))^{5/2}} dx$$

**Optimal.** Leaf size=329

$$\frac{c^2 \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{c \sec(a+bx)}}{\sqrt{bd^2}\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}} - \frac{c^2 \tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{c \sec(a+bx)}}{\sqrt{bd^2}\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}} - \frac{c^2 \sqrt{c \sec(a+bx)} \log\left(\frac{1 - \sqrt{2}\sqrt{\tan(a+bx)}}{1 + \sqrt{2}\sqrt{\tan(a+bx)}}\right)}{2\sqrt{bd^2}\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}}$$

```
[Out] (2*c*(c*Sec[a + b*x])^(3/2))/(3*b*d*(d*Csc[a + b*x])^(3/2)) + (c^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (c^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (c^2*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (c^2*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

**Rubi [A]** time = 0.216921, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2624, 2629, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{c^2 \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{c \sec(a+bx)}}{\sqrt{bd^2}\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}} - \frac{c^2 \tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{c \sec(a+bx)}}{\sqrt{bd^2}\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}} - \frac{c^2 \sqrt{c \sec(a+bx)} \log\left(\frac{1 - \sqrt{2}\sqrt{\tan(a+bx)}}{1 + \sqrt{2}\sqrt{\tan(a+bx)}}\right)}{2\sqrt{bd^2}\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(5/2)/(d*Csc[a + b*x])^(5/2), x]
```

```
[Out] (2*c*(c*Sec[a + b*x])^(3/2))/(3*b*d*(d*Csc[a + b*x])^(3/2)) + (c^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (c^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (c^2*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (c^2*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

#### Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
```

$$\frac{1}{(f*a*(n - 1)), x} + \text{Dist}[(b^2*(m + 1))/(a^2*(n - 1)), \text{Int}[(a*\text{Csc}[e + f*x])^{m+2}*(b*\text{Sec}[e + f*x])^{n-2}], x], x] /; \text{FreeQ}[\{a, b, e, f\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{LtQ}[m, -1] \&\& \text{IntegersQ}[2*m, 2*n]$$

### Rule 2629

$$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[(a*\text{Csc}[e + f*x])^m*(b*\text{Sec}[e + f*x])^n/\text{Tan}[e + f*x]^n, \text{Int}[\text{Tan}[e + f*x]^n, x], x] /; \text{FreeQ}[\{a, b, e, f, m, n\}, x] \&\& \text{!IntegerQ}[n] \&\& \text{EqQ}[m + n, 0]$$

### Rule 3476

$$\text{Int}[(b_.)*\text{tan}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[b/d, \text{Subst}[\text{Int}[x^n/(b^2 + x^2), x], x, b*\text{Tan}[c + d*x]], x] /; \text{FreeQ}[\{b, c, d, n\}, x] \&\& \text{!IntegerQ}[n]$$

### Rule 329

$$\text{Int}[(c_.)*(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{k*(m+1)-1}*(a + (b*x^{k*n}))/c^n]^p, x], x, (c*x)^{1/k}], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{RationQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$$

### Rule 297

$$\text{Int}[(x_)^2/((a_) + (b_.)*(x_)^4), x\_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1/(2*s), \text{Int}[(r + s*x^2)/(a + b*x^4), x], x] - \text{Dist}[1/(2*s), \text{Int}[(r - s*x^2)/(a + b*x^4), x], x]] /; \text{FreeQ}[\{a, b\}, x] \&\& (\text{GtQ}[a/b, 0] \text{ || } (\text{PosQ}[a/b] \&\& \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \&\& \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, b]]))$$

### Rule 1162

$$\text{Int}[(d_) + (e_.)*(x_)^2/((a_) + (c_.)*(x_)^4), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$$

### Rule 617

$$\text{Int}[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]^{-1}, x\_Symbol] \rightarrow \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \text{ || } \text{!RationalQ}[b^2 - 4*a*c])] /; \text{Free}$$

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

### Rule 204

$\text{Int}[\{(a_) + (b_.) \cdot (x_)^2\}^{-1}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2] \cdot x] / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2]), x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rule 1165

$\text{Int}[\{(d_) + (e_.) \cdot (x_)^2\} / \{(a_) + (c_.) \cdot (x_)^4\}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2d/e, 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x) / \text{Simp}[d/e + qx - x^2, x], x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x) / \text{Simp}[d/e - qx - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{NegQ}[d \cdot e]$

### Rule 628

$\text{Int}[\{(d_) + (e_.) \cdot (x_)\} / \{(a_.) + (b_.) \cdot (x_) + (c_.) \cdot (x_)^2\}, x\_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]) / b, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - b^2, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{(c \sec(a + bx))^{5/2}}{(d \csc(a + bx))^{5/2}} dx &= \frac{2c(c \sec(a + bx))^{3/2}}{3bd(d \csc(a + bx))^{3/2}} - \frac{c^2 \int \frac{\sqrt{c \sec(a+bx)}}{\sqrt{d \csc(a+bx)}} dx}{d^2} \\
&= \frac{2c(c \sec(a + bx))^{3/2}}{3bd(d \csc(a + bx))^{3/2}} - \frac{(c^2 \sqrt{c \sec(a + bx)}) \int \sqrt{\tan(a + bx)} dx}{d^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}} \\
&= \frac{2c(c \sec(a + bx))^{3/2}}{3bd(d \csc(a + bx))^{3/2}} - \frac{(c^2 \sqrt{c \sec(a + bx)}) \text{Subst} \left( \int \frac{\sqrt{x}}{1+x^2} dx, x, \tan(a + bx) \right)}{bd^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}} \\
&= \frac{2c(c \sec(a + bx))^{3/2}}{3bd(d \csc(a + bx))^{3/2}} - \frac{(2c^2 \sqrt{c \sec(a + bx)}) \text{Subst} \left( \int \frac{x^2}{1+x^4} dx, x, \sqrt{\tan(a + bx)} \right)}{bd^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}} \\
&= \frac{2c(c \sec(a + bx))^{3/2}}{3bd(d \csc(a + bx))^{3/2}} + \frac{(c^2 \sqrt{c \sec(a + bx)}) \text{Subst} \left( \int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(a + bx)} \right)}{bd^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}} - \frac{(c^2 \sqrt{c \sec(a + bx)}) \text{Subst} \left( \int \frac{1}{1-x^2} dx, x, \sqrt{\tan(a + bx)} \right)}{2bd^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}} \\
&= \frac{2c(c \sec(a + bx))^{3/2}}{3bd(d \csc(a + bx))^{3/2}} - \frac{c^2 \log \left( 1 - \sqrt{2} \sqrt{\tan(a + bx)} + \tan(a + bx) \right) \sqrt{c \sec(a + bx)}}{2\sqrt{2}bd^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}} + \frac{c^2 \log \left( 1 + \sqrt{2} \sqrt{\tan(a + bx)} + \tan(a + bx) \right) \sqrt{c \sec(a + bx)}}{2\sqrt{2}bd^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}} \\
&= \frac{2c(c \sec(a + bx))^{3/2}}{3bd(d \csc(a + bx))^{3/2}} + \frac{c^2 \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(a + bx)} \right) \sqrt{c \sec(a + bx)}}{\sqrt{2}bd^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}} - \frac{c^2 \tan^{-1} \left( 1 + \sqrt{2} \sqrt{\tan(a + bx)} \right) \sqrt{c \sec(a + bx)}}{\sqrt{2}bd^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}}
\end{aligned}$$

**Mathematica [A]** time = 1.73026, size = 280, normalized size = 0.85

$$\frac{c(c \sec(a + bx))^{3/2} \left( 8 \cos(a + bx) \sqrt[4]{\cot^2(a + bx)} \cot(a + bx) - 8 \sqrt[4]{\cot^2(a + bx)} \csc(a + bx) + 3\sqrt{2} \cos(a + bx) \cot(a + bx) \right)}{\sqrt{2}bd^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*Sec[a + b\*x])^(5/2)/(d\*Csc[a + b\*x])^(5/2), x]

[Out] -(c\*(6\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4)]\*Cos[a + b\*x]\*Cot[a + b\*x] - 6\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4)]\*Cos[a + b\*x]\*Cot[a + b\*x] + 8\*Cos[a + b\*x]\*Cot[a + b\*x]\*(Cot[a + b\*x]^2)^(1/4) - 8\*(Cot[a + b\*x]^2)^(1/4)\*Csc[a + b\*x] + 3\*Sqrt[2]\*Cos[a + b\*x]\*Cot[a + b\*x]\*Log[1 - Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4) + Sqrt[Cot[a + b\*x]^2]] - 3\*Sqrt[2]\*Cos[a + b\*x]\*Cot[a + b\*x]\*Log[1 + Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4) + Sqrt[Cot[a + b\*x]^2]])/(sqrt(2)\*b\*d^2\*sqrt(d\*csc(a + b\*x))\*sqrt(tan(a + b\*x)))

$b*x]^2]]*(c*\text{Sec}[a + b*x])^{(3/2)})/(12*b*d^2*(\text{Cot}[a + b*x]^2)^{(1/4)}*\text{Sqrt}[d*\text{Csc}[a + b*x]])$

**Maple [C]** time = 0.172, size = 544, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((c*\text{sec}(b*x+a))^{(5/2)})/(d*\text{csc}(b*x+a))^{(5/2)}, x)$

[Out]  $\frac{1}{6} \frac{1}{b^2} \sqrt{\frac{3I \cos(bx+a) * (-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{(1/2)} * ((-1 + \cos(bx+a) + \sin(bx+a)) / \sin(bx+a))^{(1/2)} * ((-1 + \cos(bx+a)) / \sin(bx+a))^{(1/2)} * \text{EllipticPi}((-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{(1/2)}, 1/2 - 1/2 * I, 1/2 * 2^{(1/2)}) - 3I \cos(bx+a) * (-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{(1/2)} * ((-1 + \cos(bx+a) + \sin(bx+a)) / \sin(bx+a))^{(1/2)} * ((-1 + \cos(bx+a)) / \sin(bx+a))^{(1/2)} * \text{EllipticPi}((-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{(1/2)}, 1/2 + 1/2 * I, 1/2 * 2^{(1/2)}) - 3 \cos(bx+a) * (-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{(1/2)} * ((-1 + \cos(bx+a) + \sin(bx+a)) / \sin(bx+a))^{(1/2)} * ((-1 + \cos(bx+a)) / \sin(bx+a))^{(1/2)} * \text{EllipticPi}((-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{(1/2)}, 1/2 - 1/2 * I, 1/2 * 2^{(1/2)}) - 3 \cos(bx+a) * (-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{(1/2)} * ((-1 + \cos(bx+a) + \sin(bx+a)) / \sin(bx+a))^{(1/2)} * ((-1 + \cos(bx+a)) / \sin(bx+a))^{(1/2)} * \text{EllipticPi}((-(-1 + \cos(bx+a) - \sin(bx+a)) / \sin(bx+a))^{(1/2)}, 1/2 + 1/2 * I, 1/2 * 2^{(1/2)}) + 2 \cos(bx+a) * 2^{(1/2)} - 2 * 2^{(1/2)})) * \cos(bx+a) * (c / \cos(bx+a))^{(5/2)} / (-1 + \cos(bx+a)) / (d / \sin(bx+a))^{(5/2)} / \sin(bx+a)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sec(bx + a))^{\frac{5}{2}}}{(d \csc(bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*\text{sec}(b*x+a))^{(5/2)})/(d*\text{csc}(b*x+a))^{(5/2)}, x, \text{algorithm}=\text{"maxima"})$

[Out]  $\text{integrate}((c*\text{sec}(b*x + a))^{(5/2)})/(d*\text{csc}(b*x + a))^{(5/2)}, x)$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(5/2)/(d\*csc(b\*x+a))^(5/2),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))\*\*(5/2)/(d\*csc(b\*x+a))\*\*(5/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c \sec(bx + a))^{\frac{5}{2}}}{(d \csc(bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*sec(b\*x+a))^(5/2)/(d\*csc(b\*x+a))^(5/2),x, algorithm="giac")

[Out] integrate((c\*sec(b\*x + a))^(5/2)/(d\*csc(b\*x + a))^(5/2), x)

$$3.254 \quad \int \frac{(d \csc(a+bx))^{9/2}}{\sqrt{c \sec(a+bx)}} dx$$

**Optimal.** Leaf size=69

$$-\frac{8cd^3(d \csc(a+bx))^{3/2}}{21b(c \sec(a+bx))^{3/2}} - \frac{2cd(d \csc(a+bx))^{7/2}}{7b(c \sec(a+bx))^{3/2}}$$

[Out]  $(-8*c*d^3*(d*Csc[a + b*x])^(3/2))/(21*b*(c*Sec[a + b*x])^(3/2)) - (2*c*d*(d*Csc[a + b*x])^(7/2))/(7*b*(c*Sec[a + b*x])^(3/2))$

**Rubi [A]** time = 0.0996172, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {2625, 2619}

$$-\frac{8cd^3(d \csc(a+bx))^{3/2}}{21b(c \sec(a+bx))^{3/2}} - \frac{2cd(d \csc(a+bx))^{7/2}}{7b(c \sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d\*Csc[a + b\*x])^(9/2)/Sqrt[c\*Sec[a + b\*x]],x]

[Out]  $(-8*c*d^3*(d*Csc[a + b*x])^(3/2))/(21*b*(c*Sec[a + b*x])^(3/2)) - (2*c*d*(d*Csc[a + b*x])^(7/2))/(7*b*(c*Sec[a + b*x])^(3/2))$

#### Rule 2625

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> -Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*(m - 1)), x] + Dist[(a^2\*(m + n - 2))/(m - 1), Int[(a\*Csc[e + f\*x])^(m - 2)\*(b\*Sec[e + f\*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && IntegersQ[2\*m, 2\*n] && !GtQ[n, m]

#### Rule 2619

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] && NeQ[n, 1]

#### Rubi steps

$$\int \frac{(d \csc(a + bx))^{9/2}}{\sqrt{c \sec(a + bx)}} dx = -\frac{2cd(d \csc(a + bx))^{7/2}}{7b(c \sec(a + bx))^{3/2}} + \frac{1}{7} (4d^2) \int \frac{(d \csc(a + bx))^{5/2}}{\sqrt{c \sec(a + bx)}} dx$$

$$= -\frac{8cd^3(d \csc(a + bx))^{3/2}}{21b(c \sec(a + bx))^{3/2}} - \frac{2cd(d \csc(a + bx))^{7/2}}{7b(c \sec(a + bx))^{3/2}}$$

**Mathematica [A]** time = 0.25782, size = 45, normalized size = 0.65

$$\frac{2cd(2 \cos(2(a + bx)) - 5)(d \csc(a + bx))^{7/2}}{21b(c \sec(a + bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(9/2)/Sqrt[c\*Sec[a + b\*x]], x]

[Out] (2\*c\*d\*(-5 + 2\*Cos[2\*(a + b\*x)])\*(d\*Csc[a + b\*x])^(7/2))/(21\*b\*(c\*Sec[a + b\*x])^(3/2))

**Maple [A]** time = 0.164, size = 54, normalized size = 0.8

$$\frac{(8 (\cos (bx + a))^2 - 14) \cos (bx + a) \sin (bx + a)}{21 b} \left( \frac{d}{\sin (bx + a)} \right)^{\frac{9}{2}} \frac{1}{\sqrt{\frac{c}{\cos (bx + a)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(9/2)/(c\*sec(b\*x+a))^(1/2), x)

[Out] 2/21/b\*(4\*cos(b\*x+a)^2-7)\*(d/sin(b\*x+a))^(9/2)\*cos(b\*x+a)\*sin(b\*x+a)/(c/cos(b\*x+a))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc (bx + a))^{\frac{9}{2}}}{\sqrt{c \sec (bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((d\*csc(b\*x+a))^(9/2)/(c\*sec(b\*x+a))^(1/2),x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(9/2)/sqrt(c\*sec(b\*x + a)), x)

**Fricas [A]** time = 2.32804, size = 185, normalized size = 2.68

$$\frac{2 \left( 4 d^4 \cos (b x + a)^4 - 7 d^4 \cos (b x + a)^2 \right) \sqrt{\frac{c}{\cos (b x + a)}} \sqrt{\frac{d}{\sin (b x + a)}}}{21 \left( b c \cos (b x + a)^2 - b c \right) \sin (b x + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(9/2)/(c\*sec(b\*x+a))^(1/2),x, algorithm="fricas")

[Out] -2/21\*(4\*d^4\*cos(b\*x + a)^4 - 7\*d^4\*cos(b\*x + a)^2)\*sqrt(c/cos(b\*x + a))\*sqrt(d/sin(b\*x + a))/((b\*c\*cos(b\*x + a)^2 - b\*c)\*sin(b\*x + a))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(9/2)/(c\*sec(b\*x+a))\*\*(1/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc (b x + a))^{\frac{9}{2}}}{\sqrt{c \sec (b x + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(9/2)/(c\*sec(b\*x+a))^(1/2),x, algorithm="giac")

[Out] integrate((d\*csc(b\*x + a))^(9/2)/sqrt(c\*sec(b\*x + a)), x)

$$3.255 \quad \int \frac{(d \csc(a+bx))^{7/2}}{\sqrt{c \sec(a+bx)}} dx$$

**Optimal.** Leaf size=128

$$-\frac{4cd^3\sqrt{d \csc(a+bx)}}{5b(c \sec(a+bx))^{3/2}} - \frac{4d^4 E\left(a+bx - \frac{\pi}{4} \middle| 2\right)}{5b\sqrt{\sin(2a+2bx)}\sqrt{c \sec(a+bx)}\sqrt{d \csc(a+bx)}} - \frac{2cd(d \csc(a+bx))^{5/2}}{5b(c \sec(a+bx))^{3/2}}$$

[Out]  $(-4*c*d^3*\text{Sqrt}[d*\text{Csc}[a + b*x]])/(5*b*(c*\text{Sec}[a + b*x])^(3/2)) - (2*c*d*(d*\text{Csc}[a + b*x])^(5/2))/(5*b*(c*\text{Sec}[a + b*x])^(3/2)) - (4*d^4*\text{EllipticE}[a - \text{Pi}/4 + b*x, 2])/(5*b*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Sqrt}[c*\text{Sec}[a + b*x]]*\text{Sqrt}[\text{Sin}[2*a + 2*b*x]])$

**Rubi [A]** time = 0.192731, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2625, 2630, 2572, 2639}

$$-\frac{4cd^3\sqrt{d \csc(a+bx)}}{5b(c \sec(a+bx))^{3/2}} - \frac{4d^4 E\left(a+bx - \frac{\pi}{4} \middle| 2\right)}{5b\sqrt{\sin(2a+2bx)}\sqrt{c \sec(a+bx)}\sqrt{d \csc(a+bx)}} - \frac{2cd(d \csc(a+bx))^{5/2}}{5b(c \sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*\text{Csc}[a + b*x])^(7/2)/\text{Sqrt}[c*\text{Sec}[a + b*x]], x]$

[Out]  $(-4*c*d^3*\text{Sqrt}[d*\text{Csc}[a + b*x]])/(5*b*(c*\text{Sec}[a + b*x])^(3/2)) - (2*c*d*(d*\text{Csc}[a + b*x])^(5/2))/(5*b*(c*\text{Sec}[a + b*x])^(3/2)) - (4*d^4*\text{EllipticE}[a - \text{Pi}/4 + b*x, 2])/(5*b*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Sqrt}[c*\text{Sec}[a + b*x]]*\text{Sqrt}[\text{Sin}[2*a + 2*b*x]])$

### Rule 2625

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^(m_.)*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^(n_.), x\_Symbol] \rightarrow -\text{Simp}[(a*b*(a*\text{Csc}[e + f*x])^(m - 1)*(b*\text{Sec}[e + f*x])^(n - 1))/(f*(m - 1)), x] + \text{Dist}[(a^2*(m + n - 2))/(m - 1), \text{Int}[(a*\text{Csc}[e + f*x])^(m - 2)*(b*\text{Sec}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, e, f, n\}, x] \&\& \text{GtQ}[m, 1] \&\& \text{IntegersQ}[2*m, 2*n] \&\& !\text{GtQ}[n, m]$

### Rule 2630

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^(m_.)*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^(n_.), x\_Symbol] \rightarrow \text{Dist}[(a*\text{Csc}[e + f*x])^m*(b*\text{Sec}[e + f*x])^n*(a*\text{Sin}[e + f*x])$

```
)^m*(b*cos[e + f*x])^n, Int[1/((a*sin[e + f*x])^m*(b*cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]
]
```

### Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]],
x_Symbol] := Dist[(Sqrt[a*Sin[e + f*x])*Sqrt[b*cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{(d \csc(a + bx))^{7/2}}{\sqrt{c \sec(a + bx)}} dx &= -\frac{2cd(d \csc(a + bx))^{5/2}}{5b(c \sec(a + bx))^{3/2}} + \frac{1}{5} (2d^2) \int \frac{(d \csc(a + bx))^{3/2}}{\sqrt{c \sec(a + bx)}} dx \\ &= -\frac{4cd^3 \sqrt{d \csc(a + bx)}}{5b(c \sec(a + bx))^{3/2}} - \frac{2cd(d \csc(a + bx))^{5/2}}{5b(c \sec(a + bx))^{3/2}} - \frac{1}{5} (4d^4) \int \frac{1}{\sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)}} dx \\ &= -\frac{4cd^3 \sqrt{d \csc(a + bx)}}{5b(c \sec(a + bx))^{3/2}} - \frac{2cd(d \csc(a + bx))^{5/2}}{5b(c \sec(a + bx))^{3/2}} - \frac{(4d^4) \int \sqrt{c \cos(a + bx)} \sqrt{d \sin(a + bx)}}{5\sqrt{c \cos(a + bx)} \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)}} dx \\ &= -\frac{4cd^3 \sqrt{d \csc(a + bx)}}{5b(c \sec(a + bx))^{3/2}} - \frac{2cd(d \csc(a + bx))^{5/2}}{5b(c \sec(a + bx))^{3/2}} - \frac{(4d^4) \int \sqrt{\sin(2a + 2bx)} dx}{5\sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} \sqrt{\sin(2a + 2bx)}} \\ &= -\frac{4cd^3 \sqrt{d \csc(a + bx)}}{5b(c \sec(a + bx))^{3/2}} - \frac{2cd(d \csc(a + bx))^{5/2}}{5b(c \sec(a + bx))^{3/2}} - \frac{4d^4 E\left(a - \frac{\pi}{4} + bx \mid 2\right)}{5b\sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} \sqrt{\sin(2a + 2bx)}} \end{aligned}$$

**Mathematica [C]** time = 1.04791, size = 104, normalized size = 0.81

$$\frac{2d^2 \tan^2(a + bx)(d \csc(a + bx))^{3/2} \left( \sin(2(a + bx)) \sqrt{-\cot^2(a + bx)} \operatorname{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \csc^2(a + bx)\right) - (c \sec(a + bx))^{3/2} \right)}{5b\sqrt{c \sec(a + bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(7/2)/Sqrt[c*Sec[a + b*x]], x]
```

```
[Out] (-2*d^2*(d*Csc[a + b*x])^(3/2)*(-((-2 + Cos[2*(a + b*x)])*Cot[a + b*x]^3) +
(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2]*
```

$\text{Sin}[2*(a + b*x)]*\text{Tan}[a + b*x]^2/(5*b*\text{Sqrt}[c*\text{Sec}[a + b*x]])$

**Maple [B]** time = 0.207, size = 992, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d*\text{csc}(b*x+a))^{7/2}/(c*\text{sec}(b*x+a))^{1/2}, x)$

[Out]  $-1/5/b*2^{1/2}*(4*\cos(b*x+a)^3*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*\text{EllipticE}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})-2*\cos(b*x+a)^3*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*\text{EllipticF}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})+4*\cos(b*x+a)^2*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*\text{EllipticE}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})-2*\cos(b*x+a)^2*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*\text{EllipticF}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})-4*\cos(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*\text{EllipticE}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})+2*\cos(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*\text{EllipticF}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})-2*\cos(b*x+a)^3*2^{1/2}-4*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*\text{EllipticE}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})+2*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*\text{EllipticF}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})+\cos(b*x+a)^2*2^{1/2}+2*\cos(b*x+a)*2^{1/2}*(d/\sin(b*x+a))^{7/2}*\sin(b*x+a)/(c/\cos(b*x+a))^{1/2}/\cos(b*x+a)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc(bx + a))^{\frac{7}{2}}}{\sqrt{c \sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")`

[Out] `integrate((d*csc(b*x + a))^(7/2)/sqrt(c*sec(b*x + a)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)} d^3 \csc(bx + a)^3}{c \sec(bx + a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*d^3*csc(b*x + a)^3/(c*sec(b*x + a)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))**(7/2)/(c*sec(b*x+a))**(1/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc(bx + a))^{\frac{7}{2}}}{\sqrt{c \sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")`

```
[Out] integrate((d*csc(b*x + a))^(7/2)/sqrt(c*sec(b*x + a)), x)
```

$$3.256 \quad \int \frac{(d \csc(a+bx))^{5/2}}{\sqrt{c \sec(a+bx)}} dx$$

Optimal. Leaf size=33

$$-\frac{2cd(d \csc(a+bx))^{3/2}}{3b(c \sec(a+bx))^{3/2}}$$

[Out]  $(-2*c*d*(d*Csc[a + b*x])^{(3/2)})/(3*b*(c*Sec[a + b*x])^{(3/2)})$

**Rubi [A]** time = 0.0480831, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {2619}

$$-\frac{2cd(d \csc(a+bx))^{3/2}}{3b(c \sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d\*Csc[a + b\*x])^(5/2)/Sqrt[c\*Sec[a + b\*x]], x]

[Out]  $(-2*c*d*(d*Csc[a + b*x])^{(3/2)})/(3*b*(c*Sec[a + b*x])^{(3/2)})$

### Rule 2619

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] && NeQ[n, 1]

### Rubi steps

$$\int \frac{(d \csc(a+bx))^{5/2}}{\sqrt{c \sec(a+bx)}} dx = -\frac{2cd(d \csc(a+bx))^{3/2}}{3b(c \sec(a+bx))^{3/2}}$$

**Mathematica [A]** time = 0.111938, size = 33, normalized size = 1.

$$-\frac{2cd(d \csc(a+bx))^{3/2}}{3b(c \sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(5/2)/Sqrt[c\*Sec[a + b\*x]], x]

[Out] (-2\*c\*d\*(d\*Csc[a + b\*x])^(3/2))/(3\*b\*(c\*Sec[a + b\*x])^(3/2))

**Maple [A]** time = 0.153, size = 42, normalized size = 1.3

$$-\frac{2 \cos (bx+a) \sin (bx+a)}{3 b} \left(\frac{d}{\sin (bx+a)}\right)^{\frac{5}{2}} \frac{1}{\sqrt{\frac{c}{\cos (bx+a)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(5/2)/(c\*sec(b\*x+a))^(1/2), x)

[Out] -2/3/b\*cos(b\*x+a)\*sin(b\*x+a)\*(d/sin(b\*x+a))^(5/2)/(c/cos(b\*x+a))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc (bx+a))^{\frac{5}{2}}}{\sqrt{c \sec (bx+a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(5/2)/(c\*sec(b\*x+a))^(1/2), x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(5/2)/sqrt(c\*sec(b\*x + a)), x)

**Fricas [A]** time = 2.06628, size = 116, normalized size = 3.52

$$-\frac{2 d^2 \sqrt{\frac{c}{\cos (bx+a)}} \sqrt{\frac{d}{\sin (bx+a)}} \cos (bx+a)^2}{3 b c \sin (bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((d\*csc(b\*x+a))^(5/2)/(c\*sec(b\*x+a))^(1/2),x, algorithm="fricas")

[Out]  $-2/3*d^2*\sqrt{c/\cos(b*x + a)}*\sqrt{d/\sin(b*x + a)}*\cos(b*x + a)^2/(b*c*\sin(b*x + a))$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(5/2)/(c\*sec(b\*x+a))\*\*(1/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc(bx + a))^{\frac{5}{2}}}{\sqrt{c \sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(5/2)/(c\*sec(b\*x+a))^(1/2),x, algorithm="giac")

[Out] integrate((d\*csc(b\*x + a))^(5/2)/sqrt(c\*sec(b\*x + a)), x)

$$3.257 \quad \int \frac{(d \csc(a+bx))^{3/2}}{\sqrt{c \sec(a+bx)}} dx$$

**Optimal.** Leaf size=89

$$-\frac{2d^2 E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c \sec(a+bx)}\sqrt{d \csc(a+bx)}} - \frac{2cd\sqrt{d \csc(a+bx)}}{b(c \sec(a+bx))^{3/2}}$$

[Out]  $(-2*c*d*\text{Sqrt}[d*\text{Csc}[a + b*x]])/(b*(c*\text{Sec}[a + b*x])^{(3/2)}) - (2*d^2*\text{EllipticE}[a - \text{Pi}/4 + b*x, 2])/(b*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Sqrt}[c*\text{Sec}[a + b*x]]*\text{Sqrt}[\text{Sin}[2*a + 2*b*x]])$

**Rubi [A]** time = 0.139264, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2625, 2630, 2572, 2639}

$$-\frac{2d^2 E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c \sec(a+bx)}\sqrt{d \csc(a+bx)}} - \frac{2cd\sqrt{d \csc(a+bx)}}{b(c \sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*\text{Csc}[a + b*x])^{(3/2)}/\text{Sqrt}[c*\text{Sec}[a + b*x]], x]$

[Out]  $(-2*c*d*\text{Sqrt}[d*\text{Csc}[a + b*x]])/(b*(c*\text{Sec}[a + b*x])^{(3/2)}) - (2*d^2*\text{EllipticE}[a - \text{Pi}/4 + b*x, 2])/(b*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Sqrt}[c*\text{Sec}[a + b*x]]*\text{Sqrt}[\text{Sin}[2*a + 2*b*x]])$

### Rule 2625

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow -\text{Simp}[(a*b*(a*\text{Csc}[e + f*x])^{(m-1)}*(b*\text{Sec}[e + f*x])^{(n-1)})/(f*(m-1)), x] + \text{Dist}[(a^{2*(m+n-2)})/(m-1), \text{Int}[(a*\text{Csc}[e + f*x])^{(m-2)}*(b*\text{Sec}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, e, f, n\}, x] \&\& \text{GtQ}[m, 1] \&\& \text{IntegersQ}[2*m, 2*n] \&\& !\text{GtQ}[n, m]$

### Rule 2630

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[(a*\text{Csc}[e + f*x])^m*(b*\text{Sec}[e + f*x])^n*(a*\text{Sin}[e + f*x])^m*(b*\text{Cos}[e + f*x])^n, \text{Int}[1/((a*\text{Sin}[e + f*x])^m*(b*\text{Cos}[e + f*x])^n), x], x] /; \text{FreeQ}\{a, b, e, f, m, n\}, x] \&\& \text{IntegerQ}[m - 1/2] \&\& \text{IntegerQ}[n - 1/2]$

]

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(d \csc(a + bx))^{3/2}}{\sqrt{c \sec(a + bx)}} dx &= -\frac{2cd\sqrt{d \csc(a + bx)}}{b(c \sec(a + bx))^{3/2}} - (2d^2) \int \frac{1}{\sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}} dx \\ &= -\frac{2cd\sqrt{d \csc(a + bx)}}{b(c \sec(a + bx))^{3/2}} - \frac{(2d^2) \int \sqrt{c \cos(a + bx)}\sqrt{d \sin(a + bx)} dx}{\sqrt{c \cos(a + bx)}\sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}\sqrt{d \sin(a + bx)}} \\ &= -\frac{2cd\sqrt{d \csc(a + bx)}}{b(c \sec(a + bx))^{3/2}} - \frac{(2d^2) \int \sqrt{\sin(2a + 2bx)} dx}{\sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}\sqrt{\sin(2a + 2bx)}} \\ &= -\frac{2cd\sqrt{d \csc(a + bx)}}{b(c \sec(a + bx))^{3/2}} - \frac{2d^2 E\left(a - \frac{\pi}{4} + bx \middle| 2\right)}{b\sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}\sqrt{\sin(2a + 2bx)}} \end{aligned}$$

**Mathematica [C]** time = 0.466088, size = 80, normalized size = 0.9

$$\frac{2d^2 \tan(a + bx) \left( \sqrt{-\cot^2(a + bx)} \operatorname{Hypergeometric2F1} \left( -\frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \csc^2(a + bx) \right) + \cot^2(a + bx) \right)}{b\sqrt{c \sec(a + bx)}\sqrt{d \csc(a + bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(3/2)/Sqrt[c*Sec[a + b*x]], x]
```

```
[Out] (-2*d^2*(Cot[a + b*x]^2 + (-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2])*Tan[a + b*x])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]])
```

**Maple [B]** time = 0.185, size = 502, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x)`

[Out]  $1/b*2^{(1/2)}*(2*\cos(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}*EllipticE((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)},1/2*2^{(1/2)})-\cos(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}*EllipticF((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)},1/2*2^{(1/2)})+2*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}*EllipticE((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)},1/2*2^{(1/2)})-((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}*EllipticF((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)},1/2*2^{(1/2)})-\cos(b*x+a)*2^{(1/2)}*(d/\sin(b*x+a))^{(3/2)}*\sin(b*x+a)/(c/\cos(b*x+a))^{(1/2)}/\cos(b*x+a)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc(bx + a))^2}{\sqrt{c \sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")`

[Out] `integrate((d*csc(b*x + a))^(3/2)/sqrt(c*sec(b*x + a)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d \csc(bx + a)}\sqrt{c \sec(bx + a)}d \csc(bx + a)}{c \sec(bx + a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(3/2)/(c\*sec(b\*x+a))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a))\*d\*csc(b\*x + a)/(c\*sec(b\*x + a)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(3/2)/(c\*sec(b\*x+a))\*\*(1/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc(bx + a))^{\frac{3}{2}}}{\sqrt{c \sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(3/2)/(c\*sec(b\*x+a))^(1/2),x, algorithm="giac")

[Out] integrate((d\*csc(b\*x + a))^(3/2)/sqrt(c\*sec(b\*x + a)), x)

$$3.258 \quad \int \frac{\sqrt{d \csc(a+bx)}}{\sqrt{c \sec(a+bx)}} dx$$

**Optimal.** Leaf size=270

$$\frac{\sqrt{\tan(a+bx)} \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{d \csc(a+bx)}}{\sqrt{2b}\sqrt{c \sec(a+bx)}} + \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{\sqrt{2b}\sqrt{c \sec(a+bx)}}$$

[Out] -((ArcTan[1 - Sqrt[2]\*Sqrt[Tan[a + b\*x]]]\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[Tan[a + b\*x]])/(Sqrt[2]\*b\*Sqrt[c\*Sec[a + b\*x]])) + (ArcTan[1 + Sqrt[2]\*Sqrt[Tan[a + b\*x]]]\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[Tan[a + b\*x]])/(Sqrt[2]\*b\*Sqrt[c\*Sec[a + b\*x]]) - (Sqrt[d\*Csc[a + b\*x]]\*Log[1 - Sqrt[2]\*Sqrt[Tan[a + b\*x]] + Tan[a + b\*x]]\*Sqrt[Tan[a + b\*x]])/(2\*Sqrt[2]\*b\*Sqrt[c\*Sec[a + b\*x]]) + (Sqrt[d\*Csc[a + b\*x]]\*Log[1 + Sqrt[2]\*Sqrt[Tan[a + b\*x]] + Tan[a + b\*x]]\*Sqrt[Tan[a + b\*x]])/(2\*Sqrt[2]\*b\*Sqrt[c\*Sec[a + b\*x]])

**Rubi [A]** time = 0.130926, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.36$ , Rules used = {2629, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt{\tan(a+bx)} \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{d \csc(a+bx)}}{\sqrt{2b}\sqrt{c \sec(a+bx)}} + \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{\sqrt{2b}\sqrt{c \sec(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*Csc[a + b\*x]]/Sqrt[c\*Sec[a + b\*x]],x]

[Out] -((ArcTan[1 - Sqrt[2]\*Sqrt[Tan[a + b\*x]]]\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[Tan[a + b\*x]])/(Sqrt[2]\*b\*Sqrt[c\*Sec[a + b\*x]])) + (ArcTan[1 + Sqrt[2]\*Sqrt[Tan[a + b\*x]]]\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[Tan[a + b\*x]])/(Sqrt[2]\*b\*Sqrt[c\*Sec[a + b\*x]]) - (Sqrt[d\*Csc[a + b\*x]]\*Log[1 - Sqrt[2]\*Sqrt[Tan[a + b\*x]] + Tan[a + b\*x]]\*Sqrt[Tan[a + b\*x]])/(2\*Sqrt[2]\*b\*Sqrt[c\*Sec[a + b\*x]]) + (Sqrt[d\*Csc[a + b\*x]]\*Log[1 + Sqrt[2]\*Sqrt[Tan[a + b\*x]] + Tan[a + b\*x]]\*Sqrt[Tan[a + b\*x]])/(2\*Sqrt[2]\*b\*Sqrt[c\*Sec[a + b\*x]])

### Rule 2629

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[((a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n)/Tan[e + f\*x]^n, Int[Tan[e + f\*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && EqQ[m + n, 0]

Rule 3476

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

Rule 329

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b]

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{d \csc(a+bx)}}{\sqrt{c \sec(a+bx)}} dx &= \frac{(\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}) \int \frac{1}{\sqrt{\tan(a+bx)}} dx}{\sqrt{c \sec(a+bx)}} \\
 &= \frac{(\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{x(1+x^2)}} dx, x, \tan(a+bx)\right)}{b\sqrt{c \sec(a+bx)}} \\
 &= \frac{(2\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}) \operatorname{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\tan(a+bx)}\right)}{b\sqrt{c \sec(a+bx)}} \\
 &= \frac{(\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}) \operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(a+bx)}\right)}{b\sqrt{c \sec(a+bx)}} + \frac{(\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}) \operatorname{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\tan(a+bx)}\right)}{b\sqrt{c \sec(a+bx)}} \\
 &= \frac{(\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}) \operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\tan(a+bx)}\right)}{2b\sqrt{c \sec(a+bx)}} + \frac{(\sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}) \operatorname{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2} dx, x, \sqrt{\tan(a+bx)}\right)}{2b\sqrt{c \sec(a+bx)}} \\
 &= -\frac{\sqrt{d \csc(a+bx)} \log\left(1 - \sqrt{2}\sqrt{\tan(a+bx)} + \tan(a+bx)\right) \sqrt{\tan(a+bx)}}{2\sqrt{2}b\sqrt{c \sec(a+bx)}} + \frac{\sqrt{d \csc(a+bx)} \log\left(1 + \sqrt{2}\sqrt{\tan(a+bx)} + \tan(a+bx)\right) \sqrt{\tan(a+bx)}}{2\sqrt{2}b\sqrt{c \sec(a+bx)}} \\
 &= -\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}}{\sqrt{2}b\sqrt{c \sec(a+bx)}} + \frac{\tan^{-1}\left(1 + \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}}{\sqrt{2}b\sqrt{c \sec(a+bx)}}
 \end{aligned}$$

**Mathematica [C]** time = 0.125277, size = 55, normalized size = 0.2

$$\frac{2 \cot(a+bx) \sqrt{d \csc(a+bx)} \operatorname{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, -\cot^2(a+bx)\right)}{3b\sqrt{c \sec(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*Csc[a + b\*x]]/Sqrt[c\*Sec[a + b\*x]], x]



[Out]  $(-2*\cot[a + b*x]*\sqrt{d*\csc[a + b*x]}*\text{Hypergeometric2F1}[3/4, 1, 7/4, -\cot[a + b*x]^2])/(3*b*\sqrt{c*\sec[a + b*x]})$

**Maple [C]** time = 0.184, size = 322, normalized size = 1.2

$$\frac{\sqrt{2}(\sin(bx+a))^2}{2b\cos(bx+a)(-1+\cos(bx+a))}\sqrt{\frac{d}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d*\csc(b*x+a))^{1/2}/(c*\sec(b*x+a))^{1/2}, x)$

[Out]  $-1/2/b*2^{1/2}*(d/\sin(b*x+a))^{1/2}*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*(I*\text{EllipticPi}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2-1/2*I, 1/2*2^{1/2})-I*\text{EllipticPi}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2+1/2*I, 1/2*2^{1/2})-2*\text{EllipticF}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})+\text{EllipticPi}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2-1/2*I, 1/2*2^{1/2})+\text{EllipticPi}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2+1/2*I, 1/2*2^{1/2}))*\sin(b*x+a)^2/(c/\cos(b*x+a))^{1/2}/\cos(b*x+a)/(-1+\cos(b*x+a))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \csc(bx+a)}}{\sqrt{c \sec(bx+a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((d*\csc(b*x+a))^{1/2}/(c*\sec(b*x+a))^{1/2}, x, \text{algorithm}=\text{"maxima"})$

[Out]  $\text{integrate}(\sqrt{d*\csc(b*x+a)}/\sqrt{c*\sec(b*x+a)}, x)$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \csc(a + bx)}}{\sqrt{c \sec(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(d*csc(a + b*x))/sqrt(c*sec(a + b*x)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \csc(bx + a)}}{\sqrt{c \sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))/sqrt(c*sec(b*x + a)), x)
```

$$3.259 \quad \int \frac{1}{\sqrt{d \csc(a+bx)} \sqrt{c \sec(a+bx)}} dx$$

**Optimal.** Leaf size=53

$$\frac{E\left(a + bx - \frac{\pi}{4} \middle| 2\right)}{b\sqrt{\sin(2a + 2bx)}\sqrt{c \sec(a + bx)}\sqrt{d \csc(a + bx)}}$$

[Out] EllipticE[a - Pi/4 + b\*x, 2]/(b\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])

**Rubi [A]** time = 0.0884838, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$ , Rules used = {2630, 2572, 2639}

$$\frac{E\left(a + bx - \frac{\pi}{4} \middle| 2\right)}{b\sqrt{\sin(2a + 2bx)}\sqrt{c \sec(a + bx)}\sqrt{d \csc(a + bx)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]),x]

[Out] EllipticE[a - Pi/4 + b\*x, 2]/(b\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])

#### Rule 2630

```
Int[(csc[(e_) + (f_)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]
```

#### Rule 2572

```
Int[Sqrt[cos[(e_) + (f_)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_) + (f_)*(x_)]] , x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

#### Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{d} \csc(a+bx) \sqrt{c} \sec(a+bx)} dx &= \frac{\int \sqrt{c} \cos(a+bx) \sqrt{d} \sin(a+bx) dx}{\sqrt{c} \cos(a+bx) \sqrt{d} \csc(a+bx) \sqrt{c} \sec(a+bx) \sqrt{d} \sin(a+bx)} \\ &= \frac{\int \sqrt{\sin(2a+2bx)} dx}{\sqrt{d} \csc(a+bx) \sqrt{c} \sec(a+bx) \sqrt{\sin(2a+2bx)}} \\ &= \frac{E\left(a - \frac{\pi}{4} + bx \mid 2\right)}{b \sqrt{d} \csc(a+bx) \sqrt{c} \sec(a+bx) \sqrt{\sin(2a+2bx)}} \end{aligned}$$

**Mathematica [C]** time = 0.218229, size = 66, normalized size = 1.25

$$\frac{\tan(a+bx) \sqrt{-\cot^2(a+bx)} \operatorname{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \csc^2(a+bx)\right)}{b \sqrt{c} \sec(a+bx) \sqrt{d} \csc(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]),x]
```

```
[Out] ((-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2]*
Tan[a + b*x])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]])
```

**Maple [B]** time = 0.172, size = 517, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x)
```

```
[Out] -1/2/b*2^(1/2)*(2*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)
*(-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a)^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))
^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2)
)-cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
```

+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1/2\*2^(1/2))+2\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticE((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1/2\*2^(1/2))-((-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1/2\*2^(1/2))+cos(b\*x+a)^2\*2^(1/2)-cos(b\*x+a)\*2^(1/2))/cos(b\*x+a)/sin(b\*x+a)/(d/sin(b\*x+a))^(1/2)/(c/cos(b\*x+a))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d} \csc(bx + a) \sqrt{c} \sec(bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*csc(b\*x+a))^(1/2)/(c\*sec(b\*x+a))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a))), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d} \csc(bx + a) \sqrt{c} \sec(bx + a)}{cd \csc(bx + a) \sec(bx + a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*csc(b\*x+a))^(1/2)/(c\*sec(b\*x+a))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a))/(c\*d\*csc(b\*x + a)\*sec(b\*x + a)), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c} \sec(a + bx) \sqrt{d} \csc(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*csc(b\*x+a))\*\*(1/2)/(c\*sec(b\*x+a))\*\*(1/2),x)

[Out] Integral(1/(sqrt(c\*sec(a + b\*x))\*sqrt(d\*csc(a + b\*x))), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d \csc (bx + a)} \sqrt{c \sec (bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*csc(b\*x+a))^(1/2)/(c\*sec(b\*x+a))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a))), x)

$$3.260 \quad \int \frac{1}{(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} dx$$

**Optimal.** Leaf size=322

$$\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{4\sqrt{2}bd^2\sqrt{c \sec(a+bx)}} + \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{4\sqrt{2}bd^2\sqrt{c \sec(a+bx)}}$$

```
[Out] -c/(2*b*d*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)) - (ArcTan[1 - Sqrt[2]
]*Sqrt[Tan[a + b*x]])*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]/(4*Sqrt[2]*b
*d^2*Sqrt[c*Sec[a + b*x]]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]])*Sqrt[d
*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]/(4*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) -
(Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*S
qrt[Tan[a + b*x]])/(8*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) + (Sqrt[d*Csc[a +
b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b*x]
])/ (8*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]])
```

**Rubi [A]** time = 0.204094, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2627, 2629, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{4\sqrt{2}bd^2\sqrt{c \sec(a+bx)}} + \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{4\sqrt{2}bd^2\sqrt{c \sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]),x]
```

```
[Out] -c/(2*b*d*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)) - (ArcTan[1 - Sqrt[2]
]*Sqrt[Tan[a + b*x]])*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]/(4*Sqrt[2]*b
*d^2*Sqrt[c*Sec[a + b*x]]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]])*Sqrt[d
*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]/(4*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) -
(Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*S
qrt[Tan[a + b*x]])/(8*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) + (Sqrt[d*Csc[a +
b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b*x]
])/ (8*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]])
```

### Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
```

)/(a\*f\*(m + n)), x] + Dist[(m + 1)/(a^2\*(m + n)), Int[(a\*Csc[e + f\*x])^(m + 2)\*(b\*Sec[e + f\*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] && NeQ[m + n, 0] && IntegersQ[2\*m, 2\*n]

### Rule 2629

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[((a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n)/Tan[e + f\*x]^n, Int[Tan[e + f\*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && EqQ[m + n, 0]

### Rule 3476

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]



Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)}} dx &= -\frac{c}{2bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{3/2}} + \frac{\int \frac{\sqrt{d \csc(a+bx)}}{\sqrt{c \sec(a+bx)}} dx}{4d^2} \\
&= -\frac{c}{2bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{3/2}} + \frac{(\sqrt{d} \csc(a + bx) \sqrt{\tan(a + bx)}) \int \frac{1}{\sqrt{\tan(a + bx)}} dx}{4d^2 \sqrt{c \sec(a + bx)}} \\
&= -\frac{c}{2bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{3/2}} + \frac{(\sqrt{d} \csc(a + bx) \sqrt{\tan(a + bx)}) \operatorname{Subst}\left(\frac{1}{\sqrt{u}}, u, \tan(a + bx)\right)}{4bd^2 \sqrt{c \sec(a + bx)}} \\
&= -\frac{c}{2bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{3/2}} + \frac{(\sqrt{d} \csc(a + bx) \sqrt{\tan(a + bx)}) \operatorname{Subst}\left(\frac{1}{\sqrt{u}}, u, \tan(a + bx)\right)}{2bd^2 \sqrt{c \sec(a + bx)}} \\
&= -\frac{c}{2bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{3/2}} + \frac{(\sqrt{d} \csc(a + bx) \sqrt{\tan(a + bx)}) \operatorname{Subst}\left(\frac{1}{\sqrt{u}}, u, \tan(a + bx)\right)}{4bd^2 \sqrt{c \sec(a + bx)}} \\
&= -\frac{c}{2bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{3/2}} + \frac{(\sqrt{d} \csc(a + bx) \sqrt{\tan(a + bx)}) \operatorname{Subst}\left(\frac{1}{\sqrt{u}}, u, \tan(a + bx)\right)}{8bd^2 \sqrt{c \sec(a + bx)}} \\
&= -\frac{c}{2bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{3/2}} - \frac{\sqrt{d} \csc(a + bx) \log\left(1 - \sqrt{2} \sqrt{\tan(a + bx)}\right)}{8\sqrt{2}bd^2 \sqrt{c \sec(a + bx)}} \\
&= -\frac{c}{2bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{3/2}} - \frac{\tan^{-1}\left(1 - \sqrt{2} \sqrt{\tan(a + bx)}\right) \sqrt{d} \csc(a + bx)}{4\sqrt{2}bd^2 \sqrt{c \sec(a + bx)}}
\end{aligned}$$

**Mathematica [C]** time = 0.255619, size = 66, normalized size = 0.2

$$\frac{\cot(a + bx) \left( \csc^2(a + bx) \operatorname{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, -\cot^2(a + bx)\right) + 3 \right)}{6b\sqrt{c \sec(a + bx)}(d \csc(a + bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d\*Csc[a + b\*x])^(3/2)\*Sqrt[c\*Sec[a + b\*x]]),x]

[Out] -(Cot[a + b\*x]\*(3 + Csc[a + b\*x]^2\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[a + b\*x]^2]))/(6\*b\*(d\*Csc[a + b\*x])^(3/2)\*Sqrt[c\*Sec[a + b\*x]])

**Maple [C]** time = 0.17, size = 668, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x)`

[Out] 
$$\begin{aligned} & -1/8/b^2^{(1/2)}*(I*\sin(b*x+a)*\text{EllipticPi}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}-I*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}*\text{EllipticPi}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})+\sin(b*x+a)*\text{EllipticPi}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)},1/2-1/2*I,1/2*2^{(1/2)})*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}-2*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}*\text{EllipticF}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)},1/2*2^{(1/2)})+\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}*\text{EllipticPi}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)},1/2+1/2*I,1/2*2^{(1/2)})+2*\cos(b*x+a)^3*2^{(1/2)}-2*\cos(b*x+a)^2*2^{(1/2)})/(-1+\cos(b*x+a))/\cos(b*x+a)/\sin(b*x+a)/(d/\sin(b*x+a))^{(3/2)}/(c/\cos(b*x+a))^{(1/2)} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d \csc(bx + a))^{\frac{3}{2}} \sqrt{c \sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((d*csc(b*x + a))^(3/2)*sqrt(c*sec(b*x + a))), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d \csc(bx + a))^{\frac{3}{2}} \sqrt{c \sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(3/2)*sqrt(c*sec(b*x + a))), x)
```

$$3.261 \quad \int \frac{1}{(d \csc(a+bx))^{5/2} \sqrt{c \sec(a+bx)}} dx$$

**Optimal.** Leaf size=95

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{c \sec(a+bx)}\sqrt{d \csc(a+bx)}} - \frac{c}{3bd(c \sec(a+bx))^{3/2}(d \csc(a+bx))^{3/2}}$$

[Out]  $-c/(3*b*d*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)) + \text{EllipticE}[a - \text{Pi}/4 + b*x, 2]/(2*b*d^2*\text{Sqrt}[d*Csc[a + b*x]]*\text{Sqrt}[c*Sec[a + b*x]]*\text{Sqrt}[\text{Sin}[2*a + 2*b*x]])$

**Rubi [A]** time = 0.141121, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2627, 2630, 2572, 2639}

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{c \sec(a+bx)}\sqrt{d \csc(a+bx)}} - \frac{c}{3bd(c \sec(a+bx))^{3/2}(d \csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/((d*Csc[a + b*x])^(5/2)*\text{Sqrt}[c*Sec[a + b*x]]), x]$

[Out]  $-c/(3*b*d*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)) + \text{EllipticE}[a - \text{Pi}/4 + b*x, 2]/(2*b*d^2*\text{Sqrt}[d*Csc[a + b*x]]*\text{Sqrt}[c*Sec[a + b*x]]*\text{Sqrt}[\text{Sin}[2*a + 2*b*x]])$

### Rule 2627

$\text{Int}[(\csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*\sec[(e_.) + (f_.)*(x_)])^(n_.), x\_Symbol] \rightarrow \text{Simp}[(b*(a*\csc[e + f*x])^(m + 1)*(b*\sec[e + f*x])^(n - 1))/(a*f*(m + n)), x] + \text{Dist}[(m + 1)/(a^2*(m + n)), \text{Int}[(a*\csc[e + f*x])^(m + 2)*(b*\sec[e + f*x])^n, x], x] /;$   $\text{FreeQ}\{a, b, e, f, n\}, x\} \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[m + n, 0] \&\& \text{IntegersQ}[2*m, 2*n]$

### Rule 2630

$\text{Int}[(\csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*\sec[(e_.) + (f_.)*(x_)])^(n_.), x\_Symbol] \rightarrow \text{Dist}[(a*\csc[e + f*x])^m*(b*\sec[e + f*x])^n*(a*\sin[e + f*x])^m*(b*\cos[e + f*x])^n, \text{Int}[1/((a*\sin[e + f*x])^m*(b*\cos[e + f*x])^n), x], x] /;$   $\text{FreeQ}\{a, b, e, f, m, n\}, x\} \&\& \text{IntegerQ}[m - 1/2] \&\& \text{IntegerQ}[n - 1/2]$

]

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] := Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]] , x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(d \csc(a + bx))^{5/2} \sqrt{c \sec(a + bx)}} dx &= -\frac{c}{3bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2}} + \frac{\int \frac{1}{\sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)}} dx}{2d^2} \\ &= -\frac{c}{3bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2}} + \frac{\int \sqrt{c \cos(a + bx)} \sqrt{d \sin(a + bx)}}{2d^2 \sqrt{c \cos(a + bx)} \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)}} dx \\ &= -\frac{c}{3bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2}} + \frac{\int \sqrt{\sin(2a + 2bx)} dx}{2d^2 \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} \sqrt{\sin(a + bx)}} \\ &= -\frac{c}{3bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2}} + \frac{E\left(a - \frac{\pi}{4} + bx \mid 2\right)}{2bd^2 \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} \sqrt{\sin(a + bx)}} \end{aligned}$$

**Mathematica [C]** time = 0.354372, size = 84, normalized size = 0.88

$$\frac{\tan(a + bx) \left( -3 \sqrt[4]{-\cot^2(a + bx)} \operatorname{Hypergeometric2F1} \left( -\frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \csc^2(a + bx) \right) + \cos(2(a + bx)) + 1 \right)}{6bd^2 \sqrt{c \sec(a + bx)} \sqrt{d \csc(a + bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Csc[a + b*x])^(5/2)*Sqrt[c*Sec[a + b*x]]),x]
```

```
[Out] -((1 + Cos[2*(a + b*x)] - 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2,
1/4, 1/2, Csc[a + b*x]^2])*Tan[a + b*x])/(6*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqr
t[c*Sec[a + b*x]])
```

**Maple [B]** time = 0.213, size = 531, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x)`

[Out] 
$$\frac{1}{12} b^2 \sqrt{2} \cos(bx+a)^4 - 6 \cos(bx+a) \left( -(-1+\cos(bx+a) - \sin(bx+a)) / \sin(bx+a) \right)^{1/2} \left( (-1+\cos(bx+a) + \sin(bx+a)) / \sin(bx+a) \right)^{1/2} \left( (-1+\cos(bx+a)) / \sin(bx+a) \right)^{1/2} \operatorname{EllipticE} \left( \frac{-(-1+\cos(bx+a) - \sin(bx+a)) / \sin(bx+a)}{\sin(bx+a)}, \frac{1}{2} \sqrt{2} \right) + 3 \cos(bx+a) \left( -(-1+\cos(bx+a) - \sin(bx+a)) / \sin(bx+a) \right)^{1/2} \left( (-1+\cos(bx+a) + \sin(bx+a)) / \sin(bx+a) \right)^{1/2} \left( (-1+\cos(bx+a)) / \sin(bx+a) \right)^{1/2} \operatorname{EllipticF} \left( \frac{-(-1+\cos(bx+a) - \sin(bx+a)) / \sin(bx+a)}{\sin(bx+a)}, \frac{1}{2} \sqrt{2} \right) - 6 \left( -(-1+\cos(bx+a) - \sin(bx+a)) / \sin(bx+a) \right)^{1/2} \left( (-1+\cos(bx+a) + \sin(bx+a)) / \sin(bx+a) \right)^{1/2} \left( (-1+\cos(bx+a)) / \sin(bx+a) \right)^{1/2} \operatorname{EllipticE} \left( \frac{-(-1+\cos(bx+a) - \sin(bx+a)) / \sin(bx+a)}{\sin(bx+a)}, \frac{1}{2} \sqrt{2} \right) + 3 \left( -(-1+\cos(bx+a) - \sin(bx+a)) / \sin(bx+a) \right)^{1/2} \left( (-1+\cos(bx+a) + \sin(bx+a)) / \sin(bx+a) \right)^{1/2} \left( (-1+\cos(bx+a)) / \sin(bx+a) \right)^{1/2} \operatorname{EllipticF} \left( \frac{-(-1+\cos(bx+a) - \sin(bx+a)) / \sin(bx+a)}{\sin(bx+a)}, \frac{1}{2} \sqrt{2} \right) - 5 \cos(bx+a)^2 \sqrt{2} + 3 \cos(bx+a)^2 \sqrt{2} / (d/\sin(bx+a))^{5/2} / (c/\cos(bx+a))^{1/2} / \cos(bx+a) / \sin(bx+a)^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d \csc(bx+a))^{\frac{5}{2}} \sqrt{c \sec(bx+a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/((d*csc(b*x + a))^(5/2)*sqrt(c*sec(b*x + a))), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{\sqrt{d} \csc(bx+a) \sqrt{c \sec(bx+a)}}{cd^3 \csc(bx+a)^3 \sec(bx+a)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c*d^3*csc(b*x + a)^3*sec(b*x + a)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d \csc(bx + a))^{\frac{5}{2}} \sqrt{c \sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(5/2)*sqrt(c*sec(b*x + a))), x)
```



$$3.262 \quad \int \frac{(d \csc(a+bx))^{11/2}}{(c \sec(a+bx))^{3/2}} dx$$

**Optimal.** Leaf size=110

$$\frac{8d^5 \sqrt{d \csc(a+bx)}}{45bc \sqrt{c \sec(a+bx)}} + \frac{2d^3 (d \csc(a+bx))^{5/2}}{45bc \sqrt{c \sec(a+bx)}} - \frac{2d (d \csc(a+bx))^{9/2}}{9bc \sqrt{c \sec(a+bx)}}$$

[Out] (8\*d^5\*Sqrt[d\*Csc[a + b\*x]])/(45\*b\*c\*Sqrt[c\*Sec[a + b\*x]]) + (2\*d^3\*(d\*Csc[a + b\*x])^(5/2))/(45\*b\*c\*Sqrt[c\*Sec[a + b\*x]]) - (2\*d\*(d\*Csc[a + b\*x])^(9/2))/(9\*b\*c\*Sqrt[c\*Sec[a + b\*x]])

**Rubi [A]** time = 0.153996, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$ , Rules used = {2623, 2625, 2619}

$$\frac{8d^5 \sqrt{d \csc(a+bx)}}{45bc \sqrt{c \sec(a+bx)}} + \frac{2d^3 (d \csc(a+bx))^{5/2}}{45bc \sqrt{c \sec(a+bx)}} - \frac{2d (d \csc(a+bx))^{9/2}}{9bc \sqrt{c \sec(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[(d\*Csc[a + b\*x])^(11/2)/(c\*Sec[a + b\*x])^(3/2),x]

[Out] (8\*d^5\*Sqrt[d\*Csc[a + b\*x]])/(45\*b\*c\*Sqrt[c\*Sec[a + b\*x]]) + (2\*d^3\*(d\*Csc[a + b\*x])^(5/2))/(45\*b\*c\*Sqrt[c\*Sec[a + b\*x]]) - (2\*d\*(d\*Csc[a + b\*x])^(9/2))/(9\*b\*c\*Sqrt[c\*Sec[a + b\*x]])

### Rule 2623

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> -Simp[(a\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1))/(f\*b\*(m - 1)), x] + Dist[(a^2\*(n + 1))/(b^2\*(m - 1)), Int[(a\*Csc[e + f\*x])^(m - 2)\*(b\*Sec[e + f\*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2\*m, 2\*n]

### Rule 2625

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> -Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*(m - 1)), x] + Dist[(a^2\*(m + n - 2))/(m - 1), Int[(a\*Csc[e + f\*x])^(m - 2)\*(b\*Sec[e + f\*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,

1] && IntegersQ[2\*m, 2\*n] && !GtQ[n, m]

### Rule 2619

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n - 1))/(f\*(n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] && NeQ[n, 1]

### Rubi steps

$$\begin{aligned} \int \frac{(d \csc(a + bx))^{11/2}}{(c \sec(a + bx))^{3/2}} dx &= -\frac{2d(d \csc(a + bx))^{9/2}}{9bc\sqrt{c \sec(a + bx)}} - \frac{d^2 \int (d \csc(a + bx))^{7/2} \sqrt{c \sec(a + bx)} dx}{9c^2} \\ &= \frac{2d^3(d \csc(a + bx))^{5/2}}{45bc\sqrt{c \sec(a + bx)}} - \frac{2d(d \csc(a + bx))^{9/2}}{9bc\sqrt{c \sec(a + bx)}} - \frac{(4d^4) \int (d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)} dx}{45c^2} \\ &= \frac{8d^5 \sqrt{d} \csc(a + bx)}{45bc\sqrt{c \sec(a + bx)}} + \frac{2d^3(d \csc(a + bx))^{5/2}}{45bc\sqrt{c \sec(a + bx)}} - \frac{2d(d \csc(a + bx))^{9/2}}{9bc\sqrt{c \sec(a + bx)}} \end{aligned}$$

**Mathematica [A]** time = 0.280943, size = 57, normalized size = 0.52

$$\frac{2d^3(2 \cos(2(a + bx)) - 7) \cot^2(a + bx)(d \csc(a + bx))^{5/2}}{45bc\sqrt{c \sec(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(11/2)/(c\*Sec[a + b\*x])^(3/2), x]

[Out] (2\*d^3\*(-7 + 2\*Cos[2\*(a + b\*x)])\*Cot[a + b\*x]^2\*(d\*Csc[a + b\*x])^(5/2))/(45\*b\*c\*Sqrt[c\*Sec[a + b\*x]])

**Maple [A]** time = 0.158, size = 54, normalized size = 0.5

$$\frac{(8(\cos(bx + a))^2 - 18) \cos(bx + a) \sin(bx + a)}{45b} \left( \frac{d}{\sin(bx + a)} \right)^{\frac{11}{2}} \left( \frac{c}{\cos(bx + a)} \right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*csc(b*x+a))^(11/2)/(c*sec(b*x+a))^(3/2),x)`

[Out]  $2/45/b*(4*\cos(b*x+a)^2-9)*(d/\sin(b*x+a))^{11/2}*\cos(b*x+a)*\sin(b*x+a)/(c/\cos(b*x+a))^{3/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc(bx + a))^{\frac{11}{2}}}{(c \sec(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))^(11/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")`

[Out] `integrate((d*csc(b*x + a))^(11/2)/(c*sec(b*x + a))^(3/2), x)`

**Fricas [A]** time = 2.21246, size = 203, normalized size = 1.85

$$\frac{2 \left( 4 d^5 \cos(bx + a)^5 - 9 d^5 \cos(bx + a)^3 \right) \sqrt{\frac{c}{\cos(bx+a)}} \sqrt{\frac{d}{\sin(bx+a)}}}{45 \left( bc^2 \cos(bx + a)^4 - 2 bc^2 \cos(bx + a)^2 + bc^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))^(11/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")`

[Out]  $2/45*(4*d^5*\cos(b*x + a)^5 - 9*d^5*\cos(b*x + a)^3)*\sqrt{c/\cos(b*x + a)}*\sqrt{d/\sin(b*x + a)}/(b*c^2*\cos(b*x + a)^4 - 2*b*c^2*\cos(b*x + a)^2 + b*c^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))**(11/2)/(c*sec(b*x+a))**(3/2),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc (bx + a))^{\frac{11}{2}}}{(c \sec (bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(11/2)/(c\*sec(b\*x+a))^(3/2),x, algorithm="giac")

[Out] integrate((d\*csc(b\*x + a))^(11/2)/(c\*sec(b\*x + a))^(3/2), x)

$$3.263 \quad \int \frac{(d \csc(a+bx))^{9/2}}{(c \sec(a+bx))^{3/2}} dx$$

**Optimal.** Leaf size=135

$$\frac{2d^4 \sqrt{\sin(2a+2bx)} \operatorname{EllipticF}\left(a+bx-\frac{\pi}{4}, 2\right) \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}{21bc^2} + \frac{2d^3 (d \csc(a+bx))^{3/2}}{21bc \sqrt{c \sec(a+bx)}} - \frac{2d (d \csc(a+bx))^{7/2}}{7bc \sqrt{c \sec(a+bx)}}$$

[Out] (2\*d^3\*(d\*Csc[a + b\*x])^(3/2))/(21\*b\*c\*Sqrt[c\*Sec[a + b\*x]]) - (2\*d\*(d\*Csc[a + b\*x])^(7/2))/(7\*b\*c\*Sqrt[c\*Sec[a + b\*x]]) - (2\*d^4\*Sqrt[d\*Csc[a + b\*x]]\*EllipticF[a - Pi/4 + b\*x, 2]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])/(21\*b\*c^2)

**Rubi [A]** time = 0.199619, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2623, 2625, 2630, 2573, 2641}

$$\frac{2d^4 \sqrt{\sin(2a+2bx)} F\left(a+bx-\frac{\pi}{4} \middle| 2\right) \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}{21bc^2} + \frac{2d^3 (d \csc(a+bx))^{3/2}}{21bc \sqrt{c \sec(a+bx)}} - \frac{2d (d \csc(a+bx))^{7/2}}{7bc \sqrt{c \sec(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[(d\*Csc[a + b\*x])^(9/2)/(c\*Sec[a + b\*x])^(3/2), x]

[Out] (2\*d^3\*(d\*Csc[a + b\*x])^(3/2))/(21\*b\*c\*Sqrt[c\*Sec[a + b\*x]]) - (2\*d\*(d\*Csc[a + b\*x])^(7/2))/(7\*b\*c\*Sqrt[c\*Sec[a + b\*x]]) - (2\*d^4\*Sqrt[d\*Csc[a + b\*x]]\*EllipticF[a - Pi/4 + b\*x, 2]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])/(21\*b\*c^2)

### Rule 2623

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> -Simp[(a\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1))/(f\*b\*(m - 1)), x] + Dist[(a^2\*(n + 1))/(b^2\*(m - 1)), Int[(a\*Csc[e + f\*x])^(m - 2)\*(b\*Sec[e + f\*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2\*m, 2\*n]

### Rule 2625

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> -Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n -

1))/(f\*(m - 1)), x] + Dist[(a^2\*(m + n - 2))/(m - 1), Int[(a\*Csc[e + f\*x])<sup>(m - 2)</sup>\*(b\*Sec[e + f\*x])<sup>n</sup>, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && IntegersQ[2\*m, 2\*n] && !GtQ[n, m]

### Rule 2630

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))<sup>(m\_)</sup>\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])<sup>(n\_)</sup>, x\_Symbol] := Dist[(a\*Csc[e + f\*x])<sup>m</sup>\*(b\*Sec[e + f\*x])<sup>n</sup>\*(a\*Sin[e + f\*x])<sup>m</sup>\*(b\*Cos[e + f\*x])<sup>n</sup>, Int[1/((a\*Sin[e + f\*x])<sup>m</sup>\*(b\*Cos[e + f\*x])<sup>n</sup>), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]

### Rule 2573

Int[1/(Sqrt[cos[(e\_.) + (f\_.)\*(x\_.)]\*(b\_.)]\*Sqrt[(a\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]]), x\_Symbol] := Dist[Sqrt[Sin[2\*e + 2\*f\*x]]/(Sqrt[a\*Sin[e + f\*x]]\*Sqrt[b\*Cos[e + f\*x]]), Int[1/Sqrt[Sin[2\*e + 2\*f\*x]], x], x] /; FreeQ[{a, b, e, f}, x]

### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] := Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{(d \csc(a + bx))^{9/2}}{(c \sec(a + bx))^{3/2}} dx &= -\frac{2d(d \csc(a + bx))^{7/2}}{7bc\sqrt{c} \sec(a + bx)} - \frac{d^2 \int (d \csc(a + bx))^{5/2} \sqrt{c \sec(a + bx)} dx}{7c^2} \\
 &= \frac{2d^3(d \csc(a + bx))^{3/2}}{21bc\sqrt{c} \sec(a + bx)} - \frac{2d(d \csc(a + bx))^{7/2}}{7bc\sqrt{c} \sec(a + bx)} - \frac{(2d^4) \int \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} dx}{21c^2} \\
 &= \frac{2d^3(d \csc(a + bx))^{3/2}}{21bc\sqrt{c} \sec(a + bx)} - \frac{2d(d \csc(a + bx))^{7/2}}{7bc\sqrt{c} \sec(a + bx)} - \frac{(2d^4 \sqrt{c \cos(a + bx)} \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)})}{21c^2} \\
 &= \frac{2d^3(d \csc(a + bx))^{3/2}}{21bc\sqrt{c} \sec(a + bx)} - \frac{2d(d \csc(a + bx))^{7/2}}{7bc\sqrt{c} \sec(a + bx)} - \frac{(2d^4 \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} \sqrt{\sin(2a + 2bx)})}{21c^2} \\
 &= \frac{2d^3(d \csc(a + bx))^{3/2}}{21bc\sqrt{c} \sec(a + bx)} - \frac{2d(d \csc(a + bx))^{7/2}}{7bc\sqrt{c} \sec(a + bx)} - \frac{2d^4 \sqrt{d \csc(a + bx)} F\left(a - \frac{\pi}{4} + bx \mid 2\right) \sqrt{c \sec(a + bx)}}{21bc^2}
 \end{aligned}$$

**Mathematica [C]** time = 1.42184, size = 119, normalized size = 0.88

$$\frac{d^3 \cos(2(a + bx))(d \csc(a + bx))^{3/2} \left( (\cos(2(a + bx)) + 5) \csc^4(a + bx) - 2(-\cot^2(a + bx))^{3/4} \sec^2(a + bx) \right) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \csc[a + b*x]^2 * \sec[a + b*x]^2\right]}{21bc \left( \csc^2(a + bx) - 2 \right) \sqrt{c \sec(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(9/2)/(c\*Sec[a + b\*x])^(3/2), x]

[Out]  $-(d^3 \cos[2(a + b*x)] * (d \csc[a + b*x])^{3/2} * ((5 + \cos[2(a + b*x)]) * \csc[a + b*x]^4 - 2 * (-\cot[a + b*x]^2)^{3/4} * \text{Hypergeometric2F1}[1/2, 3/4, 3/2, \csc[a + b*x]^2 * \sec[a + b*x]^2]) / (21 * b * c * (-2 + \csc[a + b*x]^2) * \text{Sqrt}[c * \sec[a + b*x]]))$

**Maple [B]** time = 0.193, size = 550, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(9/2)/(c\*sec(b\*x+a))^(3/2), x)

[Out]  $\frac{1}{21} \frac{1}{b^2} 2^{1/2} * (2 * \cos(b*x+a)^3 * \sin(b*x+a) * (-(-1 + \cos(b*x+a) - \sin(b*x+a)) / \sin(b*x+a))^{1/2} * ((-1 + \cos(b*x+a) + \sin(b*x+a)) / \sin(b*x+a))^{1/2} * ((-1 + \cos(b*x+a)) / \sin(b*x+a))^{1/2} * \text{EllipticF}((-(-1 + \cos(b*x+a) - \sin(b*x+a)) / \sin(b*x+a))^{1/2}, 1/2 * 2^{1/2}) + 2 * \cos(b*x+a)^2 * \sin(b*x+a) * (-(-1 + \cos(b*x+a) - \sin(b*x+a)) / \sin(b*x+a))^{1/2} * ((-1 + \cos(b*x+a) + \sin(b*x+a)) / \sin(b*x+a))^{1/2} * ((-1 + \cos(b*x+a)) / \sin(b*x+a))^{1/2} * \text{EllipticF}((-(-1 + \cos(b*x+a) - \sin(b*x+a)) / \sin(b*x+a))^{1/2}, 1/2 * 2^{1/2}) - 2 * \cos(b*x+a) * \sin(b*x+a) * (-(-1 + \cos(b*x+a) - \sin(b*x+a)) / \sin(b*x+a))^{1/2} * ((-1 + \cos(b*x+a) + \sin(b*x+a)) / \sin(b*x+a))^{1/2} * ((-1 + \cos(b*x+a)) / \sin(b*x+a))^{1/2} * \text{EllipticF}((-(-1 + \cos(b*x+a) - \sin(b*x+a)) / \sin(b*x+a))^{1/2}, 1/2 * 2^{1/2}) - 2 * \sin(b*x+a) * (-(-1 + \cos(b*x+a) - \sin(b*x+a)) / \sin(b*x+a))^{1/2} * ((-1 + \cos(b*x+a) + \sin(b*x+a)) / \sin(b*x+a))^{1/2} * ((-1 + \cos(b*x+a)) / \sin(b*x+a))^{1/2} * \text{EllipticF}((-(-1 + \cos(b*x+a) - \sin(b*x+a)) / \sin(b*x+a))^{1/2}, 1/2 * 2^{1/2}) - \cos(b*x+a)^3 * 2^{1/2} - 2 * \cos(b*x+a) * 2^{1/2}) * (d / \sin(b*x+a))^{9/2} * \sin(b*x+a) / \cos(b*x+a)^2 / (c / \cos(b*x+a))^{3/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc (bx + a))^{\frac{9}{2}}}{(c \sec (bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(9/2)/(c\*sec(b\*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(9/2)/(c\*sec(b\*x + a))^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\sqrt{d \csc (bx + a)} \sqrt{c \sec (bx + a)} d^4 \csc (bx + a)^4}{c^2 \sec (bx + a)^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(9/2)/(c\*sec(b\*x+a))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a))\*d^4\*csc(b\*x + a)^4/(c^2\*sec(b\*x + a)^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(9/2)/(c\*sec(b\*x+a))\*\*(3/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc (bx + a))^{\frac{9}{2}}}{(c \sec (bx + a))^{\frac{3}{2}}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)/(c*sec(b*x + a))^(3/2), x)
```

$$3.264 \quad \int \frac{(d \csc(a+bx))^{7/2}}{(c \sec(a+bx))^{3/2}} dx$$

Optimal. Leaf size=33

$$-\frac{2cd(d \csc(a+bx))^{5/2}}{5b(c \sec(a+bx))^{5/2}}$$

[Out]  $(-2*c*d*(d*Csc[a + b*x])^(5/2))/(5*b*(c*Sec[a + b*x])^(5/2))$

**Rubi [A]** time = 0.0533232, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {2619}

$$-\frac{2cd(d \csc(a+bx))^{5/2}}{5b(c \sec(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*Csc[a + b*x])^(7/2)/(c*Sec[a + b*x])^(3/2), x]$

[Out]  $(-2*c*d*(d*Csc[a + b*x])^(5/2))/(5*b*(c*Sec[a + b*x])^(5/2))$

### Rule 2619

$\text{Int}[(\csc[(e_.) + (f_.)*(x_.)]*(a_.))^(m_.)*((b_.)*\sec[(e_.) + (f_.)*(x_.)])^(n_.), x\_Symbol] \rightarrow \text{Simp}[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1))/(f*(n - 1)), x] /;$  FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] && NeQ[n, 1]

### Rubi steps

$$\int \frac{(d \csc(a+bx))^{7/2}}{(c \sec(a+bx))^{3/2}} dx = -\frac{2cd(d \csc(a+bx))^{5/2}}{5b(c \sec(a+bx))^{5/2}}$$

**Mathematica [A]** time = 0.133327, size = 45, normalized size = 1.36

$$-\frac{2d^3 \cot^2(a+bx)\sqrt{d \csc(a+bx)}}{5bc\sqrt{c \sec(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(7/2)/(c\*Sec[a + b\*x])^(3/2),x]

[Out]  $(-2*d^3*\cot[a + b*x]^2*\sqrt{d*Csc[a + b*x]})/(5*b*c*\sqrt{c*Sec[a + b*x]})$

**Maple [A]** time = 0.142, size = 42, normalized size = 1.3

$$-\frac{2 \cos (bx+a) \sin (bx+a)}{5 b}\left(\frac{d}{\sin (bx+a)}\right)^{\frac{7}{2}}\left(\frac{c}{\cos (bx+a)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(7/2)/(c\*sec(b\*x+a))^(3/2),x)

[Out]  $-2/5/b*\cos(b*x+a)*\sin(b*x+a)*(d/\sin(b*x+a))^(7/2)/(c/\cos(b*x+a))^(3/2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc (bx+a))^{\frac{7}{2}}}{(c \sec (bx+a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(7/2)/(c\*sec(b\*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(7/2)/(c\*sec(b\*x + a))^(3/2), x)

**Fricas [B]** time = 1.81826, size = 131, normalized size = 3.97

$$\frac{2 d^3 \sqrt{\frac{c}{\cos (bx+a)}} \sqrt{\frac{d}{\sin (bx+a)}} \cos (bx+a)^3}{5\left(b c^2 \cos (bx+a)^2-b c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*d^3*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)^3/(b*c^2*cos
(b*x + a)^2 - b*c^2)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(7/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc(bx + a))^{\frac{7}{2}}}{(c \sec(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)/(c*sec(b*x + a))^(3/2), x)
```

$$3.265 \quad \int \frac{(d \csc(a+bx))^{5/2}}{(c \sec(a+bx))^{3/2}} dx$$

**Optimal.** Leaf size=98

$$\frac{d^2 \sqrt{\sin(2a+2bx)} \text{EllipticF}\left(a+bx-\frac{\pi}{4}, 2\right) \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}{3bc^2} - \frac{2d(d \csc(a+bx))^{3/2}}{3bc \sqrt{c \sec(a+bx)}}$$

[Out]  $(-2*d*(d*Csc[a + b*x])^(3/2))/(3*b*c*Sqrt[c*Sec[a + b*x]]) - (d^2*Sqrt[d*Cs$   
 $c[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a$   
 $+ 2*b*x]])/(3*b*c^2)$

**Rubi [A]** time = 0.147519, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2623, 2630, 2573, 2641}

$$\frac{d^2 \sqrt{\sin(2a+2bx)} F\left(a+bx-\frac{\pi}{4} \middle| 2\right) \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}{3bc^2} - \frac{2d(d \csc(a+bx))^{3/2}}{3bc \sqrt{c \sec(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[(d\*Csc[a + b\*x])^(5/2)/(c\*Sec[a + b\*x])^(3/2), x]

[Out]  $(-2*d*(d*Csc[a + b*x])^(3/2))/(3*b*c*Sqrt[c*Sec[a + b*x]]) - (d^2*Sqrt[d*Cs$   
 $c[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a$   
 $+ 2*b*x]])/(3*b*c^2)$

### Rule 2623

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> -Simp[(a\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1))/(f\*b\*(m - 1)), x] + Dist[(a^2\*(n + 1))/(b^2\*(m - 1)), Int[(a\*Csc[e + f\*x])^(m - 2)\*(b\*Sec[e + f\*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2\*m, 2\*n]

### Rule 2630

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Dist[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n\*(a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]

]

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_.)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_.)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(d \csc(a + bx))^{5/2}}{(c \sec(a + bx))^{3/2}} dx &= -\frac{2d(d \csc(a + bx))^{3/2}}{3bc\sqrt{c \sec(a + bx)}} - \frac{d^2 \int \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} dx}{3c^2} \\ &= -\frac{2d(d \csc(a + bx))^{3/2}}{3bc\sqrt{c \sec(a + bx)}} - \frac{(d^2 \sqrt{c \cos(a + bx)} \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} \sqrt{d \sin(a + bx)}) \int \frac{1}{\sqrt{c \cos(a + bx)}} dx}{3c^2} \\ &= -\frac{2d(d \csc(a + bx))^{3/2}}{3bc\sqrt{c \sec(a + bx)}} - \frac{(d^2 \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} \sqrt{\sin(2a + 2bx)}) \int \frac{1}{\sqrt{\sin(2a + 2bx)}} dx}{3c^2} \\ &= -\frac{2d(d \csc(a + bx))^{3/2}}{3bc\sqrt{c \sec(a + bx)}} - \frac{d^2 \sqrt{d \csc(a + bx)} F\left(a - \frac{\pi}{4} + bx \mid 2\right) \sqrt{c \sec(a + bx)} \sqrt{\sin(2a + 2bx)}}{3bc^2} \end{aligned}$$

**Mathematica [C]** time = 0.798222, size = 105, normalized size = 1.07

$$\frac{d \cos(2(a + bx)) \sec^3(a + bx) (d \csc(a + bx))^{3/2} \left(2 \cot^2(a + bx) - (-\cot^2(a + bx))^{3/4}\right) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \csc^2(a + bx)\right)}{3b \left(\csc^2(a + bx) - 2\right) (c \sec(a + bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(5/2)/(c*Sec[a + b*x])^(3/2), x]
```

```
[Out] -(d*Cos[2*(a + b*x)]*(d*Csc[a + b*x])^(3/2)*(2*Cot[a + b*x]^2 - (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2])*Sec[a + b*x]^3)/(3*b*(-2 + Csc[a + b*x]^2)*(c*Sec[a + b*x])^(3/2))
```

---

**Maple [B]** time = 0.173, size = 290, normalized size = 3.

$$\frac{\sqrt{2} \sin (b x+a)}{3 b(\cos (b x+a))^{2}}\left(\cos (b x+a) \sin (b x+a) \sqrt{\frac{-1+\cos (b x+a)-\sin (b x+a)}{\sin (b x+a)}} \sqrt{\frac{-1+\cos (b x+a)+\sin (b x+a)}{\sin (b x+a)}} \sqrt{\frac{-1+\cos (b x+a)-\sin (b x+a)}{\sin (b x+a)}} \sqrt{\frac{-1+\cos (b x+a)+\sin (b x+a)}{\sin (b x+a)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x)`

[Out]  $-1/3/b*2^{(1/2)}*(\cos(b*x+a)*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}*EllipticF((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)},1/2*2^{(1/2)})+\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}*EllipticF((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)},1/2*2^{(1/2)})+\cos(b*x+a)*2^{(1/2)}*(d/\sin(b*x+a))^{(5/2)}*\sin(b*x+a)/\cos(b*x+a)^2/(c/\cos(b*x+a))^{(3/2)}$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc (b x+a))^{5/2}}{(c \sec (b x+a))^{3/2}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")`

[Out] `integrate((d*csc(b*x + a))^(5/2)/(c*sec(b*x + a))^(3/2), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d \csc (b x+a)} \sqrt{c \sec (b x+a)} d^2 \csc (b x+a)^2}{c^2 \sec (b x+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*d^2*csc(b*x + a)^2/(c^2*
sec(b*x + a)^2), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc(bx + a))^{\frac{5}{2}}}{(c \sec(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)/(c*sec(b*x + a))^(3/2), x)
```



$$3.266 \quad \int \frac{(d \csc(a+bx))^{3/2}}{(c \sec(a+bx))^{3/2}} dx$$

**Optimal.** Leaf size=327

$$\frac{d^2 \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{c \sec(a+bx)}}{\sqrt{2bc^2}\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}} - \frac{d^2 \tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{c \sec(a+bx)}}{\sqrt{2bc^2}\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}} - \frac{d^2 \sqrt{c \sec(a+bx)} \log\left(\frac{1 - \sqrt{2}\sqrt{\tan(a+bx)}}{1 + \sqrt{2}\sqrt{\tan(a+bx)}}\right)}{2\sqrt{2bc^2}}$$

```
[Out] (-2*d*Sqrt[d*Csc[a + b*x]])/(b*c*Sqrt[c*Sec[a + b*x]]) + (d^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (d^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (d^2*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(2*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (d^2*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(2*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

**Rubi [A]** time = 0.215005, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2623, 2629, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{d^2 \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{c \sec(a+bx)}}{\sqrt{2bc^2}\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}} - \frac{d^2 \tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{c \sec(a+bx)}}{\sqrt{2bc^2}\sqrt{\tan(a+bx)}\sqrt{d \csc(a+bx)}} - \frac{d^2 \sqrt{c \sec(a+bx)} \log\left(\frac{1 - \sqrt{2}\sqrt{\tan(a+bx)}}{1 + \sqrt{2}\sqrt{\tan(a+bx)}}\right)}{2\sqrt{2bc^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(3/2)/(c*Sec[a + b*x])^(3/2), x]
```

```
[Out] (-2*d*Sqrt[d*Csc[a + b*x]])/(b*c*Sqrt[c*Sec[a + b*x]]) + (d^2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (d^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (d^2*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(2*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (d^2*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(2*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

### Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1))
```

)/(f\*b\*(m - 1)), x] + Dist[(a^2\*(n + 1))/(b^2\*(m - 1)), Int[(a\*Csc[e + f\*x])^(m - 2)\*(b\*Sec[e + f\*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2\*m, 2\*n]

### Rule 2629

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[((a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n)/Tan[e + f\*x]^n, Int[Tan[e + f\*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && EqQ[m + n, 0]

### Rule 3476

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

### Rule 204

$\text{Int}[\frac{(a_+) + (b_+)(x_+)^2}{(a_+ + (b_+)(x_+)^2)^{-1}}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\frac{\text{Rt}[-b, 2]x}{\text{Rt}[-a, 2]}] / \frac{\text{Rt}[-a, 2]}{\text{Rt}[-a, 2] \text{Rt}[-b, 2]}], x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rule 1165

$\text{Int}[\frac{(d_+) + (e_+)(x_+)^2}{(a_+) + (c_+)(x_+)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-2d/e, 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2d - ae^2, 0] \ \&\& \ \text{NegQ}[d^2e]$

### Rule 628

$\text{Int}[\frac{(d_+) + (e_+)(x_+)}{(a_+) + (b_+)(x_+) + (c_+)(x_+)^2}, x\_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{(d \csc(a + bx))^{3/2}}{(c \sec(a + bx))^{3/2}} dx &= -\frac{2d\sqrt{d} \csc(a + bx)}{bc\sqrt{c} \sec(a + bx)} - \frac{d^2 \int \frac{\sqrt{c \sec(a+bx)}}{\sqrt{d} \csc(a+bx)} dx}{c^2} \\
&= -\frac{2d\sqrt{d} \csc(a + bx)}{bc\sqrt{c} \sec(a + bx)} - \frac{(d^2 \sqrt{c} \sec(a + bx)) \int \sqrt{\tan(a + bx)} dx}{c^2 \sqrt{d} \csc(a + bx) \sqrt{\tan(a + bx)}} \\
&= -\frac{2d\sqrt{d} \csc(a + bx)}{bc\sqrt{c} \sec(a + bx)} - \frac{(d^2 \sqrt{c} \sec(a + bx)) \text{Subst} \left( \int \frac{\sqrt{x}}{1+x^2} dx, x, \tan(a + bx) \right)}{bc^2 \sqrt{d} \csc(a + bx) \sqrt{\tan(a + bx)}} \\
&= -\frac{2d\sqrt{d} \csc(a + bx)}{bc\sqrt{c} \sec(a + bx)} - \frac{(2d^2 \sqrt{c} \sec(a + bx)) \text{Subst} \left( \int \frac{x^2}{1+x^4} dx, x, \sqrt{\tan(a + bx)} \right)}{bc^2 \sqrt{d} \csc(a + bx) \sqrt{\tan(a + bx)}} \\
&= -\frac{2d\sqrt{d} \csc(a + bx)}{bc\sqrt{c} \sec(a + bx)} + \frac{(d^2 \sqrt{c} \sec(a + bx)) \text{Subst} \left( \int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(a + bx)} \right)}{bc^2 \sqrt{d} \csc(a + bx) \sqrt{\tan(a + bx)}} - \frac{(d^2 \sqrt{c} \sec(a + bx)) \text{Subst} \left( \int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\tan(a + bx)} \right)}{2bc^2 \sqrt{d} \csc(a + bx) \sqrt{\tan(a + bx)}} \\
&= -\frac{2d\sqrt{d} \csc(a + bx)}{bc\sqrt{c} \sec(a + bx)} - \frac{d^2 \log \left( 1 - \sqrt{2} \sqrt{\tan(a + bx)} + \tan(a + bx) \right) \sqrt{c} \sec(a + bx)}{2\sqrt{2}bc^2 \sqrt{d} \csc(a + bx) \sqrt{\tan(a + bx)}} + \frac{d^2 \log \left( 1 + \sqrt{2} \sqrt{\tan(a + bx)} + \tan(a + bx) \right) \sqrt{c} \sec(a + bx)}{2\sqrt{2}bc^2 \sqrt{d} \csc(a + bx) \sqrt{\tan(a + bx)}} \\
&= -\frac{2d\sqrt{d} \csc(a + bx)}{bc\sqrt{c} \sec(a + bx)} + \frac{d^2 \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(a + bx)} \right) \sqrt{c} \sec(a + bx)}{\sqrt{2}bc^2 \sqrt{d} \csc(a + bx) \sqrt{\tan(a + bx)}} - \frac{d^2 \tan^{-1} \left( 1 + \sqrt{2} \sqrt{\tan(a + bx)} \right) \sqrt{c} \sec(a + bx)}{\sqrt{2}bc^2 \sqrt{d} \csc(a + bx) \sqrt{\tan(a + bx)}}
\end{aligned}$$

**Mathematica [A]** time = 0.634882, size = 199, normalized size = 0.61

$$\frac{d\sqrt{d} \csc(a + bx) \left( 8\sqrt[4]{\cot^2(a + bx)} + \sqrt{2} \log \left( \sqrt{\cot^2(a + bx)} - \sqrt{2} \sqrt[4]{\cot^2(a + bx)} + 1 \right) - \sqrt{2} \log \left( \sqrt{\cot^2(a + bx)} + \sqrt{2} \sqrt[4]{\cot^2(a + bx)} + 1 \right) \right)}{4bc\sqrt[4]{\cot^2(a + bx)} \sqrt{c} \sec(a + bx)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(3/2)/(c\*Sec[a + b\*x])^(3/2), x]

[Out] -(d\*Sqrt[d\*Csc[a + b\*x]]\*(2\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4)] - 2\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4)] + 8\*(Cot[a + b\*x]^2)^(1/4) + Sqrt[2]\*Log[1 - Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4) + Sqrt[Cot[a + b\*x]^2]] - Sqrt[2]\*Log[1 + Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4) + Sqrt[Cot[a + b\*x]^2]]))/(4\*b\*c\*(Cot[a + b\*x]^2)^(1/4)\*Sqrt[c\*Sec[a + b\*x]])

---

**Maple [C]** time = 0.178, size = 975, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (d \csc(bx+a))^{3/2} / (c \sec(bx+a))^{3/2} dx$

[Out] 
$$-1/2/b^2^{1/2} * (I \cos(bx+a) * \text{EllipticPi}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2-1/2*I, 1/2*2^{1/2})) * (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} - I \cos(bx+a) * (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} * \text{EllipticPi}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2+1/2*I, 1/2*2^{1/2}) - \cos(bx+a) * (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} * \text{EllipticPi}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2-1/2*I, 1/2*2^{1/2}) - \cos(bx+a) * (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} * \text{EllipticPi}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2+1/2*I, 1/2*2^{1/2}) + I * \text{EllipticPi}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2-1/2*I, 1/2*2^{1/2}) * (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} - I * (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} * \text{EllipticPi}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2+1/2*I, 1/2*2^{1/2}) - \text{EllipticPi}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2-1/2*I, 1/2*2^{1/2}) * (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} - (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} * \text{EllipticPi}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2+1/2*I, 1/2*2^{1/2}) + 2 * \cos(bx+a) * 2^{1/2} * (d/\sin(bx+a))^{3/2} * \sin(bx+a) / \cos(bx+a)^{2/2} / (c/\cos(bx+a))^{3/2}$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc(bx+a))^{\frac{3}{2}}}{(c \sec(bx+a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(3/2)/(c\*sec(b\*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(3/2)/(c\*sec(b\*x + a))^(3/2), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(3/2)/(c\*sec(b\*x+a))^(3/2),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(3/2)/(c\*sec(b\*x+a))\*\*(3/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc (bx + a))^{\frac{3}{2}}}{(c \sec (bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(3/2)/(c\*sec(b\*x+a))^(3/2),x, algorithm="giac")

[Out] integrate((d\*csc(b\*x + a))^(3/2)/(c\*sec(b\*x + a))^(3/2), x)

$$3.267 \quad \int \frac{\sqrt{d \csc(a+bx)}}{(c \sec(a+bx))^{3/2}} dx$$

**Optimal.** Leaf size=92

$$\frac{\sqrt{\sin(2a+2bx)} \operatorname{EllipticF}\left(a+bx-\frac{\pi}{4}, 2\right) \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}{2bc^2} + \frac{d}{bc \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}$$

[Out] d/(b\*c\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]) + (Sqrt[d\*Csc[a + b\*x]]\*EllipticF[a - Pi/4 + b\*x, 2]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])/(2\*b\*c^2)

**Rubi [A]** time = 0.142954, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2628, 2630, 2573, 2641}

$$\frac{\sqrt{\sin(2a+2bx)} F\left(a+bx-\frac{\pi}{4} \middle| 2\right) \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}{2bc^2} + \frac{d}{bc \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*Csc[a + b\*x]]/(c\*Sec[a + b\*x])^(3/2), x]

[Out] d/(b\*c\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]) + (Sqrt[d\*Csc[a + b\*x]]\*EllipticF[a - Pi/4 + b\*x, 2]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])/(2\*b\*c^2)

### Rule 2628

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> -Simp[(a\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1))/(b\*f\*(m + n)), x] + Dist[(n + 1)/(b^2\*(m + n)), Int[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && NeQ[m + n, 0] && IntegersQ[2\*m, 2\*n]

### Rule 2630

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n\*(a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]

]

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{d \csc(a+bx)}}{(c \sec(a+bx))^{3/2}} dx &= \frac{d}{bc\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}} + \frac{\int \sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)} dx}{2c^2} \\ &= \frac{d}{bc\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}} + \frac{(\sqrt{c \cos(a+bx)}\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}\sqrt{d \sin(a+bx)})}{2c^2} \\ &= \frac{d}{bc\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}} + \frac{(\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}\sqrt{\sin(2a+2bx)})}{2c^2} \int \frac{1}{\sqrt{\sin(2a+2bx)}} \\ &= \frac{d}{bc\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}} + \frac{\sqrt{d \csc(a+bx)}F\left(a - \frac{\pi}{4} + bx \middle| 2\right) \sqrt{c \sec(a+bx)}\sqrt{\sin(2a+2bx)}}{2bc^2} \end{aligned}$$

**Mathematica [C]** time = 0.641334, size = 84, normalized size = 0.91

$$\frac{d \sec^3(a+bx) \left( -(-\cot^2(a+bx))^{3/4} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \csc^2(a+bx)\right) + \cos(2(a+bx)) + 1 \right)}{2b(c \sec(a+bx))^{3/2} \sqrt{d \csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[a + b*x]]/(c*Sec[a + b*x])^(3/2), x]
```

```
[Out] (d*(1 + Cos[2*(a + b*x)] - (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2])*Sec[a + b*x]^3)/(2*b*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2))
```



---

**Maple [A]** time = 0.19, size = 195, normalized size = 2.1

$$\frac{\sqrt{2} \sin(bx + a)}{2b(-1 + \cos(bx + a))(\cos(bx + a))^2} \left( -\sin(bx + a) \sqrt{\frac{-1 + \cos(bx + a) - \sin(bx + a)}{\sin(bx + a)}} \sqrt{\frac{-1 + \cos(bx + a) + \sin(bx + a)}{\sin(bx + a)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(1/2)/(c\*sec(b\*x+a))^(3/2), x)

[Out]  $\frac{1}{2} b^{-2} \sqrt{2} \sin(bx + a) \left( -\sin(bx + a) \sqrt{\frac{-1 + \cos(bx + a) - \sin(bx + a)}{\sin(bx + a)}} \sqrt{\frac{-1 + \cos(bx + a) + \sin(bx + a)}{\sin(bx + a)}} \right) \operatorname{EllipticF}\left(\sqrt{\frac{-1 + \cos(bx + a) - \sin(bx + a)}{\sin(bx + a)}}, \frac{1}{2} \sqrt{2}\right) + \frac{\cos(bx + a)^2 \sqrt{2} - \cos(bx + a)}{2c \cos(bx + a)^{3/2}}$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \csc(bx + a)}}{(c \sec(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(1/2)/(c\*sec(b\*x+a))^(3/2), x, algorithm="maxima")

[Out] integrate(sqrt(d\*csc(b\*x + a))/(c\*sec(b\*x + a))^(3/2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{d \csc(bx + a)} \sqrt{c \sec(bx + a)}}{c^2 \sec(bx + a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(1/2)/(c\*sec(b\*x+a))^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a))/(c^2\*sec(b\*x + a)^2), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d} \csc(a + bx)}{(c \sec(a + bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(1/2)/(c\*sec(b\*x+a))\*\*(3/2), x)

[Out] Integral(sqrt(d\*csc(a + b\*x))/(c\*sec(a + b\*x))\*\*(3/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d} \csc(bx + a)}{(c \sec(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(1/2)/(c\*sec(b\*x+a))^(3/2), x, algorithm="giac")

[Out] integrate(sqrt(d\*csc(b\*x + a))/(c\*sec(b\*x + a))^(3/2), x)

$$3.268 \quad \int \frac{1}{\sqrt{d} \csc(a+bx)(c \sec(a+bx))^{3/2}} dx$$

**Optimal.** Leaf size=322

$$\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{c \sec(a+bx)}}{4\sqrt{2}bc^2\sqrt{\tan(a+bx)}\sqrt{d} \csc(a+bx)} + \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{c \sec(a+bx)}}{4\sqrt{2}bc^2\sqrt{\tan(a+bx)}\sqrt{d} \csc(a+bx)} + \frac{\sqrt{c \sec(a+bx)} \log(\tan(a+bx))}{8\sqrt{2}bc^2\sqrt{\tan(a+bx)}}$$

```
[Out] d/(2*b*c*(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]) - (ArcTan[1 - Sqrt[2]
*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[d*Csc[a +
b*x]]*Sqrt[Tan[a + b*x]]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*
Sec[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) +
(Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(
8*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (Log[1 + Sqrt[2]
*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(8*Sqrt[2]*b*c^2*
Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

**Rubi [A]** time = 0.213415, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2628, 2629, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{c \sec(a+bx)}}{4\sqrt{2}bc^2\sqrt{\tan(a+bx)}\sqrt{d} \csc(a+bx)} + \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{c \sec(a+bx)}}{4\sqrt{2}bc^2\sqrt{\tan(a+bx)}\sqrt{d} \csc(a+bx)} + \frac{\sqrt{c \sec(a+bx)} \log(\tan(a+bx))}{8\sqrt{2}bc^2\sqrt{\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)),x]
```

```
[Out] d/(2*b*c*(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]) - (ArcTan[1 - Sqrt[2]
*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[d*Csc[a +
b*x]]*Sqrt[Tan[a + b*x]]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*
Sec[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) +
(Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(
8*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (Log[1 + Sqrt[2]
*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(8*Sqrt[2]*b*c^2*
Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

### Rule 2628

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
```

))/(b\*f\*(m + n)), x] + Dist[(n + 1)/(b^2\*(m + n)), Int[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && NeQ[m + n, 0] && IntegersQ[2\*m, 2\*n]

### Rule 2629

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^m\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^n, x\_Symbol] := Dist[((a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n)/Tan[e + f\*x]^n, Int[Tan[e + f\*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && EqQ[m + n, 0]

### Rule 3476

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^n, x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 329

Int[((c\_.)\*(x\_))^m\*((a\_) + (b\_.)\*(x\_)^(n\_))^p, x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n))/c^n)^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

### Rule 204

$\text{Int}[\frac{(a_1 + (b_1)x^2)^{-1}}{x}, x\_Symbol] \rightarrow -\text{Simp}[\frac{\text{ArcTan}[\frac{\text{Rt}[-b, 2]x}{\text{Rt}[-a, 2] + \text{Rt}[-a, 2]\text{Rt}[-b, 2]}}]{\text{Rt}[-a, 2] + \text{Rt}[-a, 2]\text{Rt}[-b, 2]}, x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rule 1165

$\text{Int}[\frac{(d_1 + (e_1)x^2)}{(a_1 + (c_1)x^4)}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[\frac{-2d_1}{e_1}, 2]\}, \text{Dist}[e/(2cq), \text{Int}[\frac{q - 2x}{\text{Simp}[d/e + qx - x^2, x]}, x], x] + \text{Dist}[e/(2cq), \text{Int}[\frac{q + 2x}{\text{Simp}[d/e - qx - x^2, x]}, x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2d^2 - ae^2, 0] \ \&\& \ \text{NegQ}[d^2e]$

### Rule 628

$\text{Int}[\frac{(d_1 + (e_1)x)}{(a_1 + (b_1)x + (c_1)x^2)}, x\_Symbol] \rightarrow \text{Simp}[\frac{d_1 \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{d \csc(a+bx)}(c \sec(a+bx))^{3/2}} dx &= \frac{d}{2bc(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} + \frac{\int \frac{\sqrt{c \sec(a+bx)}}{\sqrt{d \csc(a+bx)}} dx}{4c^2} \\
&= \frac{d}{2bc(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} + \frac{\sqrt{c \sec(a+bx)} \int \sqrt{\tan(a+bx)} dx}{4c^2 \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\
&= \frac{d}{2bc(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} + \frac{\sqrt{c \sec(a+bx)} \operatorname{Subst}\left(\int \frac{\sqrt{x}}{1+x^2} dx, x, \tan(a+bx)\right)}{4bc^2 \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\
&= \frac{d}{2bc(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} + \frac{\sqrt{c \sec(a+bx)} \operatorname{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{\tan(a+bx)}\right)}{2bc^2 \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\
&= \frac{d}{2bc(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} - \frac{\sqrt{c \sec(a+bx)} \operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(a+bx)}\right)}{4bc^2 \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\
&= \frac{d}{2bc(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} + \frac{\sqrt{c \sec(a+bx)} \operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\tan(a+bx)}\right)}{8bc^2 \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\
&= \frac{d}{2bc(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} + \frac{\log\left(1 - \sqrt{2}\sqrt{\tan(a+bx)} + \tan(a+bx)\right)}{8\sqrt{2}bc^2 \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}} \\
&= \frac{d}{2bc(d \csc(a+bx))^{3/2} \sqrt{c \sec(a+bx)}} - \frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{c \sec(a+bx)}}{4\sqrt{2}bc^2 \sqrt{d \csc(a+bx)} \sqrt{\tan(a+bx)}}
\end{aligned}$$

**Mathematica [A]** time = 1.97279, size = 223, normalized size = 0.69

$$\frac{\sqrt{d \csc(a+bx)} \left( 4 \sqrt[4]{\cot^2(a+bx)} - 4 \cos(2(a+bx)) \sqrt[4]{\cot^2(a+bx)} + \sqrt{2} \log \left( \sqrt{\cot^2(a+bx)} - \sqrt{2} \sqrt[4]{\cot^2(a+bx)} + 1 \right) - \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(a+bx)} \right) \sqrt{c \sec(a+bx)} \right)}{16bcd \sqrt[4]{\cot^2(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d\*Csc[a + b\*x]]\*(c\*Sec[a + b\*x])^(3/2)),x]

[Out] (Sqrt[d\*Csc[a + b\*x]]\*(2\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4)] - 2\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4)] + 4\*(Cot[a + b\*x]^2)^(1/4) - 4\*Cos[2\*(a + b\*x)]\*(Cot[a + b\*x]^2)^(1/4) + Sqrt[2]\*Log[1 - Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4) + Sqrt[Cot[a + b\*x]^2]] - Sqrt[2]\*Log[1 + Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4) + Sqrt[Cot[a + b\*x]^2]]))/(16\*b\*c\*d\*(Cot[a + b\*x]^2)^(1/4)\*Sqrt[c\*Sec[a + b\*x]])

---

**Maple [C]** time = 0.19, size = 526, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2), x)`

[Out]  $\frac{1}{8}b^{2^{1/2}}(-I\text{EllipticPi}(\frac{-(-1+\cos(bx+a)-\sin(bx+a))}{\sin(bx+a)})^{1/2}, \frac{1}{2}-\frac{1}{2}I, \frac{1}{2}2^{1/2}) * (-(-1+\cos(bx+a)-\sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1+\cos(bx+a)+\sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1+\cos(bx+a)) / \sin(bx+a))^{1/2} + I * (-(-1+\cos(bx+a)-\sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1+\cos(bx+a)+\sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1+\cos(bx+a)) / \sin(bx+a))^{1/2} * \text{EllipticPi}(\frac{-(-1+\cos(bx+a)-\sin(bx+a))}{\sin(bx+a)})^{1/2}, \frac{1}{2}+\frac{1}{2}I, \frac{1}{2}2^{1/2}) + \text{EllipticPi}(\frac{-(-1+\cos(bx+a)-\sin(bx+a))}{\sin(bx+a)})^{1/2}, \frac{1}{2}-\frac{1}{2}I, \frac{1}{2}2^{1/2}) * (-(-1+\cos(bx+a)-\sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1+\cos(bx+a)+\sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1+\cos(bx+a)) / \sin(bx+a))^{1/2} + (-(-1+\cos(bx+a)-\sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1+\cos(bx+a)+\sin(bx+a)) / \sin(bx+a))^{1/2} * ((-1+\cos(bx+a)) / \sin(bx+a))^{1/2} * \text{EllipticPi}(\frac{-(-1+\cos(bx+a)-\sin(bx+a))}{\sin(bx+a)})^{1/2}, \frac{1}{2}+\frac{1}{2}I, \frac{1}{2}2^{1/2}) + 2*\cos(bx+a)^{2^{1/2}} - 2*\cos(bx+a)*2^{1/2} * \sin(bx+a) / (-1+\cos(bx+a)) / \cos(bx+a)^2 / (d/\sin(bx+a))^{1/2} / (c/\cos(bx+a))^{3/2}$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d} \csc(bx+a) (c \sec(bx+a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2), x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(3/2)), x)`

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d} \csc(bx+a) (c \sec(bx+a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(3/2)), x)
```



$$3.269 \quad \int \frac{1}{(d \csc(a+bx))^{3/2} (c \sec(a+bx))^{3/2}} dx$$

**Optimal.** Leaf size=135

$$\frac{\sqrt{\sin(2a+2bx)} \operatorname{EllipticF}\left(a+bx-\frac{\pi}{4}, 2\right) \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}{12bc^2d^2} - \frac{c}{3bd(c \sec(a+bx))^{5/2} \sqrt{d \csc(a+bx)}} + \frac{1}{6bcd \sqrt{c \sec(a+bx)}}$$

[Out]  $-c/(3*b*d*\sqrt{d*\operatorname{Csc}[a+b*x]}*(c*\operatorname{Sec}[a+b*x])^{5/2}) + 1/(6*b*c*d*\sqrt{d*\operatorname{Csc}[a+b*x]}*\sqrt{c*\operatorname{Sec}[a+b*x]}) + (\sqrt{d*\operatorname{Csc}[a+b*x]}*\operatorname{EllipticF}[a-Pi/4+b*x, 2]*\sqrt{c*\operatorname{Sec}[a+b*x]}*\sqrt{\operatorname{Sin}[2*a+2*b*x]})/(12*b*c^2*d^2)$

**Rubi [A]** time = 0.207116, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2627, 2628, 2630, 2573, 2641}

$$\frac{\sqrt{\sin(2a+2bx)} F\left(a+bx-\frac{\pi}{4} \middle| 2\right) \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}}{12bc^2d^2} - \frac{c}{3bd(c \sec(a+bx))^{5/2} \sqrt{d \csc(a+bx)}} + \frac{1}{6bcd \sqrt{c \sec(a+bx)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[1/((d*\operatorname{Csc}[a+b*x])^{3/2}*(c*\operatorname{Sec}[a+b*x])^{3/2}), x]$

[Out]  $-c/(3*b*d*\sqrt{d*\operatorname{Csc}[a+b*x]}*(c*\operatorname{Sec}[a+b*x])^{5/2}) + 1/(6*b*c*d*\sqrt{d*\operatorname{Csc}[a+b*x]}*\sqrt{c*\operatorname{Sec}[a+b*x]}) + (\sqrt{d*\operatorname{Csc}[a+b*x]}*\operatorname{EllipticF}[a-Pi/4+b*x, 2]*\sqrt{c*\operatorname{Sec}[a+b*x]}*\sqrt{\operatorname{Sin}[2*a+2*b*x]})/(12*b*c^2*d^2)$

### Rule 2627

$\operatorname{Int}[(\operatorname{csc}[e_.] + (f_.)*(x_.)*(a_.)^{(m_.)}*((b_.)*\operatorname{sec}[e_.] + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(b*(a*\operatorname{Csc}[e+f*x])^{(m+1)}*(b*\operatorname{Sec}[e+f*x])^{(n-1)})/(a*f*(m+n)), x] + \operatorname{Dist}[(m+1)/(a^{2*(m+n)}), \operatorname{Int}[(a*\operatorname{Csc}[e+f*x])^{(m+2)}*(b*\operatorname{Sec}[e+f*x])^n, x], x] /;$   $\operatorname{FreeQ}\{a, b, e, f, n\}, x$  &&  $\operatorname{LtQ}[m, -1]$  &&  $\operatorname{NeQ}[m+n, 0]$  &&  $\operatorname{IntegersQ}[2*m, 2*n]$

### Rule 2628

$\operatorname{Int}[(\operatorname{csc}[e_.] + (f_.)*(x_.)*(a_.)^{(m_.)}*((b_.)*\operatorname{sec}[e_.] + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow -\operatorname{Simp}[(a*(a*\operatorname{Csc}[e+f*x])^{(m-1)}*(b*\operatorname{Sec}[e+f*x])^{(n+1)})/(b*f*(m+n)), x] + \operatorname{Dist}[(n+1)/(b^{2*(m+n)}), \operatorname{Int}[(a*\operatorname{Csc}[e+f*x])^m*(b*\operatorname{Sec}[e+f*x])^{(n+2)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, e, f, m\}, x$  &&  $\operatorname{LtQ}[n, -1]$

&& NeQ[m + n, 0] && IntegersQ[2\*m, 2\*n]

### Rule 2630

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]
```

### Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

### Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{3/2}} dx &= -\frac{c}{3bd\sqrt{d \csc(a + bx)}(c \sec(a + bx))^{5/2}} + \frac{\int \frac{\sqrt{d \csc(a + bx)}}{(c \sec(a + bx))^{3/2}} dx}{6d^2} \\ &= -\frac{c}{3bd\sqrt{d \csc(a + bx)}(c \sec(a + bx))^{5/2}} + \frac{1}{6bcd\sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}} + \\ &= -\frac{c}{3bd\sqrt{d \csc(a + bx)}(c \sec(a + bx))^{5/2}} + \frac{1}{6bcd\sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}} + \\ &= -\frac{c}{3bd\sqrt{d \csc(a + bx)}(c \sec(a + bx))^{5/2}} + \frac{1}{6bcd\sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}} + \\ &= -\frac{c}{3bd\sqrt{d \csc(a + bx)}(c \sec(a + bx))^{5/2}} + \frac{1}{6bcd\sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}} + \end{aligned}$$

**Mathematica [C]** time = 0.554515, size = 89, normalized size = 0.66

$$\frac{\frac{\csc^2(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \csc^2(a+bx)\right)}{\sqrt[4]{-\cot^2(a+bx)}} - 2\cos(2(a+bx))}{12bcd\sqrt{c}\sec(a+bx)\sqrt{d}\csc(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d\*Csc[a + b\*x])^(3/2)\*(c\*Sec[a + b\*x])^(3/2)),x]

[Out] (-2\*Cos[2\*(a + b\*x)] + (Csc[a + b\*x]^2\*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b\*x]^2])/(-Cot[a + b\*x]^2)^(1/4))/(12\*b\*c\*d\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]])

**Maple [A]** time = 0.181, size = 222, normalized size = 1.6

$$\frac{\sqrt{2}}{12b(-1 + \cos(bx + a))(\cos(bx + a))^2 \sin(bx + a)} \left( 2\sqrt{2}(\cos(bx + a))^4 + \sin(bx + a) \sqrt{-\frac{-1 + \cos(bx + a) - \sin(bx + a)}{\sin(bx + a)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*csc(b\*x+a))^(3/2)/(c\*sec(b\*x+a))^(3/2),x)

[Out] -1/12/b\*2^(1/2)\*(2\*2^(1/2)\*cos(b\*x+a)^4+sin(b\*x+a)\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1/2\*2^(1/2))-2\*cos(b\*x+a)^3\*2^(1/2)-cos(b\*x+a)^2\*2^(1/2)+cos(b\*x+a)\*2^(1/2))/(-1+cos(b\*x+a))/cos(b\*x+a)^2/sin(b\*x+a)/(c/cos(b\*x+a))^(3/2)/(d/sin(b\*x+a))^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d \csc(bx + a))^{\frac{3}{2}} (c \sec(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(3/2)), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d} \csc(bx + a) \sqrt{c \sec(bx + a)}}{c^2 d^2 \csc(bx + a)^2 \sec(bx + a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c^2*d^2*csc(b*x + a)^2*sec(b*x + a)^2), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d \csc(bx + a))^{\frac{3}{2}} (c \sec(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(3/2)), x)
```

$$3.270 \quad \int \frac{1}{(d \csc(a+bx))^{5/2} (c \sec(a+bx))^{3/2}} dx$$

**Optimal.** Leaf size=371

$$\frac{3 \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(a+bx)} \right) \sqrt{c \sec(a+bx)}}{32 \sqrt{2} bc^2 d^2 \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}} + \frac{3 \tan^{-1} \left( \sqrt{2} \sqrt{\tan(a+bx)} + 1 \right) \sqrt{c \sec(a+bx)}}{32 \sqrt{2} bc^2 d^2 \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}} + \frac{3 \sqrt{c \sec(a+bx)} \log \left( \dots \right)}{64 \sqrt{2} bc^2 d^2 \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}$$

[Out]  $-c/(4*b*d*(d*Csc[a + b*x])^{(3/2)}*(c*Sec[a + b*x])^{(5/2)}) + 3/(16*b*c*d*(d*Csc[a + b*x])^{(3/2)}*Sqrt[c*Sec[a + b*x]]) - (3*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]])*Sqrt[c*Sec[a + b*x]])/(32*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]])*Sqrt[c*Sec[a + b*x]])/(32*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (3*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(64*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (3*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(64*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])$

**Rubi [A]** time = 0.297432, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.44$ , Rules used = {2627, 2628, 2629, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3 \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(a+bx)} \right) \sqrt{c \sec(a+bx)}}{32 \sqrt{2} bc^2 d^2 \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}} + \frac{3 \tan^{-1} \left( \sqrt{2} \sqrt{\tan(a+bx)} + 1 \right) \sqrt{c \sec(a+bx)}}{32 \sqrt{2} bc^2 d^2 \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}} + \frac{3 \sqrt{c \sec(a+bx)} \log \left( \dots \right)}{64 \sqrt{2} bc^2 d^2 \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[1/((d\*Csc[a + b\*x])^(5/2)\*(c\*Sec[a + b\*x])^(3/2)),x]

[Out]  $-c/(4*b*d*(d*Csc[a + b*x])^{(3/2)}*(c*Sec[a + b*x])^{(5/2)}) + 3/(16*b*c*d*(d*Csc[a + b*x])^{(3/2)}*Sqrt[c*Sec[a + b*x]]) - (3*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]])*Sqrt[c*Sec[a + b*x]])/(32*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]])*Sqrt[c*Sec[a + b*x]])/(32*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (3*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(64*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (3*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(64*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])$

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

### Rule 2628

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1))/(b*f*(m + n)), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

### Rule 2629

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[((a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n)/Tan[e + f*x]^n, Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && EqQ[m + n, 0]
```

### Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{1}{(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{3/2}} dx &= -\frac{c}{4bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{5/2}} + \frac{3 \int \frac{1}{\sqrt{d \csc(a+bx)} (c \sec(a+bx))^{3/2}} dx}{8d^2} \\
&= -\frac{c}{4bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{5/2}} + \frac{3}{16bcd(d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)}} \\
&= -\frac{c}{4bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{5/2}} + \frac{3}{16bcd(d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)}} \\
&= -\frac{c}{4bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{5/2}} + \frac{3}{16bcd(d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)}} \\
&= -\frac{c}{4bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{5/2}} + \frac{3}{16bcd(d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)}} \\
&= -\frac{c}{4bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{5/2}} + \frac{3}{16bcd(d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)}} \\
&= -\frac{c}{4bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{5/2}} + \frac{3}{16bcd(d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)}} \\
&= -\frac{c}{4bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{5/2}} + \frac{3}{16bcd(d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)}} \\
&= -\frac{c}{4bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{5/2}} + \frac{3}{16bcd(d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)}} \\
&= -\frac{c}{4bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{5/2}} + \frac{3}{16bcd(d \csc(a + bx))^{3/2} \sqrt{c \sec(a + bx)}}
\end{aligned}$$

**Mathematica [A]** time = 2.30413, size = 246, normalized size = 0.66

$$\sqrt{d \csc(a + bx)} \left( 8 \sqrt[4]{\cot^2(a + bx)} - 12 \cos(2(a + bx)) \sqrt[4]{\cot^2(a + bx)} + 4 \cos(4(a + bx)) \sqrt[4]{\cot^2(a + bx)} + 3\sqrt{2} \log \left( \sqrt{\cot^2(a + bx)} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((d\*Csc[a + b\*x])^(5/2)\*(c\*Sec[a + b\*x])^(3/2)),x]

[Out] (Sqrt[d\*Csc[a + b\*x]]\*(6\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4)] - 6\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4)] + 8\*(Cot[a + b\*x]^2)^(1/4) - 12\*Cos[2\*(a + b\*x)]\*(Cot[a + b\*x]^2)^(1/4) + 4\*Cos[4\*(a + b\*x)]\*(Cot[a + b\*x]^2)^(1/4) + 3\*Sqrt[2]\*Log[1 - Sqrt[2]\*(Cot[a + b\*x]^2)^(1/4) +

```
Sqrt[Cot[a + b*x]^2]] - 3*Sqrt[2]*Log[1 + Sqrt[2]*(Cot[a + b*x]^2)^(1/4) +
Sqrt[Cot[a + b*x]^2]])/(128*b*c*d^3*(Cot[a + b*x]^2)^(1/4)*Sqrt[c*Sec[a +
b*x]])
```

**Maple [C]** time = 0.174, size = 556, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2), x)
```

```
[Out] -1/64/b*2^(1/2)*(8*2^(1/2)*cos(b*x+a)^4-3*I*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2), 1/2+1/2*I, 1/2*2^(1/2))+3*I*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2), 1/2-1/2*I, 1/2*2^(1/2))*((-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)-8*cos(b*x+a)^3*2^(1/2)-3*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2), 1/2+1/2*I, 1/2*2^(1/2))-3*EllipticPi((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2), 1/2-1/2*I, 1/2*2^(1/2))*((-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)-6*cos(b*x+a)^2*2^(1/2)+6*cos(b*x+a)*2^(1/2))/(-1+cos(b*x+a))/cos(b*x+a)^2/sin(b*x+a)/(d/sin(b*x+a))^(5/2)/(c/cos(b*x+a))^(3/2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d \csc(bx + a))^{\frac{5}{2}} (c \sec(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2), x, algorithm="maxima")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(3/2)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*csc(b\*x+a))^(5/2)/(c\*sec(b\*x+a))^(3/2),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*csc(b\*x+a))\*\*(5/2)/(c\*sec(b\*x+a))\*\*(3/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d \csc(bx + a))^{\frac{5}{2}} (c \sec(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*csc(b\*x+a))^(5/2)/(c\*sec(b\*x+a))^(3/2),x, algorithm="giac")

[Out] integrate(1/((d\*csc(b\*x + a))^(5/2)\*(c\*sec(b\*x + a))^(3/2)), x)

$$3.271 \quad \int \frac{(d \csc(a+bx))^{9/2}}{(c \sec(a+bx))^{5/2}} dx$$

Optimal. Leaf size=33

$$-\frac{2cd(d \csc(a+bx))^{7/2}}{7b(c \sec(a+bx))^{7/2}}$$

[Out]  $(-2*c*d*(d*Csc[a + b*x])^(7/2))/(7*b*(c*Sec[a + b*x])^(7/2))$

**Rubi [A]** time = 0.0540944, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$ , Rules used = {2619}

$$-\frac{2cd(d \csc(a+bx))^{7/2}}{7b(c \sec(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*Csc[a + b*x])^(9/2)/(c*Sec[a + b*x])^(5/2), x]$

[Out]  $(-2*c*d*(d*Csc[a + b*x])^(7/2))/(7*b*(c*Sec[a + b*x])^(7/2))$

### Rule 2619

$\text{Int}[(\csc[(e_.) + (f_.)*(x_.)]*(a_.))^(m_.)*((b_.)*\sec[(e_.) + (f_.)*(x_.)])^(n_.), x\_Symbol] \rightarrow \text{Simp}[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1))/(f*(n - 1)), x] /;$  FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] && NeQ[n, 1]

### Rubi steps

$$\int \frac{(d \csc(a+bx))^{9/2}}{(c \sec(a+bx))^{5/2}} dx = -\frac{2cd(d \csc(a+bx))^{7/2}}{7b(c \sec(a+bx))^{7/2}}$$

**Mathematica [A]** time = 0.158198, size = 45, normalized size = 1.36

$$-\frac{2d^4 \cot^3(a+bx)\sqrt{d \csc(a+bx)}}{7bc^2\sqrt{c \sec(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(9/2)/(c\*Sec[a + b\*x])^(5/2),x]

[Out]  $(-2*d^4*\cot[a + b*x]^3*\sqrt{d*\csc[a + b*x]}]/(7*b*c^2*\sqrt{c*\sec[a + b*x]})$

**Maple [A]** time = 0.138, size = 42, normalized size = 1.3

$$-\frac{2 \cos (bx+a) \sin (bx+a)}{7 b}\left(\frac{d}{\sin (bx+a)}\right)^{\frac{9}{2}}\left(\frac{c}{\cos (bx+a)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(9/2)/(c\*sec(b\*x+a))^(5/2),x)

[Out]  $-2/7/b*\cos(b*x+a)*\sin(b*x+a)*(d/\sin(b*x+a))^(9/2)/(c/\cos(b*x+a))^(5/2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc (bx+a))^{\frac{9}{2}}}{(c \sec (bx+a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(9/2)/(c\*sec(b\*x+a))^(5/2),x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(9/2)/(c\*sec(b\*x + a))^(5/2), x)

**Fricas [B]** time = 2.30939, size = 151, normalized size = 4.58

$$\frac{2 d^4 \sqrt{\frac{c}{\cos (bx+a)}} \sqrt{\frac{d}{\sin (bx+a)}} \cos (bx+a)^4}{7\left(b c^3 \cos (bx+a)^2-b c^3\right) \sin (bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*d^4*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)^4/((b*c^3*cos(b*x + a)^2 - b*c^3)*sin(b*x + a))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(9/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc(bx + a))^{\frac{9}{2}}}{(c \sec(bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)/(c*sec(b*x + a))^(5/2), x)
```

$$3.272 \quad \int \frac{(d \csc(a+bx))^{7/2}}{(c \sec(a+bx))^{5/2}} dx$$

**Optimal.** Leaf size=135

$$\frac{6d^4 E\left(a + bx - \frac{\pi}{4} \middle| 2\right)}{5bc^2 \sqrt{\sin(2a + 2bx)} \sqrt{c \sec(a + bx)} \sqrt{d \csc(a + bx)}} + \frac{6d^3 \sqrt{d \csc(a + bx)}}{5bc(c \sec(a + bx))^{3/2}} - \frac{2d(d \csc(a + bx))^{5/2}}{5bc(c \sec(a + bx))^{3/2}}$$

[Out] (6\*d^3\*Sqrt[d\*Csc[a + b\*x]]/(5\*b\*c\*(c\*Sec[a + b\*x])^(3/2)) - (2\*d\*(d\*Csc[a + b\*x])^(5/2))/(5\*b\*c\*(c\*Sec[a + b\*x])^(3/2)) + (6\*d^4\*EllipticE[a - Pi/4 + b\*x, 2])/(5\*b\*c^2\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]]))

**Rubi [A]** time = 0.202232, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2623, 2625, 2630, 2572, 2639}

$$\frac{6d^4 E\left(a + bx - \frac{\pi}{4} \middle| 2\right)}{5bc^2 \sqrt{\sin(2a + 2bx)} \sqrt{c \sec(a + bx)} \sqrt{d \csc(a + bx)}} + \frac{6d^3 \sqrt{d \csc(a + bx)}}{5bc(c \sec(a + bx))^{3/2}} - \frac{2d(d \csc(a + bx))^{5/2}}{5bc(c \sec(a + bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d\*Csc[a + b\*x])^(7/2)/(c\*Sec[a + b\*x])^(5/2), x]

[Out] (6\*d^3\*Sqrt[d\*Csc[a + b\*x]]/(5\*b\*c\*(c\*Sec[a + b\*x])^(3/2)) - (2\*d\*(d\*Csc[a + b\*x])^(5/2))/(5\*b\*c\*(c\*Sec[a + b\*x])^(3/2)) + (6\*d^4\*EllipticE[a - Pi/4 + b\*x, 2])/(5\*b\*c^2\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]]))

### Rule 2623

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^(n\_)), x\_Symbol] :> -Simp[(a\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1))/(f\*b\*(m - 1)), x] + Dist[(a^2\*(n + 1))/(b^2\*(m - 1)), Int[(a\*Csc[e + f\*x])^(m - 2)\*(b\*Sec[e + f\*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2\*m, 2\*n]

### Rule 2625

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^(n\_)), x\_Symbol] :> -Simp[(a\*b\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n -

1))/(f\*(m - 1)), x] + Dist[(a^2\*(m + n - 2))/(m - 1), Int[(a\*Csc[e + f\*x])<sup>(m - 2)</sup>\*(b\*Sec[e + f\*x])<sup>n</sup>, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && IntegerQ[2\*m, 2\*n] && !GtQ[n, m]

### Rule 2630

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))<sup>(m\_)</sup>\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])<sup>(n\_)</sup>, x\_Symbol] := Dist[(a\*Csc[e + f\*x])<sup>m</sup>\*(b\*Sec[e + f\*x])<sup>n</sup>\*(a\*Sin[e + f\*x])<sup>m</sup>\*(b\*Cos[e + f\*x])<sup>n</sup>, Int[1/((a\*Sin[e + f\*x])<sup>m</sup>\*(b\*Cos[e + f\*x])<sup>n</sup>), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]

### Rule 2572

Int[Sqrt[cos[(e\_.) + (f\_.)\*(x\_.)]\*(b\_.)]\*Sqrt[(a\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]], x\_Symbol] := Dist[(Sqrt[a\*Sin[e + f\*x]]\*Sqrt[b\*Cos[e + f\*x]])/Sqrt[Sin[2\*e + 2\*f\*x]], Int[Sqrt[Sin[2\*e + 2\*f\*x]], x], x] /; FreeQ[{a, b, e, f}, x]

### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{(d \csc(a + bx))^{7/2}}{(c \sec(a + bx))^{5/2}} dx &= -\frac{2d(d \csc(a + bx))^{5/2}}{5bc(c \sec(a + bx))^{3/2}} - \frac{(3d^2) \int \frac{(d \csc(a + bx))^{3/2}}{\sqrt{c \sec(a + bx)}} dx}{5c^2} \\
 &= \frac{6d^3 \sqrt{d \csc(a + bx)}}{5bc(c \sec(a + bx))^{3/2}} - \frac{2d(d \csc(a + bx))^{5/2}}{5bc(c \sec(a + bx))^{3/2}} + \frac{(6d^4) \int \frac{1}{\sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)}} dx}{5c^2} \\
 &= \frac{6d^3 \sqrt{d \csc(a + bx)}}{5bc(c \sec(a + bx))^{3/2}} - \frac{2d(d \csc(a + bx))^{5/2}}{5bc(c \sec(a + bx))^{3/2}} + \frac{(6d^4) \int \sqrt{c \cos(a + bx)} \sqrt{d \sin(a + bx)}}{5c^2 \sqrt{c \cos(a + bx)} \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)}} \\
 &= \frac{6d^3 \sqrt{d \csc(a + bx)}}{5bc(c \sec(a + bx))^{3/2}} - \frac{2d(d \csc(a + bx))^{5/2}}{5bc(c \sec(a + bx))^{3/2}} + \frac{(6d^4) \int \sqrt{\sin(2a + 2bx)} dx}{5c^2 \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} \sqrt{\sin(2a + 2bx)}} \\
 &= \frac{6d^3 \sqrt{d \csc(a + bx)}}{5bc(c \sec(a + bx))^{3/2}} - \frac{2d(d \csc(a + bx))^{5/2}}{5bc(c \sec(a + bx))^{3/2}} + \frac{6d^4 E\left(a - \frac{\pi}{4} + bx \mid 2\right)}{5bc^2 \sqrt{d \csc(a + bx)} \sqrt{c \sec(a + bx)} \sqrt{\sin(2a + 2bx)}}
 \end{aligned}$$



**Mathematica [C]** time = 1.80213, size = 101, normalized size = 0.75

$$\frac{d^5 \sqrt{c \sec(a + bx)} \left( 6 \sqrt[4]{-\cot^2(a + bx)} \operatorname{Hypergeometric2F1} \left( -\frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \csc^2(a + bx) \right) + (1 - 3 \cos(2(a + bx))) \cot^2(a + bx) \right)}{5bc^3(d \csc(a + bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(7/2)/(c\*Sec[a + b\*x])^(5/2),x]

[Out] (d^5\*((1 - 3\*Cos[2\*(a + b\*x)])\*Cot[a + b\*x]^2\*Csc[a + b\*x]^2 + 6\*(-Cot[a + b\*x]^2)^(1/4)\*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b\*x]^2])\*Sqrt[c\*Sec[a + b\*x]])/(5\*b\*c^3\*(d\*Csc[a + b\*x])^(3/2))

**Maple [B]** time = 0.196, size = 993, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*csc(b\*x+a))^(7/2)/(c\*sec(b\*x+a))^(5/2),x)

[Out] 1/5/b\*2^(1/2)\*(6\*cos(b\*x+a)^3\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2))\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticE((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1/2\*2^(1/2))-3\*cos(b\*x+a)^3\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1/2\*2^(1/2))+6\*cos(b\*x+a)^2\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticE((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1/2\*2^(1/2))-3\*cos(b\*x+a)^2\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1/2\*2^(1/2))-6\*cos(b\*x+a)\*((-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticE((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1/2\*2^(1/2))+3\*cos(b\*x+a)\*((-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1/2\*2^(1/2))-3\*cos(b\*x+a)^3\*2^(1/2)-6\*(-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))^(1/2)\*((-1+cos(b\*x+a))/sin(b\*x+a))^(1/2)\*EllipticE((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))^(1/2),1

$$\begin{aligned} & /2*2^{(1/2)}+3*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a) \\ & )+\sin(b*x+a))/\sin(b*x+a))^{(1/2)}*((-1+\cos(b*x+a))/\sin(b*x+a))^{(1/2)}*\text{Elliptic} \\ & \text{F}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{(1/2)},1/2*2^{(1/2)}-\cos(b*x+a)^2* \\ & 2^{(1/2)}+3*\cos(b*x+a)*2^{(1/2)})*(d/\sin(b*x+a))^{(7/2)}*\sin(b*x+a)/\cos(b*x+a)^3/ \\ & (c/\cos(b*x+a))^{(5/2)} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc (bx + a))^{\frac{7}{2}}}{(c \sec (bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(7/2)/(c\*sec(b\*x+a))^(5/2),x, algorithm="maxima")

[Out] integrate((d\*csc(b\*x + a))^(7/2)/(c\*sec(b\*x + a))^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d \csc (bx + a)}\sqrt{c \sec (bx + a)}d^3 \csc (bx + a)^3}{c^3 \sec (bx + a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(7/2)/(c\*sec(b\*x+a))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a))\*d^3\*csc(b\*x + a)^3/(c^3\*sec(b\*x + a)^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))\*\*(7/2)/(c\*sec(b\*x+a))\*\*(5/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc(bx + a))^{\frac{7}{2}}}{(c \sec(bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*csc(b\*x+a))^(7/2)/(c\*sec(b\*x+a))^(5/2),x, algorithm="giac")

[Out] integrate((d\*csc(b\*x + a))^(7/2)/(c\*sec(b\*x + a))^(5/2), x)

$$3.273 \quad \int \frac{(d \csc(a+bx))^{5/2}}{(c \sec(a+bx))^{5/2}} dx$$

**Optimal.** Leaf size=329

$$\frac{d^2 \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{\sqrt{2bc^2} \sqrt{c \sec(a+bx)}} - \frac{d^2 \tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{\sqrt{2bc^2} \sqrt{c \sec(a+bx)}}$$

[Out]  $(-2*d*(d*\text{Csc}[a + b*x])^{(3/2)})/(3*b*c*(c*\text{Sec}[a + b*x])^{(3/2)}) + (d^2*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[a + b*x]]]*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Sqrt}[\text{Tan}[a + b*x]])/(\text{Sqrt}[2]*b*c^2*\text{Sqrt}[c*\text{Sec}[a + b*x]]) - (d^2*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[a + b*x]]]*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Sqrt}[\text{Tan}[a + b*x]])/(\text{Sqrt}[2]*b*c^2*\text{Sqrt}[c*\text{Sec}[a + b*x]]) + (d^2*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[a + b*x]] + \text{Tan}[a + b*x]]*\text{Sqrt}[\text{Tan}[a + b*x]])/(2*\text{Sqrt}[2]*b*c^2*\text{Sqrt}[c*\text{Sec}[a + b*x]]) - (d^2*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[a + b*x]] + \text{Tan}[a + b*x]]*\text{Sqrt}[\text{Tan}[a + b*x]])/(2*\text{Sqrt}[2]*b*c^2*\text{Sqrt}[c*\text{Sec}[a + b*x]])$

**Rubi [A]** time = 0.214465, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2623, 2629, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^2 \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{\sqrt{2bc^2} \sqrt{c \sec(a+bx)}} - \frac{d^2 \tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{\sqrt{2bc^2} \sqrt{c \sec(a+bx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*\text{Csc}[a + b*x])^{(5/2)}/(c*\text{Sec}[a + b*x])^{(5/2)}, x]$

[Out]  $(-2*d*(d*\text{Csc}[a + b*x])^{(3/2)})/(3*b*c*(c*\text{Sec}[a + b*x])^{(3/2)}) + (d^2*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[a + b*x]]]*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Sqrt}[\text{Tan}[a + b*x]])/(\text{Sqrt}[2]*b*c^2*\text{Sqrt}[c*\text{Sec}[a + b*x]]) - (d^2*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[a + b*x]]]*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Sqrt}[\text{Tan}[a + b*x]])/(\text{Sqrt}[2]*b*c^2*\text{Sqrt}[c*\text{Sec}[a + b*x]]) + (d^2*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[a + b*x]] + \text{Tan}[a + b*x]]*\text{Sqrt}[\text{Tan}[a + b*x]])/(2*\text{Sqrt}[2]*b*c^2*\text{Sqrt}[c*\text{Sec}[a + b*x]]) - (d^2*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[a + b*x]] + \text{Tan}[a + b*x]]*\text{Sqrt}[\text{Tan}[a + b*x]])/(2*\text{Sqrt}[2]*b*c^2*\text{Sqrt}[c*\text{Sec}[a + b*x]])$

### Rule 2623

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow -\text{Simp}[(a*(a*\text{Csc}[e + f*x])^{(m - 1)}*(b*\text{Sec}[e + f*x])^{(n + 1)})]$

)/(f\*b\*(m - 1)), x] + Dist[(a^2\*(n + 1))/(b^2\*(m - 1)), Int[(a\*Csc[e + f\*x])^(m - 2)\*(b\*Sec[e + f\*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2\*m, 2\*n]

### Rule 2629

Int[(csc[(e\_) + (f\_)\*(x\_)]\*(a\_))^(m\_)\*((b\_)\*sec[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[((a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n)/Tan[e + f\*x]^n, Int[Tan[e + f\*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && EqQ[m + n, 0]

### Rule 3476

Int[((b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 329

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d \csc(a + bx))^{5/2}}{(c \sec(a + bx))^{5/2}} dx &= -\frac{2d(d \csc(a + bx))^{3/2}}{3bc(c \sec(a + bx))^{3/2}} - \frac{d^2 \int \frac{\sqrt{d \csc(a+bx)}}{\sqrt{c \sec(a+bx)}} dx}{c^2} \\
&= -\frac{2d(d \csc(a + bx))^{3/2}}{3bc(c \sec(a + bx))^{3/2}} - \frac{(d^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}) \int \frac{1}{\sqrt{\tan(a+bx)}} dx}{c^2 \sqrt{c \sec(a + bx)}} \\
&= -\frac{2d(d \csc(a + bx))^{3/2}}{3bc(c \sec(a + bx))^{3/2}} - \frac{(d^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}) \text{Subst} \left( \int \frac{1}{\sqrt{x(1+x^2)}} dx, x, \tan(a + bx) \right)}{bc^2 \sqrt{c \sec(a + bx)}} \\
&= -\frac{2d(d \csc(a + bx))^{3/2}}{3bc(c \sec(a + bx))^{3/2}} - \frac{(2d^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}) \text{Subst} \left( \int \frac{1}{1+x^4} dx, x, \sqrt{\tan(a + bx)} \right)}{bc^2 \sqrt{c \sec(a + bx)}} \\
&= -\frac{2d(d \csc(a + bx))^{3/2}}{3bc(c \sec(a + bx))^{3/2}} - \frac{(d^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}) \text{Subst} \left( \int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(a + bx)} \right)}{bc^2 \sqrt{c \sec(a + bx)}} \\
&= -\frac{2d(d \csc(a + bx))^{3/2}}{3bc(c \sec(a + bx))^{3/2}} - \frac{(d^2 \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}) \text{Subst} \left( \int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\tan(a + bx)} \right)}{2bc^2 \sqrt{c \sec(a + bx)}} \\
&= -\frac{2d(d \csc(a + bx))^{3/2}}{3bc(c \sec(a + bx))^{3/2}} + \frac{d^2 \sqrt{d \csc(a + bx)} \log \left( 1 - \sqrt{2} \sqrt{\tan(a + bx)} + \tan(a + bx) \right) \sqrt{\tan(a + bx)}}{2\sqrt{2}bc^2 \sqrt{c \sec(a + bx)}} \\
&= -\frac{2d(d \csc(a + bx))^{3/2}}{3bc(c \sec(a + bx))^{3/2}} + \frac{d^2 \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(a + bx)} \right) \sqrt{d \csc(a + bx)} \sqrt{\tan(a + bx)}}{\sqrt{2}bc^2 \sqrt{c \sec(a + bx)}} - \frac{d^2}{c^2}
\end{aligned}$$

**Mathematica [C]** time = 0.200299, size = 55, normalized size = 0.17

$$\frac{2d(d \csc(a + bx))^{3/2} \left( \text{Hypergeometric2F1} \left( \frac{3}{4}, 1, \frac{7}{4}, -\cot^2(a + bx) \right) - 1 \right)}{3bc(c \sec(a + bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*Csc[a + b\*x])^(5/2)/(c\*Sec[a + b\*x])^(5/2),x]

[Out] (2\*d\*(d\*Csc[a + b\*x])^(3/2)\*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[a + b\*x]^2]))/(3\*b\*c\*(c\*Sec[a + b\*x])^(3/2))

**Maple [C]** time = 0.179, size = 1259, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d*\csc(b*x+a))^{5/2}/(c*\sec(b*x+a))^{5/2},x)$

[Out] 
$$\begin{aligned} & -1/6/b^2^{1/2}*(-3*I*\cos(b*x+a)*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2} \\ & *((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2} \\ & *EllipticPi((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2},1/2+1/2*I,1/2*2^{1/2}) \\ & +3*I*\cos(b*x+a)*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2} \\ & *((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2} \\ & *EllipticPi((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2},1/2-1/2*I,1/2*2^{1/2}) \\ & +3*\cos(b*x+a)*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2} \\ & *((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2} \\ & *EllipticPi((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2},1/2+1/2*I,1/2*2^{1/2}) \\ & +3*\cos(b*x+a)*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2} \\ & *((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2} \\ & *EllipticPi((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2},1/2-1/2*I,1/2*2^{1/2}) \\ & -6*\cos(b*x+a)*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2} \\ & *((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2} \\ & *EllipticF((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2},1/2*2^{1/2}) \\ & -3*I*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2} \\ & *((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2} \\ & *EllipticPi((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2},1/2+1/2*I,1/2*2^{1/2}) \\ & +3*I*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2} \\ & *((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2} \\ & *EllipticPi((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2},1/2-1/2*I,1/2*2^{1/2}) \\ & +3*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2} \\ & *((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2} \\ & *EllipticPi((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2},1/2+1/2*I,1/2*2^{1/2}) \\ & +3*\sin(b*x+a)*EllipticPi((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2},1/2-1/2*I,1/2*2^{1/2}) \\ & *(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2} \\ & *((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2} \\ & -6*\sin(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2} \\ & *((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2} \\ & *EllipticF((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2},1/2*2^{1/2}) \\ & +2*\cos(b*x+a)^2*2^{1/2}*(d/\sin(b*x+a))^{5/2}*\sin(b*x+a)/\cos(b*x+a)^3/(c/\cos(b*x+a))^{5/2} \end{aligned}$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc(bx + a))^{\frac{5}{2}}}{(c \sec(bx + a))^{\frac{5}{2}}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)/(c*sec(b*x + a))^(5/2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc(bx + a))^{\frac{5}{2}}}{(c \sec(bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)/(c*sec(b*x + a))^(5/2), x)
```

$$3.274 \quad \int \frac{(d \csc(a+bx))^{3/2}}{(c \sec(a+bx))^{5/2}} dx$$

**Optimal.** Leaf size=94

$$-\frac{3d^2 E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{bc^2 \sqrt{\sin(2a+2bx)} \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}} - \frac{2d \sqrt{d \csc(a+bx)}}{bc(c \sec(a+bx))^{3/2}}$$

[Out]  $(-2*d*\text{Sqrt}[d*\text{Csc}[a + b*x]])/(b*c*(c*\text{Sec}[a + b*x])^{(3/2)}) - (3*d^2*\text{EllipticE}[a - \text{Pi}/4 + b*x, 2])/(b*c^2*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Sqrt}[c*\text{Sec}[a + b*x]]*\text{Sqrt}[\text{Sin}[2*a + 2*b*x]])$

**Rubi [A]** time = 0.146961, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2623, 2630, 2572, 2639}

$$-\frac{3d^2 E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{bc^2 \sqrt{\sin(2a+2bx)} \sqrt{c \sec(a+bx)} \sqrt{d \csc(a+bx)}} - \frac{2d \sqrt{d \csc(a+bx)}}{bc(c \sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*\text{Csc}[a + b*x])^{(3/2)}/(c*\text{Sec}[a + b*x])^{(5/2)}, x]$

[Out]  $(-2*d*\text{Sqrt}[d*\text{Csc}[a + b*x]])/(b*c*(c*\text{Sec}[a + b*x])^{(3/2)}) - (3*d^2*\text{EllipticE}[a - \text{Pi}/4 + b*x, 2])/(b*c^2*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Sqrt}[c*\text{Sec}[a + b*x]]*\text{Sqrt}[\text{Sin}[2*a + 2*b*x]])$

### Rule 2623

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \text{ :> } -\text{Simp}[(a*(a*\text{Csc}[e + f*x])^{(m-1)}*(b*\text{Sec}[e + f*x])^{(n+1)})/(f*b*(m-1)), x] + \text{Dist}[(a^2*(n+1))/(b^2*(m-1)), \text{Int}[(a*\text{Csc}[e + f*x])^{(m-2)}*(b*\text{Sec}[e + f*x])^{(n+2)}, x], x] \text{ /; } \text{FreeQ}\{a, b, e, f\}, x\} \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegersQ}[2*m, 2*n]$

### Rule 2630

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*((b_.)*\text{sec}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \text{ :> } \text{Dist}[(a*\text{Csc}[e + f*x])^m*(b*\text{Sec}[e + f*x])^n*(a*\text{Sin}[e + f*x])^m*(b*\text{Cos}[e + f*x])^n, \text{Int}[1/((a*\text{Sin}[e + f*x])^m*(b*\text{Cos}[e + f*x])^n), x], x] \text{ /; } \text{FreeQ}\{a, b, e, f, m, n\}, x\} \ \&\& \ \text{IntegerQ}[m - 1/2] \ \&\& \ \text{IntegerQ}[n - 1/2]$

]

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(d \csc(a + bx))^{3/2}}{(c \sec(a + bx))^{5/2}} dx &= -\frac{2d\sqrt{d \csc(a + bx)}}{bc(c \sec(a + bx))^{3/2}} - \frac{(3d^2) \int \frac{1}{\sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}} dx}{c^2} \\ &= -\frac{2d\sqrt{d \csc(a + bx)}}{bc(c \sec(a + bx))^{3/2}} - \frac{(3d^2) \int \sqrt{c \cos(a + bx)}\sqrt{d \sin(a + bx)} dx}{c^2\sqrt{c \cos(a + bx)}\sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}\sqrt{d \sin(a + bx)}} \\ &= -\frac{2d\sqrt{d \csc(a + bx)}}{bc(c \sec(a + bx))^{3/2}} - \frac{(3d^2) \int \sqrt{\sin(2a + 2bx)} dx}{c^2\sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}\sqrt{\sin(2a + 2bx)}} \\ &= -\frac{2d\sqrt{d \csc(a + bx)}}{bc(c \sec(a + bx))^{3/2}} - \frac{3d^2 E\left(a - \frac{\pi}{4} + bx \mid 2\right)}{bc^2\sqrt{d \csc(a + bx)}\sqrt{c \sec(a + bx)}\sqrt{\sin(2a + 2bx)}} \end{aligned}$$

**Mathematica [C]** time = 0.660887, size = 80, normalized size = 0.85

$$\frac{d^3 \sqrt{c \sec(a + bx)} \left( 3 \sqrt{-\cot^2(a + bx)} \operatorname{Hypergeometric2F1} \left( -\frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \csc^2(a + bx) \right) + 2 \cot^2(a + bx) \right)}{bc^3 (d \csc(a + bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(3/2)/(c*Sec[a + b*x])^(5/2), x]
```

```
[Out] -((d^3*(2*Cot[a + b*x]^2 + 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(b*c^3*(d*Csc[a + b*x])^(3/2)))
```

**Maple [B]** time = 0.169, size = 515, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d*\csc(b*x+a))^{3/2}/(c*\sec(b*x+a))^{5/2}, x)$

[Out]  $\frac{1}{2}b^{1/2}*(6*\cos(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*\text{EllipticE}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})-3*\cos(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*\text{EllipticF}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})+6*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*\text{EllipticE}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})-3*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*\text{EllipticF}((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})+\cos(b*x+a)^2*2^{1/2}-3*\cos(b*x+a)*2^{1/2})*(d/\sin(b*x+a))^{3/2}*\sin(b*x+a)/\cos(b*x+a)^3/(c/\cos(b*x+a))^{5/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc(bx + a))^{\frac{3}{2}}}{(c \sec(bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((d*\csc(b*x+a))^{3/2}/(c*\sec(b*x+a))^{5/2}, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}((d*\csc(b*x + a))^{3/2}/(c*\sec(b*x + a))^{5/2}, x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d \csc(bx + a)}\sqrt{c \sec(bx + a)}d \csc(bx + a)}{c^3 \sec(bx + a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*d*csc(b*x + a)/(c^3*sec(b*x + a)^3), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d \csc(bx + a))^{\frac{3}{2}}}{(c \sec(bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)/(c*sec(b*x + a))^(5/2), x)
```

$$3.275 \quad \int \frac{\sqrt{d} \csc(a+bx)}{(c \sec(a+bx))^{5/2}} dx$$

**Optimal.** Leaf size=322

$$\frac{3 \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{\tan(a+bx)} \sqrt{d} \csc(a+bx)}{4\sqrt{2}bc^2\sqrt{c} \sec(a+bx)} + \frac{3 \tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{\tan(a+bx)} \sqrt{d} \csc(a+bx)}{4\sqrt{2}bc^2\sqrt{c} \sec(a+bx)}$$

```
[Out] d/(2*b*c*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)) - (3*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]) - (3*Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b*x]])/(8*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]) + (3*Sqrt[d*Csc[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b*x]])/(8*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]])
```

**Rubi [A]** time = 0.203437, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2628, 2629, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{3 \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{\tan(a+bx)} \sqrt{d} \csc(a+bx)}{4\sqrt{2}bc^2\sqrt{c} \sec(a+bx)} + \frac{3 \tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+bx)} + 1\right) \sqrt{\tan(a+bx)} \sqrt{d} \csc(a+bx)}{4\sqrt{2}bc^2\sqrt{c} \sec(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[a + b*x]]/(c*Sec[a + b*x])^(5/2), x]
```

```
[Out] d/(2*b*c*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)) - (3*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]) - (3*Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b*x]])/(8*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]) + (3*Sqrt[d*Csc[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b*x]])/(8*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]])
```

**Rule 2628**

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1
```

))/ (b\*f\*(m + n)), x] + Dist[(n + 1)/(b^2\*(m + n)), Int[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && NeQ[m + n, 0] && IntegersQ[2\*m, 2\*n]

### Rule 2629

Int[(csc[(e\_) + (f\_)\*(x\_)]\*(a\_))^(m\_)\*((b\_)\*sec[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :=> Dist[((a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n)/Tan[e + f\*x]^n, Int[Tan[e + f\*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && EqQ[m + n, 0]

### Rule 3476

Int[((b\_)\*tan[(c\_) + (d\_)\*(x\_)])^(n\_), x\_Symbol] :=> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

### Rule 329

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :=> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n))/c^n)^p, x], x, (c\*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^4)^(-1), x\_Symbol] :=> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] :=> With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] :=> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps



$$\begin{aligned}
\int \frac{\sqrt{d \csc(a+bx)}}{(c \sec(a+bx))^{5/2}} dx &= \frac{d}{2bc\sqrt{d} \csc(a+bx)(c \sec(a+bx))^{3/2}} + \frac{3 \int \frac{\sqrt{d \csc(a+bx)}}{\sqrt{c \sec(a+bx)}} dx}{4c^2} \\
&= \frac{d}{2bc\sqrt{d} \csc(a+bx)(c \sec(a+bx))^{3/2}} + \frac{(3\sqrt{d} \csc(a+bx)\sqrt{\tan(a+bx)}) \int \frac{1}{\sqrt{\tan(a+bx)}} dx}{4c^2\sqrt{c \sec(a+bx)}} \\
&= \frac{d}{2bc\sqrt{d} \csc(a+bx)(c \sec(a+bx))^{3/2}} + \frac{(3\sqrt{d} \csc(a+bx)\sqrt{\tan(a+bx)}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{x(1+x^2)}} dx, \sqrt{\tan(a+bx)}\right)}{4bc^2\sqrt{c \sec(a+bx)}} \\
&= \frac{d}{2bc\sqrt{d} \csc(a+bx)(c \sec(a+bx))^{3/2}} + \frac{(3\sqrt{d} \csc(a+bx)\sqrt{\tan(a+bx)}) \operatorname{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\tan(a+bx)}\right)}{2bc^2\sqrt{c \sec(a+bx)}} \\
&= \frac{d}{2bc\sqrt{d} \csc(a+bx)(c \sec(a+bx))^{3/2}} + \frac{(3\sqrt{d} \csc(a+bx)\sqrt{\tan(a+bx)}) \operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(a+bx)}\right)}{4bc^2\sqrt{c \sec(a+bx)}} \\
&= \frac{d}{2bc\sqrt{d} \csc(a+bx)(c \sec(a+bx))^{3/2}} + \frac{(3\sqrt{d} \csc(a+bx)\sqrt{\tan(a+bx)}) \operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, \frac{1-\sqrt{2}\sqrt{\tan(a+bx)}}{2}, \sqrt{\tan(a+bx)}\right)}{8bc^2\sqrt{c \sec(a+bx)}} \\
&= \frac{d}{2bc\sqrt{d} \csc(a+bx)(c \sec(a+bx))^{3/2}} - \frac{3\sqrt{d} \csc(a+bx) \log\left(1 - \sqrt{2}\sqrt{\tan(a+bx)} + \tan(a+bx)\right)}{8\sqrt{2}bc^2\sqrt{c \sec(a+bx)}} \\
&= \frac{d}{2bc\sqrt{d} \csc(a+bx)(c \sec(a+bx))^{3/2}} - \frac{3 \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+bx)}\right) \sqrt{d} \csc(a+bx) \sqrt{\tan(a+bx)}}{4\sqrt{2}bc^2\sqrt{c \sec(a+bx)}}
\end{aligned}$$

**Mathematica [C]** time = 0.220849, size = 70, normalized size = 0.22

$$\frac{\cot(a+bx)\sqrt{d \csc(a+bx)} \left( 2 \operatorname{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, -\cot^2(a+bx)\right) + \cos(2(a+bx)) - 1 \right)}{4bc^2\sqrt{c \sec(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*Csc[a + b\*x]]/(c\*Sec[a + b\*x])^(5/2), x]

[Out] -(Cot[a + b\*x]\*Sqrt[d\*Csc[a + b\*x]]\*(-1 + Cos[2\*(a + b\*x)] + 2\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[a + b\*x]^2]))/(4\*b\*c^2\*Sqrt[c\*Sec[a + b\*x]])

**Maple [C]** time = 0.175, size = 668, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x)`

[Out]  $\frac{1}{8}b^{-2} \left( \frac{1}{2} \right) * (3I \sin(bx+a) * (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} * \text{EllipticPi}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2+1/2*I, 1/2*2^{1/2}) - 3I \sin(bx+a) * (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} * \text{EllipticPi}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2-1/2*I, 1/2*2^{1/2}) - 3 \sin(bx+a) * (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} * \text{EllipticPi}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2+1/2*I, 1/2*2^{1/2}) + 6 \sin(bx+a) * (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} * \text{EllipticF}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2*2^{1/2}) - 3 \sin(bx+a) * \text{EllipticPi}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2-1/2*I, 1/2*2^{1/2}) * (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} * ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} + 2 \cos(bx+a)^3 * 2^{1/2} - 2 \cos(bx+a)^2 * 2^{1/2} * (d/\sin(bx+a))^{1/2} * \sin(bx+a) / (-1+\cos(bx+a)) / \cos(bx+a)^3 / (c/\cos(bx+a))^{5/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d} \csc(bx+a)}{(c \sec(bx+a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(d*csc(b*x + a))/(c*sec(b*x + a))^(5/2), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \csc(bx + a)}}{(c \sec(bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))/(c*sec(b*x + a))^(5/2), x)
```

$$3.276 \quad \int \frac{1}{\sqrt{d} \csc(a+bx)(c \sec(a+bx))^{5/2}} dx$$

**Optimal.** Leaf size=95

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bc^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}} + \frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

[Out] d/(3\*b\*c\*(d\*Csc[a + b\*x])^(3/2)\*(c\*Sec[a + b\*x])^(3/2)) + EllipticE[a - Pi/4 + b\*x, 2]/(2\*b\*c^2\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])

**Rubi [A]** time = 0.141849, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$ , Rules used = {2628, 2630, 2572, 2639}

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bc^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}} + \frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d\*Csc[a + b\*x]]\*(c\*Sec[a + b\*x])^(5/2)),x]

[Out] d/(3\*b\*c\*(d\*Csc[a + b\*x])^(3/2)\*(c\*Sec[a + b\*x])^(3/2)) + EllipticE[a - Pi/4 + b\*x, 2]/(2\*b\*c^2\*Sqrt[d\*Csc[a + b\*x]]\*Sqrt[c\*Sec[a + b\*x]]\*Sqrt[Sin[2\*a + 2\*b\*x]])

### Rule 2628

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> -Simp[(a\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1))/(b\*f\*(m + n)), x] + Dist[(n + 1)/(b^2\*(m + n)), Int[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && NeQ[m + n, 0] && IntegersQ[2\*m, 2\*n]

### Rule 2630

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Dist[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n\*(a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]

]

Rule 2572

Int[Sqrt[cos[(e\_.) + (f\_.)\*(x\_)]\*(b\_.)]\*Sqrt[(a\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] :> Dist[(Sqrt[a\*Sin[e + f\*x]]\*Sqrt[b\*Cos[e + f\*x]])/Sqrt[Sin[2\*e + 2\*f\*x]], Int[Sqrt[Sin[2\*e + 2\*f\*x]], x], x] /; FreeQ[{a, b, e, f}, x]

Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] :> Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{d \csc(a+bx)}(c \sec(a+bx))^{5/2}} dx &= \frac{d}{3bc(d \csc(a+bx))^{3/2}(c \sec(a+bx))^{3/2}} + \frac{\int \frac{1}{\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}} dx}{2c^2} \\ &= \frac{d}{3bc(d \csc(a+bx))^{3/2}(c \sec(a+bx))^{3/2}} + \frac{\int \sqrt{c \cos(a+bx)}\sqrt{d \sin(a+bx)} dx}{2c^2\sqrt{c \cos(a+bx)}\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}} \\ &= \frac{d}{3bc(d \csc(a+bx))^{3/2}(c \sec(a+bx))^{3/2}} + \frac{\int \sqrt{\sin(2a+2bx)} dx}{2c^2\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}\sqrt{\sin(2a+2bx)}} \\ &= \frac{d}{3bc(d \csc(a+bx))^{3/2}(c \sec(a+bx))^{3/2}} + \frac{E\left(a - \frac{\pi}{4} + bx \mid 2\right)}{2bc^2\sqrt{d \csc(a+bx)}\sqrt{c \sec(a+bx)}\sqrt{\sin(2a+2bx)}} \end{aligned}$$

**Mathematica [C]** time = 0.363158, size = 79, normalized size = 0.83

$$\frac{d\sqrt{c \sec(a+bx)} \left( 3\sqrt[4]{-\cot^2(a+bx)} \text{Hypergeometric2F1} \left( -\frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \csc^2(a+bx) \right) + \cos(2(a+bx)) + 1 \right)}{6bc^3(d \csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d\*Csc[a + b\*x]]\*(c\*Sec[a + b\*x])^(5/2)),x]

[Out] (d\*(1 + Cos[2\*(a + b\*x)] + 3\*(-Cot[a + b\*x]^2)^(1/4)\*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b\*x]^2])\*Sqrt[c\*Sec[a + b\*x]])/(6\*b\*c^3\*(d\*Csc[a + b\*x])^(3/2))

**Maple [B]** time = 0.192, size = 530, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/(d*\csc(b*x+a))^{1/2}/(c*\sec(b*x+a))^{5/2}, x)$

[Out] 
$$-1/12/b^2^{1/2}*(2*2^{1/2}*\cos(b*x+a)^4+6*\cos(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*EllipticE((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})-3*\cos(b*x+a)*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*EllipticF((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})+6*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*EllipticE((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})-3*(-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a)+\sin(b*x+a))/\sin(b*x+a))^{1/2}*((-1+\cos(b*x+a))/\sin(b*x+a))^{1/2}*EllipticF((-(-1+\cos(b*x+a)-\sin(b*x+a))/\sin(b*x+a))^{1/2}, 1/2*2^{1/2})+\cos(b*x+a)^2*2^{1/2}-3*\cos(b*x+a)*2^{1/2})/\cos(b*x+a)^3/\sin(b*x+a)/(d/\sin(b*x+a))^{1/2}/(c/\cos(b*x+a))^{5/2}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d} \csc(bx+a) (c \sec(bx+a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(1/(d*\csc(b*x+a))^{1/2}/(c*\sec(b*x+a))^{5/2}, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}(1/(\text{sqrt}(d*\csc(b*x+a))*(c*\sec(b*x+a))^{5/2}), x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d} \csc(bx+a) \sqrt{c \sec(bx+a)}}{c^3 d \csc(bx+a) \sec(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c^3*d*csc(b*x + a)*sec(b*x + a)^3), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d} \csc(bx + a) (c \sec(bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(5/2)), x)
```

$$3.277 \quad \int \frac{1}{(d \csc(a+bx))^{3/2} (c \sec(a+bx))^{5/2}} dx$$

**Optimal.** Leaf size=371

$$\frac{3 \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(a+bx)} \right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{32 \sqrt{2} b c^2 d^2 \sqrt{c \sec(a+bx)}} + \frac{3 \tan^{-1} \left( \sqrt{2} \sqrt{\tan(a+bx)} + 1 \right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{32 \sqrt{2} b c^2 d^2 \sqrt{c \sec(a+bx)}}$$

[Out]  $-c/(4*b*d*\text{Sqrt}[d*\text{Csc}[a + b*x]]*(c*\text{Sec}[a + b*x])^{(7/2)}) + 1/(16*b*c*d*\text{Sqrt}[d*\text{Csc}[a + b*x]]*(c*\text{Sec}[a + b*x])^{(3/2)}) - (3*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[a + b*x]])*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Sqrt}[\text{Tan}[a + b*x]])/(32*\text{Sqrt}[2]*b*c^2*d^2*\text{Sqrt}[c*\text{Sec}[a + b*x]]) + (3*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[a + b*x]])*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Sqrt}[\text{Tan}[a + b*x]])/(32*\text{Sqrt}[2]*b*c^2*d^2*\text{Sqrt}[c*\text{Sec}[a + b*x]]) - (3*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[a + b*x]] + \text{Tan}[a + b*x]]*\text{Sqrt}[\text{Tan}[a + b*x]])/(64*\text{Sqrt}[2]*b*c^2*d^2*\text{Sqrt}[c*\text{Sec}[a + b*x]]) + (3*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[a + b*x]] + \text{Tan}[a + b*x]]*\text{Sqrt}[\text{Tan}[a + b*x]])/(64*\text{Sqrt}[2]*b*c^2*d^2*\text{Sqrt}[c*\text{Sec}[a + b*x]])$

**Rubi [A]** time = 0.290564, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.44$ , Rules used = {2627, 2628, 2629, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{3 \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(a+bx)} \right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{32 \sqrt{2} b c^2 d^2 \sqrt{c \sec(a+bx)}} + \frac{3 \tan^{-1} \left( \sqrt{2} \sqrt{\tan(a+bx)} + 1 \right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{32 \sqrt{2} b c^2 d^2 \sqrt{c \sec(a+bx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/((d*\text{Csc}[a + b*x])^{(3/2)}*(c*\text{Sec}[a + b*x])^{(5/2)}), x]$

[Out]  $-c/(4*b*d*\text{Sqrt}[d*\text{Csc}[a + b*x]]*(c*\text{Sec}[a + b*x])^{(7/2)}) + 1/(16*b*c*d*\text{Sqrt}[d*\text{Csc}[a + b*x]]*(c*\text{Sec}[a + b*x])^{(3/2)}) - (3*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[a + b*x]])*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Sqrt}[\text{Tan}[a + b*x]])/(32*\text{Sqrt}[2]*b*c^2*d^2*\text{Sqrt}[c*\text{Sec}[a + b*x]]) + (3*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[a + b*x]])*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Sqrt}[\text{Tan}[a + b*x]])/(32*\text{Sqrt}[2]*b*c^2*d^2*\text{Sqrt}[c*\text{Sec}[a + b*x]]) - (3*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[a + b*x]] + \text{Tan}[a + b*x]]*\text{Sqrt}[\text{Tan}[a + b*x]])/(64*\text{Sqrt}[2]*b*c^2*d^2*\text{Sqrt}[c*\text{Sec}[a + b*x]]) + (3*\text{Sqrt}[d*\text{Csc}[a + b*x]]*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Tan}[a + b*x]] + \text{Tan}[a + b*x]]*\text{Sqrt}[\text{Tan}[a + b*x]])/(64*\text{Sqrt}[2]*b*c^2*d^2*\text{Sqrt}[c*\text{Sec}[a + b*x]])$

Rule 2627



```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

### Rule 2628

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1))/(b*f*(m + n)), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

### Rule 2629

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[((a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n)/Tan[e + f*x]^n, Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && EqQ[m + n, 0]
```

### Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{5/2}} dx &= -\frac{c}{4bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{7/2}} + \frac{\int \frac{\sqrt{d} \csc(a+bx)}{(c \sec(a+bx))^{5/2}} dx}{8d^2} \\
&= -\frac{c}{4bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{7/2}} + \frac{1}{16bcd\sqrt{d} \csc(a + bx)(c \sec(a + bx))} \\
&= -\frac{c}{4bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{7/2}} + \frac{1}{16bcd\sqrt{d} \csc(a + bx)(c \sec(a + bx))} \\
&= -\frac{c}{4bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{7/2}} + \frac{1}{16bcd\sqrt{d} \csc(a + bx)(c \sec(a + bx))} \\
&= -\frac{c}{4bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{7/2}} + \frac{1}{16bcd\sqrt{d} \csc(a + bx)(c \sec(a + bx))} \\
&= -\frac{c}{4bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{7/2}} + \frac{1}{16bcd\sqrt{d} \csc(a + bx)(c \sec(a + bx))} \\
&= -\frac{c}{4bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{7/2}} + \frac{1}{16bcd\sqrt{d} \csc(a + bx)(c \sec(a + bx))} \\
&= -\frac{c}{4bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{7/2}} + \frac{1}{16bcd\sqrt{d} \csc(a + bx)(c \sec(a + bx))} \\
&= -\frac{c}{4bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{7/2}} + \frac{1}{16bcd\sqrt{d} \csc(a + bx)(c \sec(a + bx))} \\
&= -\frac{c}{4bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{7/2}} + \frac{1}{16bcd\sqrt{d} \csc(a + bx)(c \sec(a + bx))} \\
&= -\frac{c}{4bd\sqrt{d} \csc(a + bx)(c \sec(a + bx))^{7/2}} + \frac{1}{16bcd\sqrt{d} \csc(a + bx)(c \sec(a + bx))}
\end{aligned}$$

**Mathematica [C]** time = 0.378894, size = 81, normalized size = 0.22

$$-\frac{\csc^3(a + bx) \left( 2\text{Hypergeometric2F1} \left( \frac{3}{4}, 1, \frac{7}{4}, -\cot^2(a + bx) \right) + \cos(2(a + bx)) - \cos(4(a + bx)) \right)}{32bc(c \sec(a + bx))^{3/2} (d \csc(a + bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d\*Csc[a + b\*x])^(3/2)\*(c\*Sec[a + b\*x])^(5/2)),x]

[Out] -(Csc[a + b\*x]^3\*(Cos[2\*(a + b\*x)] - Cos[4\*(a + b\*x)] + 2\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[a + b\*x]^2]))/(32\*b\*c\*(d\*Csc[a + b\*x])^(3/2)\*(c\*Sec[a + b\*x])^(3/2))

---

**Maple [C]** time = 0.183, size = 696, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x)`

[Out] 
$$\frac{1}{64} b^{-1/2} (3I \sin(bx+a) (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} \operatorname{EllipticPi}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2+1/2I, 1/2*2^{1/2}) - 3I \sin(bx+a) (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} \operatorname{EllipticPi}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2-1/2I, 1/2*2^{1/2}) - 8*2^{1/2} \cos(bx+a)^5 - 3\sin(bx+a) (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} \operatorname{EllipticPi}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2+1/2I, 1/2*2^{1/2}) + 6\sin(bx+a) (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} \operatorname{EllipticF}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2*2^{1/2}) - 3\sin(bx+a) \operatorname{EllipticPi}((-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2}, 1/2-1/2I, 1/2*2^{1/2}) * (-(-1+\cos(bx+a)-\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a)+\sin(bx+a))/\sin(bx+a))^{1/2} ((-1+\cos(bx+a))/\sin(bx+a))^{1/2} + 8*2^{1/2} \cos(bx+a)^4 + 2\cos(bx+a)^3 * 2^{1/2} - 2\cos(bx+a)^2 * 2^{1/2}) / ((-1+\cos(bx+a))/\cos(bx+a))^3 / \sin(bx+a) / (d/\sin(bx+a))^{3/2} / (c/\cos(bx+a))^{5/2}$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d \csc(bx+a))^{\frac{3}{2}} (c \sec(bx+a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")`

[Out] `integrate(1/((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(5/2)), x)`

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(5/2),x)
```

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d \csc(bx + a))^{\frac{3}{2}} (c \sec(bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(5/2)), x)
```

$$3.278 \quad \int \frac{1}{(d \csc(a+bx))^{5/2} (c \sec(a+bx))^{5/2}} dx$$

**Optimal.** Leaf size=135

$$\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{20bc^2d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}} - \frac{c}{5bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{3/2}} + \frac{1}{10bcd(c\sec(a+bx))^{3/2}}$$

[Out]  $-c/(5*b*d*(d*Csc[a + b*x])^{(3/2)}*(c*Sec[a + b*x])^{(7/2)}) + 1/(10*b*c*d*(d*Csc[a + b*x])^{(3/2)}*(c*Sec[a + b*x])^{(3/2)}) + (3*EllipticE[a - Pi/4 + b*x, 2])/ (20*b*c^2*d^2*sqrt[d*Csc[a + b*x]]*sqrt[c*Sec[a + b*x]]*sqrt[Sin[2*a + 2*b*x]])$

**Rubi [A]** time = 0.205576, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2627, 2628, 2630, 2572, 2639}

$$\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{20bc^2d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}} - \frac{c}{5bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{3/2}} + \frac{1}{10bcd(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((d\*Csc[a + b\*x])^(5/2)\*(c\*Sec[a + b\*x])^(5/2)),x]

[Out]  $-c/(5*b*d*(d*Csc[a + b*x])^{(3/2)}*(c*Sec[a + b*x])^{(7/2)}) + 1/(10*b*c*d*(d*Csc[a + b*x])^{(3/2)}*(c*Sec[a + b*x])^{(3/2)}) + (3*EllipticE[a - Pi/4 + b*x, 2])/ (20*b*c^2*d^2*sqrt[d*Csc[a + b*x]]*sqrt[c*Sec[a + b*x]]*sqrt[Sin[2*a + 2*b*x]])$

### Rule 2627

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Simp[(b\*(a\*Csc[e + f\*x])^(m + 1)\*(b\*Sec[e + f\*x])^(n - 1))/(a\*f\*(m + n)), x] + Dist[(m + 1)/(a^2\*(m + n)), Int[(a\*Csc[e + f\*x])^(m + 2)\*(b\*Sec[e + f\*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] && NeQ[m + n, 0] && IntegersQ[2\*m, 2\*n]

### Rule 2628

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> -Simp[(a\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1))/(b\*f\*(m + n)), x] + Dist[(n + 1)/(b^2\*(m + n)), Int[(a\*Csc[e + f\*x])^m\*(

$b*\text{Sec}[e + f*x]^{(n + 2)}, x], x] /;$  FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]  
&& NeQ[m + n, 0] && IntegersQ[2\*m, 2\*n]

### Rule 2630

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^(n\_)), x\_Symbol] := Dist[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n\*(a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2]

### Rule 2572

Int[Sqrt[cos[(e\_.) + (f\_.)\*(x\_)]\*(b\_.)]\*Sqrt[(a\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]], x\_Symbol] := Dist[(Sqrt[a\*Sin[e + f\*x]]\*Sqrt[b\*Cos[e + f\*x]])/Sqrt[Sin[2\*e + 2\*f\*x]], Int[Sqrt[Sin[2\*e + 2\*f\*x]], x], x] /; FreeQ[{a, b, e, f}, x]

### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned} \int \frac{1}{(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{5/2}} dx &= -\frac{c}{5bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{7/2}} + \frac{3 \int \frac{1}{\sqrt{d \csc(a + bx)} (c \sec(a + bx))^{5/2}} dx}{10d^2} \\ &= -\frac{c}{5bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{7/2}} + \frac{1}{10bcd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{7/2}} \\ &= -\frac{c}{5bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{7/2}} + \frac{1}{10bcd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{7/2}} \\ &= -\frac{c}{5bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{7/2}} + \frac{1}{10bcd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{7/2}} \\ &= -\frac{c}{5bd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{7/2}} + \frac{1}{10bcd(d \csc(a + bx))^{3/2} (c \sec(a + bx))^{7/2}} \end{aligned}$$

**Mathematica [C]** time = 0.657982, size = 90, normalized size = 0.67

$$\frac{\sqrt{c \sec(a + bx)} \left( 3 \sqrt[4]{-\cot^2(a + bx)} \text{Hypergeometric2F1} \left( -\frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \csc^2(a + bx) \right) - 2 \cos^2(a + bx) \cos(2(a + bx)) \right)}{20bc^3d(d \csc(a + bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(5/2)),x]
```

```
[Out] ((-2*Cos[a + b*x]^2*Cos[2*(a + b*x)] + 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(20*b*c^3*d*(d*Csc[a + b*x])^(3/2))
```

**Maple [B]** time = 0.165, size = 544, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x)
```

```
[Out] 1/40/b*2^(1/2)*(4*2^(1/2)*cos(b*x+a)^6+3*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-6*cos(b*x+a)*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-6*2^(1/2)*cos(b*x+a)^4+3*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-6*(-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(-1+cos(b*x+a)-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)^2*2^(1/2)+3*cos(b*x+a)*2^(1/2)/cos(b*x+a)^3/sin(b*x+a)^3/(d/sin(b*x+a))^(5/2)/(c/cos(b*x+a))^(5/2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d \csc(bx + a))^{\frac{5}{2}} (c \sec(bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```



[Out] integrate(1/((d\*csc(b\*x + a))^(5/2)\*(c\*sec(b\*x + a))^(5/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d} \csc(bx + a) \sqrt{c \sec(bx + a)}}{c^3 d^3 \csc(bx + a)^3 \sec(bx + a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*csc(b\*x+a))^(5/2)/(c\*sec(b\*x+a))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(d\*csc(b\*x + a))\*sqrt(c\*sec(b\*x + a))/(c^3\*d^3\*csc(b\*x + a)^3\*sec(b\*x + a)^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*csc(b\*x+a))\*\*(5/2)/(c\*sec(b\*x+a))\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d \csc(bx + a))^{\frac{5}{2}} (c \sec(bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*csc(b\*x+a))^(5/2)/(c\*sec(b\*x+a))^(5/2),x, algorithm="giac")

[Out] integrate(1/((d\*csc(b\*x + a))^(5/2)\*(c\*sec(b\*x + a))^(5/2)), x)

$$3.279 \quad \int \frac{1}{(d \csc(a+bx))^{7/2} (c \sec(a+bx))^{5/2}} dx$$

**Optimal.** Leaf size=406

$$\frac{5 \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(a+bx)} \right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{128 \sqrt{2} b c^2 d^4 \sqrt{c \sec(a+bx)}} + \frac{5 \tan^{-1} \left( \sqrt{2} \sqrt{\tan(a+bx)} + 1 \right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{128 \sqrt{2} b c^2 d^4 \sqrt{c \sec(a+bx)}}$$

```
[Out] -c/(6*b*d*(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(7/2)) - (5*c)/(48*b*d^3*
Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(7/2)) + 5/(192*b*c*d^3*Sqrt[d*Csc[a
+ b*x]]*(c*Sec[a + b*x])^(3/2)) - (5*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]
*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(128*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec
[a + b*x]]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]
]*Sqrt[Tan[a + b*x]])/(128*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec[a + b*x]]) - (5*Sqr
t[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[T
an[a + b*x]])/(256*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec[a + b*x]]) + (5*Sqrt[d*Csc[
a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b
*x]])/(256*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec[a + b*x]])
```

---

**Rubi [A]** time = 0.389599, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.44$ , Rules used = {2627, 2628, 2629, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{5 \tan^{-1} \left( 1 - \sqrt{2} \sqrt{\tan(a+bx)} \right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{128 \sqrt{2} b c^2 d^4 \sqrt{c \sec(a+bx)}} + \frac{5 \tan^{-1} \left( \sqrt{2} \sqrt{\tan(a+bx)} + 1 \right) \sqrt{\tan(a+bx)} \sqrt{d \csc(a+bx)}}{128 \sqrt{2} b c^2 d^4 \sqrt{c \sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(5/2)),x]
```

```
[Out] -c/(6*b*d*(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(7/2)) - (5*c)/(48*b*d^3*
Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(7/2)) + 5/(192*b*c*d^3*Sqrt[d*Csc[a
+ b*x]]*(c*Sec[a + b*x])^(3/2)) - (5*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]
*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(128*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec
[a + b*x]]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]
]*Sqrt[Tan[a + b*x]])/(128*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec[a + b*x]]) - (5*Sqr
t[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[T
an[a + b*x]])/(256*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec[a + b*x]]) + (5*Sqrt[d*Csc[
a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b
*x]])/(256*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec[a + b*x]])
```

Rule 2627

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Simp[(b\*(a\*Csc[e + f\*x])^(m + 1)\*(b\*Sec[e + f\*x])^(n - 1))/(a\*f\*(m + n)), x] + Dist[(m + 1)/(a^2\*(m + n)), Int[(a\*Csc[e + f\*x])^(m + 2)\*(b\*Sec[e + f\*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] && NeQ[m + n, 0] && IntegersQ[2\*m, 2\*n]

Rule 2628

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := -Simp[(a\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1))/(b\*f\*(m + n)), x] + Dist[(n + 1)/(b^2\*(m + n)), Int[(a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && NeQ[m + n, 0] && IntegersQ[2\*m, 2\*n]

Rule 2629

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[((a\*Csc[e + f\*x])^m\*(b\*Sec[e + f\*x])^n)/Tan[e + f\*x]^n, Int[Tan[e + f\*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && EqQ[m + n, 0]

Rule 3476

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 329

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 211

Int[((a\_.) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(d \csc(a + bx))^{7/2} (c \sec(a + bx))^{5/2}} dx &= -\frac{c}{6bd(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{7/2}} + \frac{5 \int \frac{1}{(d \csc(a+bx))^{3/2} (c \sec(a+bx))^{5/2}} dx}{12d^2} \\
&= -\frac{c}{6bd(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{7/2}} - \frac{5c}{48bd^3 \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{7/2}} \\
&= -\frac{c}{6bd(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{7/2}} - \frac{5c}{48bd^3 \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{7/2}} \\
&= -\frac{c}{6bd(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{7/2}} - \frac{5c}{48bd^3 \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{7/2}} \\
&= -\frac{c}{6bd(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{7/2}} - \frac{5c}{48bd^3 \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{7/2}} \\
&= -\frac{c}{6bd(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{7/2}} - \frac{5c}{48bd^3 \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{7/2}} \\
&= -\frac{c}{6bd(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{7/2}} - \frac{5c}{48bd^3 \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{7/2}} \\
&= -\frac{c}{6bd(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{7/2}} - \frac{5c}{48bd^3 \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{7/2}} \\
&= -\frac{c}{6bd(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{7/2}} - \frac{5c}{48bd^3 \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{7/2}} \\
&= -\frac{c}{6bd(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{7/2}} - \frac{5c}{48bd^3 \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{7/2}} \\
&= -\frac{c}{6bd(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{7/2}} - \frac{5c}{48bd^3 \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{7/2}} \\
&= -\frac{c}{6bd(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{7/2}} - \frac{5c}{48bd^3 \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{7/2}} \\
&= -\frac{c}{6bd(d \csc(a + bx))^{5/2} (c \sec(a + bx))^{7/2}} - \frac{5c}{48bd^3 \sqrt{d \csc(a + bx)} (c \sec(a + bx))^{7/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.411069, size = 95, normalized size = 0.23

$$\frac{\cot(a + bx) \sqrt{d \csc(a + bx)} \left( 10 \operatorname{Hypergeometric2F1} \left( \frac{3}{4}, 1, \frac{7}{4}, -\cot^2(a + bx) \right) + 3 \cos(2(a + bx)) - 9 \cos(4(a + bx)) + 2 \cos(6(a + bx)) \right)}{384bc^2d^4 \sqrt{c \sec(a + bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d\*Csc[a + b\*x])^(7/2)\*(c\*Sec[a + b\*x])^(5/2)),x]

[Out] -(Cot[a + b\*x]\*Sqrt[d\*Csc[a + b\*x]]\*(4 + 3\*Cos[2\*(a + b\*x)] - 9\*Cos[4\*(a + b\*x)] + 2\*Cos[6\*(a + b\*x)] + 10\*Hypergeometric2F1[3/4, 1, 7/4, -Cot[a + b\*x]

]<sup>2</sup>))/((384\*b\*c<sup>2</sup>\*d<sup>4</sup>\*Sqrt[c\*Sec[a + b\*x]]))

**Maple [C]** time = 0.221, size = 722, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*csc(b\*x+a))<sup>(7/2)</sup>/(c\*sec(b\*x+a))<sup>(5/2)</sup>,x)

[Out] 1/768/b\*2<sup>(1/2)</sup>\*(64\*2<sup>(1/2)</sup>\*cos(b\*x+a)<sup>7</sup>-64\*2<sup>(1/2)</sup>\*cos(b\*x+a)<sup>6</sup>-104\*2<sup>(1/2)</sup>\*cos(b\*x+a)<sup>5</sup>+15\*I\*((-1+cos(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>\*((-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>\*EllipticPi((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>,1/2+1/2\*I,1/2\*2<sup>(1/2)</sup>)\*sin(b\*x+a)-15\*I\*((-1+cos(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>\*((-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>\*EllipticPi((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>,1/2-1/2\*I,1/2\*2<sup>(1/2)</sup>)\*sin(b\*x+a)+104\*2<sup>(1/2)</sup>\*cos(b\*x+a)<sup>4</sup>-15\*sin(b\*x+a)\*((-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>\*((-1+cos(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>\*EllipticPi((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>,1/2+1/2\*I,1/2\*2<sup>(1/2)</sup>)-15\*sin(b\*x+a)\*EllipticPi((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>,1/2-1/2\*I,1/2\*2<sup>(1/2)</sup>)\*((-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>\*((-1+cos(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>+30\*sin(b\*x+a)\*((-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>\*((-1+cos(b\*x+a)+sin(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>\*((-1+cos(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>\*EllipticF((-(-1+cos(b\*x+a)-sin(b\*x+a))/sin(b\*x+a))<sup>(1/2)</sup>,1/2\*2<sup>(1/2)</sup>)+10\*cos(b\*x+a)<sup>3</sup>\*2<sup>(1/2)</sup>-10\*cos(b\*x+a)<sup>2</sup>\*2<sup>(1/2)</sup>)/(-1+cos(b\*x+a))/cos(b\*x+a)<sup>3</sup>/sin(b\*x+a)<sup>3</sup>/(d/sin(b\*x+a))<sup>(7/2)</sup>/(c/cos(b\*x+a))<sup>(5/2)</sup>

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d \csc (bx + a))^{\frac{7}{2}} (c \sec (bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*csc(b\*x+a))<sup>(7/2)</sup>/(c\*sec(b\*x+a))<sup>(5/2)</sup>,x, algorithm="maxima")

[Out] integrate(1/((d\*csc(b\*x + a))^(7/2)\*(c\*sec(b\*x + a))^(5/2)), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*csc(b\*x+a))^(7/2)/(c\*sec(b\*x+a))^(5/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*csc(b\*x+a))\*\*(7/2)/(c\*sec(b\*x+a))\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d \csc(bx + a))^{\frac{7}{2}} (c \sec(bx + a))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*csc(b\*x+a))^(7/2)/(c\*sec(b\*x+a))^(5/2),x, algorithm="giac")

[Out] integrate(1/((d\*csc(b\*x + a))^(7/2)\*(c\*sec(b\*x + a))^(5/2)), x)

### 3.280 $\int \csc^n(e + fx) \sec^m(e + fx) dx$

**Optimal.** Leaf size=81

$$\frac{\cos^2(e + fx)^{\frac{m+1}{2}} \sec^{m+1}(e + fx) \csc^{n-1}(e + fx) \operatorname{Hypergeometric2F1}\left(\frac{m+1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e + fx)\right)}{f(1-n)}$$

[Out] ((Cos[e + f\*x]^2)^((1 + m)/2)\*Csc[e + f\*x]^(-1 + n)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*Sec[e + f\*x]^(1 + m))/(f\*(1 - n))

**Rubi [A]** time = 0.073849, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2631, 2577}

$$\frac{\cos^2(e + fx)^{\frac{m+1}{2}} \sec^{m+1}(e + fx) \csc^{n-1}(e + fx) {}_2F_1\left(\frac{m+1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f\*x]^n\*Sec[e + f\*x]^m,x]

[Out] ((Cos[e + f\*x]^2)^((1 + m)/2)\*Csc[e + f\*x]^(-1 + n)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*Sec[e + f\*x]^(1 + m))/(f\*(1 - n))

#### Rule 2631

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Dist[(a^2\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1))\*(a\*Sin[e + f\*x])^(m - 1)\*(b\*Cos[e + f\*x])^(n + 1))/b^2, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !SimplerQ[-m, -n]

#### Rule 2577

Int[(cos[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^(n\_)\*((a\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_), x\_Symbol] :> Simp[(b^(2\*IntPart[(n - 1)/2] + 1)\*(b\*Cos[e + f\*x])^(2\*FracPart[(n - 1)/2]))\*(a\*Sin[e + f\*x])^(m + 1)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f\*x]^2)]/(a\*f\*(m + 1)\*(Cos[e + f\*x]^2)^FracPart[



$(n - 1)/2]$ ),  $x]$  /; FreeQ[{a, b, e, f, m, n}, x]

### Rubi steps

$$\int \csc^n(e + fx) \sec^m(e + fx) dx = \left( \cos^{1+m}(e + fx) \csc^{-1+n}(e + fx) \sec^{1+m}(e + fx) \sin^{-1+n}(e + fx) \right) \int \cos^{-m}(e + fx) \sin^{1+n}(e + fx) dx$$

$$= \frac{\cos^2(e + fx)^{\frac{1+m}{2}} \csc^{-1+n}(e + fx) {}_2F_1\left(\frac{1+m}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right) \sec^{1+m}(e + fx)}{f(1-n)}$$

**Mathematica [C]** time = 2.24891, size = 278, normalized size = 3.43

$$\frac{(n-3) \sec^m(e + fx) \csc^{n-1}(e + fx)}{f(n-1) \left( (n-3) F_1\left(\frac{1}{2} - \frac{n}{2}; m, -m - n + 1; \frac{3}{2} - \frac{n}{2}; \tan^2\left(\frac{1}{2}(e + fx)\right), -\tan^2\left(\frac{1}{2}(e + fx)\right)\right) - 2 \tan^2\left(\frac{1}{2}(e + fx)\right) \right) (m + n)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[e + f\*x]^n\*Sec[e + f\*x]^m,x]

[Out] -((( -3 + n)\*AppellF1[1/2 - n/2, m, 1 - m - n, 3/2 - n/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2]\*Csc[e + f\*x]^(-1 + n)\*Sec[e + f\*x]^m)/(f\*(-1 + n))\*(( -3 + n)\*AppellF1[1/2 - n/2, m, 1 - m - n, 3/2 - n/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2] - 2\*(( -1 + m + n)\*AppellF1[3/2 - n/2, m, 2 - m - n, 5/2 - n/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2] + m\*AppellF1[3/2 - n/2, 1 + m, 1 - m - n, 5/2 - n/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2])\*Tan[(e + f\*x)/2]^2))

**Maple [F]** time = 0.565, size = 0, normalized size = 0.

$$\int (\csc(fx + e))^n (\sec(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f\*x+e)^n\*sec(f\*x+e)^m,x)

[Out] int(csc(f\*x+e)^n\*sec(f\*x+e)^m,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc (fx + e)^n \sec (fx + e)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f\*x+e)^n\*sec(f\*x+e)^m,x, algorithm="maxima")

[Out] integrate(csc(f\*x + e)^n\*sec(f\*x + e)^m, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\csc (fx + e)^n \sec (fx + e)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f\*x+e)^n\*sec(f\*x+e)^m,x, algorithm="fricas")

[Out] integral(csc(f\*x + e)^n\*sec(f\*x + e)^m, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc^n (e + fx) \sec^m (e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f\*x+e)\*\*n\*sec(f\*x+e)\*\*m,x)

[Out] Integral(csc(e + f\*x)\*\*n\*sec(e + f\*x)\*\*m, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc (fx + e)^n \sec (fx + e)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^n*sec(f*x+e)^m,x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^n*sec(f*x + e)^m, x)
```

### 3.281 $\int \csc^n(e + fx)(a \sec(e + fx))^m dx$

**Optimal.** Leaf size=86

$$\frac{\cos^2(e + fx)^{\frac{m+1}{2}} \csc^{n-1}(e + fx)(a \sec(e + fx))^{m+1} \text{Hypergeometric2F1}\left(\frac{m+1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e + fx)\right)}{af(1-n)}$$

[Out] ((Cos[e + f\*x]^2)^((1 + m)/2)\*Csc[e + f\*x]^(-1 + n)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*(a\*Sec[e + f\*x])^(1 + m))/(a\*f\*(1 - n))

**Rubi [A]** time = 0.0864896, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2631, 2577}

$$\frac{\cos^2(e + fx)^{\frac{m+1}{2}} \csc^{n-1}(e + fx)(a \sec(e + fx))^{m+1} {}_2F_1\left(\frac{m+1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right)}{af(1-n)}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f\*x]^n\*(a\*Sec[e + f\*x])^m,x]

[Out] ((Cos[e + f\*x]^2)^((1 + m)/2)\*Csc[e + f\*x]^(-1 + n)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*(a\*Sec[e + f\*x])^(1 + m))/(a\*f\*(1 - n))

#### Rule 2631

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[(a^2\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1)\*(a\*Sin[e + f\*x])^(m - 1)\*(b\*Cos[e + f\*x])^(n + 1))/b^2, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !SimplerQ[-m, -n]

#### Rule 2577

Int[(cos[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^(n\_)\*((a\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_), x\_Symbol] := Simp[(b^(2\*IntPart[(n - 1)/2] + 1)\*(b\*Cos[e + f\*x])^(2\*FracPart[(n - 1)/2])\*(a\*Sin[e + f\*x])^(m + 1)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f\*x]^2)]/(a\*f\*(m + 1)\*(Cos[e + f\*x]^2)^FracPart[

$(n - 1)/2]$ ),  $x]$  /; FreeQ[{a, b, e, f, m, n}, x]

### Rubi steps

$$\int \csc^n(e + fx)(a \sec(e + fx))^m dx = \frac{\left((a \cos(e + fx))^{1+m} \csc^{-1+n}(e + fx)(a \sec(e + fx))^{1+m} \sin^{-1+n}(e + fx)\right) \int (a \cos(e + fx))^{1+m} \csc^{-1+n}(e + fx) dx}{a^2}$$

$$= \frac{\cos^2(e + fx)^{\frac{1+m}{2}} \csc^{-1+n}(e + fx) {}_2F_1\left(\frac{1+m}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right) (a \sec(e + fx))^{1+m}}{af(1-n)}$$

**Mathematica [C]** time = 0.549284, size = 280, normalized size = 3.26

$$\frac{(n-3) \csc^{n-1}(e+fx)(a \sec(e+fx))^m}{f(n-1) \left( (n-3) {}_2F_1\left(\frac{1}{2} - \frac{n}{2}; m, -m-n+1; \frac{3}{2} - \frac{n}{2}; \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right) - 2 \tan^2\left(\frac{1}{2}(e+fx)\right) \left( (m+n) \csc^{n-1}(e+fx) \right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[e + f\*x]^n\*(a\*Sec[e + f\*x])^m,x]

[Out] -((((-3 + n)\*AppellF1[1/2 - n/2, m, 1 - m - n, 3/2 - n/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2]\*Csc[e + f\*x]^(-1 + n)\*(a\*Sec[e + f\*x])^m)/(f\*(-1 + n))\*((-3 + n)\*AppellF1[1/2 - n/2, m, 1 - m - n, 3/2 - n/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2] - 2\*((-1 + m + n)\*AppellF1[3/2 - n/2, m, 2 - m - n, 5/2 - n/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2] + m\*AppellF1[3/2 - n/2, 1 + m, 1 - m - n, 5/2 - n/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2])\*Tan[(e + f\*x)/2]^2))

**Maple [F]** time = 0.509, size = 0, normalized size = 0.

$$\int (\csc(fx + e))^n (a \sec(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f\*x+e)^n\*(a\*sec(f\*x+e))^m,x)

[Out] int(csc(f\*x+e)^n\*(a\*sec(f\*x+e))^m,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (a \sec (fx + e))^m \csc (fx + e)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f\*x+e)^n\*(a\*sec(f\*x+e))^m,x, algorithm="maxima")

[Out] integrate((a\*sec(f\*x + e))^m\*csc(f\*x + e)^n, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a \sec (fx + e)\right)^m \csc (fx + e)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f\*x+e)^n\*(a\*sec(f\*x+e))^m,x, algorithm="fricas")

[Out] integral((a\*sec(f\*x + e))^m\*csc(f\*x + e)^n, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a \sec (e + fx))^m \csc^n (e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f\*x+e)\*\*n\*(a\*sec(f\*x+e))\*\*m,x)

[Out] Integral((a\*sec(e + f\*x))\*\*m\*csc(e + f\*x)\*\*n, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (a \sec (fx + e))^m \csc (fx + e)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^n*(a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e))^m*csc(f*x + e)^n, x)
```

### 3.282 $\int (b \csc(e + fx))^n \sec^m(e + fx) dx$

**Optimal.** Leaf size=84

$$\frac{b \cos^2(e + fx)^{\frac{m+1}{2}} \sec^{m+1}(e + fx) (b \csc(e + fx))^{n-1} \text{Hypergeometric2F1}\left(\frac{m+1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e + fx)\right)}{f(1-n)}$$

[Out] (b\*(Cos[e + f\*x]^2)^((1 + m)/2)\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*Sec[e + f\*x]^(1 + m))/(f\*(1 - n))

**Rubi [A]** time = 0.0868945, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2631, 2577}

$$\frac{b \cos^2(e + fx)^{\frac{m+1}{2}} \sec^{m+1}(e + fx) (b \csc(e + fx))^{n-1} {}_2F_1\left(\frac{m+1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Csc[e + f\*x])^n\*Sec[e + f\*x]^m,x]

[Out] (b\*(Cos[e + f\*x]^2)^((1 + m)/2)\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*Sec[e + f\*x]^(1 + m))/(f\*(1 - n))

#### Rule 2631

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^m]\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Dist[(a^2\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1))\*(a\*Sin[e + f\*x])^(m - 1)\*(b\*Cos[e + f\*x])^(n + 1))/b^2, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !SimplerQ[-m, -n]

#### Rule 2577

Int[(cos[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^n]\*((a\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_), x\_Symbol] :> Simp[(b^(2\*IntPart[(n - 1)/2] + 1)\*(b\*Cos[e + f\*x])^(2\*FracPart[(n - 1)/2])\*(a\*Sin[e + f\*x])^(m + 1)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f\*x]^2])/(a\*f\*(m + 1)\*(Cos[e + f\*x]^2)^FracPart[



$(n - 1)/2]$ ),  $x]$  /; FreeQ[{a, b, e, f, m, n}, x]

### Rubi steps

$$\int (b \csc(e + fx))^n \sec^m(e + fx) dx = \left( b^2 \cos^{1+m}(e + fx) (b \csc(e + fx))^{-1+n} \sec^{1+m}(e + fx) (b \sin(e + fx))^{-1+n} \right) \int \cos^{-n} \\ = \frac{b \cos^2(e + fx)^{\frac{1+m}{2}} (b \csc(e + fx))^{-1+n} {}_2F_1\left(\frac{1+m}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right) \sec^{1+m}(e + fx)}{f(1-n)}$$

**Mathematica [C]** time = 0.311531, size = 281, normalized size = 3.35

$$\frac{b(n-3) \sec^m(e + fx) (b \csc(e + fx))^{n-1} F_1\left(\frac{1-n}{2}; m, -m-n+1; \frac{3-n}{2}; \tan^2\left(\frac{1}{2}(e + fx)\right), -\tan^2\left(\frac{1}{2}(e + fx)\right)\right) - 2 \tan^2\left(\frac{1}{2}(e + fx)\right) \left((m+n-1)\right)}{f(n-1)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(b\*Csc[e + f\*x])^n\*Sec[e + f\*x]^m,x]

[Out] -((b\*(-3 + n)\*AppellF1[(1 - n)/2, m, 1 - m - n, (3 - n)/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2]\*(b\*Csc[e + f\*x])^(-1 + n)\*Sec[e + f\*x]^m)/(f\*(-1 + n)\*((-3 + n)\*AppellF1[(1 - n)/2, m, 1 - m - n, (3 - n)/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2] - 2\*(-1 + m + n)\*AppellF1[(3 - n)/2, m, 2 - m - n, (5 - n)/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2] + m\*AppellF1[(3 - n)/2, 1 + m, 1 - m - n, (5 - n)/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2])\*Tan[(e + f\*x)/2]^2))

**Maple [F]** time = 0.529, size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n (\sec(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*csc(f\*x+e))^n\*sec(f\*x+e)^m,x)

[Out] int((b\*csc(f\*x+e))^n\*sec(f\*x+e)^m,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \sec(fx + e)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^m,x, algorithm="maxima")

[Out] integrate((b\*csc(f\*x + e))^n\*sec(f\*x + e)^m, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \csc(fx + e)\right)^n \sec(fx + e)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^m,x, algorithm="fricas")

[Out] integral((b\*csc(f\*x + e))^n\*sec(f\*x + e)^m, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(e + fx))^n \sec^m(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))\*\*n\*sec(f\*x+e)\*\*m,x)

[Out] Integral((b\*csc(e + f\*x))\*\*n\*sec(e + f\*x)\*\*m, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \sec(fx + e)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^m, x)
```

### 3.283 $\int (b \csc(e + fx))^n (a \sec(e + fx))^m dx$

**Optimal.** Leaf size=89

$$\frac{b \cos^2(e + fx)^{\frac{m+1}{2}} (a \sec(e + fx))^{m+1} (b \csc(e + fx))^{n-1} \text{Hypergeometric2F1}\left(\frac{m+1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e + fx)\right)}{af(1-n)}$$

[Out] (b\*(Cos[e + f\*x]^2)^((1 + m)/2)\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*(a\*Sec[e + f\*x])^(1 + m))/(a\*f\*(1 - n))

**Rubi [A]** time = 0.0942339, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2631, 2577}

$$\frac{b \cos^2(e + fx)^{\frac{m+1}{2}} (a \sec(e + fx))^{m+1} (b \csc(e + fx))^{n-1} {}_2F_1\left(\frac{m+1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right)}{af(1-n)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Csc[e + f\*x])^n\*(a\*Sec[e + f\*x])^m,x]

[Out] (b\*(Cos[e + f\*x]^2)^((1 + m)/2)\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*(a\*Sec[e + f\*x])^(1 + m))/(a\*f\*(1 - n))

#### Rule 2631

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^m]\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[(a^2\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1))\*(a\*Sin[e + f\*x])^(m - 1)\*(b\*Cos[e + f\*x])^(n + 1))/b^2, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !SimplerQ[-m, -n]

#### Rule 2577

Int[(cos[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^n]\*((a\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_), x\_Symbol] := Simp[(b^(2\*IntPart[(n - 1)/2] + 1)\*(b\*Cos[e + f\*x])^(2\*FracPart[(n - 1)/2])\*(a\*Sin[e + f\*x])^(m + 1)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f\*x]^2])/(a\*f\*(m + 1)\*(Cos[e + f\*x]^2)^FracPart[

$(n - 1)/2]$ ),  $x]$  /; FreeQ[{a, b, e, f, m, n}, x]

### Rubi steps

$$\int (b \csc(e + fx))^n (a \sec(e + fx))^m dx = \frac{(b^2 (a \cos(e + fx))^{1+m} (b \csc(e + fx))^{-1+n} (a \sec(e + fx))^{1+m} (b \sin(e + fx))^{-1+n})}{a^2}$$

$$= \frac{b \cos^2(e + fx)^{\frac{1+m}{2}} (b \csc(e + fx))^{-1+n} {}_2F_1\left(\frac{1+m}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right) (a \sec(e + fx))^m}{af(1-n)}$$

**Mathematica [C]** time = 0.187567, size = 283, normalized size = 3.18

$$\frac{b(n-3)(a \sec(e + fx))^m (b \csc(e + fx))^{n-3}}{f(n-1) \left( (n-3) F_1\left(\frac{1-n}{2}; m, -m-n+1; \frac{3-n}{2}; \tan^2\left(\frac{1}{2}(e + fx)\right), -\tan^2\left(\frac{1}{2}(e + fx)\right)\right) - 2 \tan^2\left(\frac{1}{2}(e + fx)\right) \left((m+n-1) \tan^2\left(\frac{1}{2}(e + fx)\right)\right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(b\*Csc[e + f\*x])^n\*(a\*Sec[e + f\*x])^m,x]

[Out] -((b\*(-3 + n)\*AppellF1[(1 - n)/2, m, 1 - m - n, (3 - n)/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2]\*(b\*Csc[e + f\*x])^(-1 + n)\*(a\*Sec[e + f\*x])^m)/(f\*(-1 + n)\*((-3 + n)\*AppellF1[(1 - n)/2, m, 1 - m - n, (3 - n)/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2] - 2\*((-1 + m + n)\*AppellF1[(3 - n)/2, m, 2 - m - n, (5 - n)/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2] + m\*AppellF1[(3 - n)/2, 1 + m, 1 - m - n, (5 - n)/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2])\*Tan[(e + f\*x)/2]^2))

**Maple [F]** time = 0.558, size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n (a \sec(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*csc(f\*x+e))^n\*(a\*sec(f\*x+e))^m,x)

[Out] int((b\*csc(f\*x+e))^n\*(a\*sec(f\*x+e))^m,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc (fx + e))^n (a \sec (fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*(a\*sec(f\*x+e))^m,x, algorithm="maxima")

[Out] integrate((b\*csc(f\*x + e))^n\*(a\*sec(f\*x + e))^m, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \csc (fx + e)\right)^n \left(a \sec (fx + e)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*(a\*sec(f\*x+e))^m,x, algorithm="fricas")

[Out] integral((b\*csc(f\*x + e))^n\*(a\*sec(f\*x + e))^m, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a \sec (e + fx))^m (b \csc (e + fx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))\*\*n\*(a\*sec(f\*x+e))\*\*m,x)

[Out] Integral((a\*sec(e + f\*x))\*\*m\*(b\*csc(e + f\*x))\*\*n, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc (fx + e))^n (a \sec (fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*(a*sec(f*x + e))^m, x)
```

### 3.284 $\int (b \csc(e + fx))^n \sec^5(e + fx) dx$

**Optimal.** Leaf size=48

$$\frac{(b \csc(e + fx))^{n+5} \text{Hypergeometric2F1}\left(3, \frac{n+5}{2}, \frac{n+7}{2}, \csc^2(e + fx)\right)}{b^5 f(n+5)}$$

[Out] ((b\*Csc[e + f\*x])^(5 + n)\*Hypergeometric2F1[3, (5 + n)/2, (7 + n)/2, Csc[e + f\*x]^2])/(b^5\*f\*(5 + n))

**Rubi [A]** time = 0.0508996, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2621, 364}

$$\frac{(b \csc(e + fx))^{n+5} {}_2F_1\left(3, \frac{n+5}{2}; \frac{n+7}{2}; \csc^2(e + fx)\right)}{b^5 f(n+5)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Csc[e + f\*x])^n\*Sec[e + f\*x]^5,x]

[Out] ((b\*Csc[e + f\*x])^(5 + n)\*Hypergeometric2F1[3, (5 + n)/2, (7 + n)/2, Csc[e + f\*x]^2])/(b^5\*f\*(5 + n))

#### Rule 2621

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)]^(n\_.), x\_Symbol] :> -Dist[(f\*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a\*Csc[e + f\*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

#### Rule 364

Int[((c\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])]/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps



$$\int (b \csc(e + fx))^n \sec^5(e + fx) dx = -\frac{\text{Subst}\left(\int \frac{x^{4+n}}{\left(-1+\frac{x^2}{b^2}\right)^3} dx, x, b \csc(e + fx)\right)}{b^5 f}$$

$$= \frac{(b \csc(e + fx))^{5+n} {}_2F_1\left(3, \frac{5+n}{2}; \frac{7+n}{2}; \csc^2(e + fx)\right)}{b^5 f(5+n)}$$

**Mathematica [A]** time = 0.0512588, size = 51, normalized size = 1.06

$$-\frac{b(b \csc(e + fx))^{n-1} \text{Hypergeometric2F1}\left(3, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e + fx)\right)}{f(n-1)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Csc[e + f\*x])^n\*Sec[e + f\*x]^5,x]

[Out] -((b\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[3, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2])/(f\*(-1 + n)))

**Maple [F]** time = 0.338, size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n (\sec(fx + e))^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*csc(f\*x+e))^n\*sec(f\*x+e)^5,x)

[Out] int((b\*csc(f\*x+e))^n\*sec(f\*x+e)^5,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \sec(fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^5,x, algorithm="maxima")

[Out] integrate((b\*csc(f\*x + e))^n\*sec(f\*x + e)^5, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \csc(fx + e)\right)^n \sec(fx + e)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^5,x, algorithm="fricas")

[Out] integral((b\*csc(f\*x + e))^n\*sec(f\*x + e)^5, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^5,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \sec(fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^5,x, algorithm="giac")

[Out] integrate((b\*csc(f\*x + e))^n\*sec(f\*x + e)^5, x)

### 3.285 $\int (b \csc(e + fx))^n \sec^3(e + fx) dx$

**Optimal.** Leaf size=49

$$-\frac{(b \csc(e + fx))^{n+3} \text{Hypergeometric2F1}\left(2, \frac{n+3}{2}, \frac{n+5}{2}, \csc^2(e + fx)\right)}{b^3 f(n+3)}$$

[Out] -(((b\*Csc[e + f\*x])^(3 + n)\*Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, Csc[e + f\*x]^2]))/(b^3\*f\*(3 + n))

**Rubi [A]** time = 0.0510699, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2621, 364}

$$-\frac{(b \csc(e + fx))^{n+3} {}_2F_1\left(2, \frac{n+3}{2}; \frac{n+5}{2}; \csc^2(e + fx)\right)}{b^3 f(n+3)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Csc[e + f\*x])^n\*Sec[e + f\*x]^3,x]

[Out] -(((b\*Csc[e + f\*x])^(3 + n)\*Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, Csc[e + f\*x]^2]))/(b^3\*f\*(3 + n))

#### Rule 2621

Int[(csc[(e\_) + (f\_)\*(x\_)]\*(a\_))^(m\_)\*sec[(e\_) + (f\_)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[(f\*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a\*Csc[e + f\*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/ (c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int (b \csc(e + fx))^n \sec^3(e + fx) dx = -\frac{\text{Subst}\left(\int \frac{x^{2+n}}{\left(-1+\frac{x^2}{b^2}\right)^2} dx, x, b \csc(e + fx)\right)}{b^3 f}$$

$$= -\frac{(b \csc(e + fx))^{3+n} {}_2F_1\left(2, \frac{3+n}{2}; \frac{5+n}{2}; \csc^2(e + fx)\right)}{b^3 f(3+n)}$$

**Mathematica [A]** time = 0.0430347, size = 51, normalized size = 1.04

$$-\frac{b(b \csc(e + fx))^{n-1} \text{Hypergeometric2F1}\left(2, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e + fx)\right)}{f(n-1)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Csc[e + f\*x])^n\*Sec[e + f\*x]^3,x]

[Out] -((b\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2])/(f\*(-1 + n)))

**Maple [F]** time = 0.323, size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n (\sec(fx + e))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*csc(f\*x+e))^n\*sec(f\*x+e)^3,x)

[Out] int((b\*csc(f\*x+e))^n\*sec(f\*x+e)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \sec(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^3,x, algorithm="maxima")

[Out] integrate((b\*csc(f\*x + e))^n\*sec(f\*x + e)^3, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \csc(fx + e)\right)^n \sec(fx + e)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^3,x, algorithm="fricas")

[Out] integral((b\*csc(f\*x + e))^n\*sec(f\*x + e)^3, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \sec(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^3,x, algorithm="giac")

[Out] integrate((b\*csc(f\*x + e))^n\*sec(f\*x + e)^3, x)

### 3.286 $\int (b \csc(e + fx))^n \sec(e + fx) dx$

**Optimal.** Leaf size=48

$$\frac{(b \csc(e + fx))^{n+1} \text{Hypergeometric2F1}\left(1, \frac{n+1}{2}, \frac{n+3}{2}, \csc^2(e + fx)\right)}{bf(n+1)}$$

[Out] ((b\*Csc[e + f\*x])^(1 + n)\*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Csc[e + f\*x]^2])/(b\*f\*(1 + n))

**Rubi [A]** time = 0.0355939, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2621, 364}

$$\frac{(b \csc(e + fx))^{n+1} {}_2F_1\left(1, \frac{n+1}{2}; \frac{n+3}{2}; \csc^2(e + fx)\right)}{bf(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Csc[e + f\*x])^n\*Sec[e + f\*x],x]

[Out] ((b\*Csc[e + f\*x])^(1 + n)\*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Csc[e + f\*x]^2])/(b\*f\*(1 + n))

#### Rule 2621

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)]^(n\_.), x\_Symbol] :> -Dist[(f\*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a\*Csc[e + f\*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

#### Rule 364

Int[((c\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])]/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int (b \csc(e + fx))^n \sec(e + fx) dx = -\frac{\text{Subst}\left(\int \frac{x^n}{-1+\frac{x^2}{b^2}} dx, x, b \csc(e + fx)\right)}{bf}$$

$$= \frac{(b \csc(e + fx))^{1+n} {}_2F_1\left(1, \frac{1+n}{2}; \frac{3+n}{2}; \csc^2(e + fx)\right)}{bf(1+n)}$$

**Mathematica [A]** time = 0.0325293, size = 51, normalized size = 1.06

$$\frac{b(b \csc(e + fx))^{n-1} \text{Hypergeometric2F1}\left(1, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e + fx)\right)}{f(n-1)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Csc[e + f\*x])^n\*Sec[e + f\*x],x]

[Out] -((b\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2])/(f\*(-1 + n)))

**Maple [F]** time = 0.31, size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \sec(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*csc(f\*x+e))^n\*sec(f\*x+e),x)

[Out] int((b\*csc(f\*x+e))^n\*sec(f\*x+e),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \sec(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e),x, algorithm="maxima")

[Out] integrate((b\*csc(f\*x + e))^n\*sec(f\*x + e), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \csc(fx + e)\right)^n \sec(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e),x, algorithm="fricas")

[Out] integral((b\*csc(f\*x + e))^n\*sec(f\*x + e), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(e + fx))^n \sec(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e),x)

[Out] Integral((b\*csc(e + f\*x))^n\*sec(e + f\*x), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \sec(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e),x, algorithm="giac")

[Out] integrate((b\*csc(f\*x + e))^n\*sec(f\*x + e), x)



$$3.287 \quad \int \cos(e + fx)(b \csc(e + fx))^n dx$$

Optimal. Leaf size=24

$$\frac{b(b \csc(e + fx))^{n-1}}{f(1 - n)}$$

[Out] (b\*(b\*Csc[e + f\*x])^(-1 + n))/(f\*(1 - n))

**Rubi [A]** time = 0.0335722, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2621, 30}

$$\frac{b(b \csc(e + fx))^{n-1}}{f(1 - n)}$$

Antiderivative was successfully verified.

[In] Int[Cos[e + f\*x]\*(b\*Csc[e + f\*x])^n,x]

[Out] (b\*(b\*Csc[e + f\*x])^(-1 + n))/(f\*(1 - n))

#### Rule 2621

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)]^(n\_.), x\_Symbol] :> -Dist[(f\*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a\*Csc[e + f\*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \cos(e + fx)(b \csc(e + fx))^n dx &= -\frac{b \operatorname{Subst}\left(\int x^{-2+n} dx, x, b \csc(e + fx)\right)}{f} \\ &= \frac{b(b \csc(e + fx))^{-1+n}}{f(1 - n)} \end{aligned}$$

**Mathematica [A]** time = 0.0249491, size = 23, normalized size = 0.96

$$-\frac{b(b \csc(e + fx))^{n-1}}{f(n-1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[e + f\*x]\*(b\*Csc[e + f\*x])^n,x]

[Out] -((b\*(b\*Csc[e + f\*x])^(-1 + n))/(f\*(-1 + n)))

**Maple [B]** time = 0.07, size = 66, normalized size = 2.8

$$-2 \frac{\tan\left(\frac{1}{2}fx + \frac{e}{2}\right)}{f(-1+n)\left(1 + \left(\tan\left(\frac{1}{2}fx + \frac{e}{2}\right)\right)^2\right)} e^{n \ln\left(\frac{1}{2} \frac{b\left(1 + \left(\tan\left(\frac{1}{2}fx + \frac{e}{2}\right)\right)^2\right)}{\tan\left(\frac{1}{2}fx + \frac{e}{2}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f\*x+e)\*(b\*csc(f\*x+e))^n,x)

[Out] -2/f/(-1+n)\*tan(1/2\*f\*x+1/2\*e)\*exp(n\*ln(1/2\*b\*(1+tan(1/2\*f\*x+1/2\*e)^2)/tan(1/2\*f\*x+1/2\*e)))/(1+tan(1/2\*f\*x+1/2\*e)^2)

**Maxima [A]** time = 1.14941, size = 39, normalized size = 1.62

$$-\frac{b^n \sin(fx + e)^{-n} \sin(fx + e)}{f(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f\*x+e)\*(b\*csc(f\*x+e))^n,x, algorithm="maxima")

[Out] -b^n\*sin(f\*x + e)^(-n)\*sin(f\*x + e)/(f\*(n - 1))

**Fricas [A]** time = 1.69949, size = 59, normalized size = 2.46

$$-\frac{\left(\frac{b}{\sin(fx+e)}\right)^n \sin(fx+e)}{fn-f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f\*x+e)\*(b\*csc(f\*x+e))^n,x, algorithm="fricas")

[Out] -(b/sin(f\*x + e))^n\*sin(f\*x + e)/(f\*n - f)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(e + fx))^n \cos(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f\*x+e)\*(b\*csc(f\*x+e))\*\*n,x)

[Out] Integral((b\*csc(e + f\*x))\*\*n\*cos(e + f\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \cos(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f\*x+e)\*(b\*csc(f\*x+e))^n,x, algorithm="giac")

[Out] integrate((b\*csc(f\*x + e))^n\*cos(f\*x + e), x)

### 3.288 $\int \cos^3(e + fx)(b \csc(e + fx))^n dx$

**Optimal.** Leaf size=52

$$\frac{b(b \csc(e + fx))^{n-1}}{f(1-n)} - \frac{b^3(b \csc(e + fx))^{n-3}}{f(3-n)}$$

[Out]  $-\left(\frac{b^3(b \csc(e + fx))^{-3+n}}{f(3-n)}\right) + \frac{b(b \csc(e + fx))^{-1+n}}{f(1-n)}$

**Rubi [A]** time = 0.0516861, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2621, 14}

$$\frac{b(b \csc(e + fx))^{n-1}}{f(1-n)} - \frac{b^3(b \csc(e + fx))^{n-3}}{f(3-n)}$$

Antiderivative was successfully verified.

[In] Int[Cos[e + f\*x]^3\*(b\*Csc[e + f\*x])^n,x]

[Out]  $-\left(\frac{b^3(b \csc(e + fx))^{-3+n}}{f(3-n)}\right) + \frac{b(b \csc(e + fx))^{-1+n}}{f(1-n)}$

#### Rule 2621

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)]^(n\_.), x\_Symbol] :> -Dist[(f\*a^n)^(-1), Subst[Int[x^(m+n-1)/(-1+x^2/a^2)^((n+1)/2), x], x, a\*Csc[e + f\*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n+1)/2] && !(IntegerQ[(m+1)/2] && LtQ[0, m, n])

#### Rule 14

Int[(u\_.)\*((c\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_.)\*(v\_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rubi steps

$$\begin{aligned}
\int \cos^3(e + fx)(b \csc(e + fx))^n dx &= -\frac{b^3 \text{Subst}\left(\int x^{-4+n}\left(-1 + \frac{x^2}{b^2}\right) dx, x, b \csc(e + fx)\right)}{f} \\
&= -\frac{b^3 \text{Subst}\left(\int \left(-x^{-4+n} + \frac{x^{-2+n}}{b^2}\right) dx, x, b \csc(e + fx)\right)}{f} \\
&= -\frac{b^3(b \csc(e + fx))^{-3+n}}{f(3-n)} + \frac{b(b \csc(e + fx))^{-1+n}}{f(1-n)}
\end{aligned}$$

**Mathematica [A]** time = 0.126489, size = 45, normalized size = 0.87

$$-\frac{b((n-1)\cos(2(e+fx)) + n-5)(b \csc(e+fx))^{n-1}}{2f(n-3)(n-1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[e + f\*x]^3\*(b\*Csc[e + f\*x])^n,x]

[Out] -(b\*(-5 + n + (-1 + n)\*Cos[2\*(e + f\*x)])\*(b\*Csc[e + f\*x])^(-1 + n))/(2\*f\*(-3 + n)\*(-1 + n))

**Maple [F]** time = 1.153, size = 0, normalized size = 0.

$$\int (\cos(fx + e))^3 (b \csc(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f\*x+e)^3\*(b\*csc(f\*x+e))^n,x)

[Out] int(cos(f\*x+e)^3\*(b\*csc(f\*x+e))^n,x)

**Maxima [A]** time = 1.1554, size = 78, normalized size = 1.5

$$\frac{\frac{b^n \sin(fx+e)^{-n} \sin(fx+e)^3}{n-3} - \frac{b^n \sin(fx+e)^{-n} \sin(fx+e)}{n-1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f\*x+e)^3\*(b\*csc(f\*x+e))^n,x, algorithm="maxima")

[Out] (b^n\*sin(f\*x + e)^(-n)\*sin(f\*x + e)^3/(n - 3) - b^n\*sin(f\*x + e)^(-n)\*sin(f\*x + e)/(n - 1))/f

**Fricas [A]** time = 1.7469, size = 115, normalized size = 2.21

$$\frac{\left((n-1)\cos(fx+e)^2 - 2\right)\left(\frac{b}{\sin(fx+e)}\right)^n \sin(fx+e)}{fn^2 - 4fn + 3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f\*x+e)^3\*(b\*csc(f\*x+e))^n,x, algorithm="fricas")

[Out] -((n - 1)\*cos(f\*x + e)^2 - 2)\*(b/sin(f\*x + e))^n\*sin(f\*x + e)/(f\*n^2 - 4\*f\*n + 3\*f)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f\*x+e)\*\*3\*(b\*csc(f\*x+e))\*\*n,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx+e))^n \cos(fx+e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^3, x)
```

### 3.289 $\int \cos^5(e + fx)(b \csc(e + fx))^n dx$

**Optimal.** Leaf size=78

$$\frac{b^5(b \csc(e + fx))^{n-5}}{f(5-n)} - \frac{2b^3(b \csc(e + fx))^{n-3}}{f(3-n)} + \frac{b(b \csc(e + fx))^{n-1}}{f(1-n)}$$

[Out]  $(b^5*(b*\text{Csc}[e + f*x])^{(-5 + n)})/(f*(5 - n)) - (2*b^3*(b*\text{Csc}[e + f*x])^{(-3 + n)})/(f*(3 - n)) + (b*(b*\text{Csc}[e + f*x])^{(-1 + n)})/(f*(1 - n))$

**Rubi [A]** time = 0.0642744, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2621, 270}

$$\frac{b^5(b \csc(e + fx))^{n-5}}{f(5-n)} - \frac{2b^3(b \csc(e + fx))^{n-3}}{f(3-n)} + \frac{b(b \csc(e + fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cos}[e + f*x]^5*(b*\text{Csc}[e + f*x])^n, x]$

[Out]  $(b^5*(b*\text{Csc}[e + f*x])^{(-5 + n)})/(f*(5 - n)) - (2*b^3*(b*\text{Csc}[e + f*x])^{(-3 + n)})/(f*(3 - n)) + (b*(b*\text{Csc}[e + f*x])^{(-1 + n)})/(f*(1 - n))$

#### Rule 2621

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(a_.))^{(m_.)}*\text{sec}[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow -\text{Dist}[(f*a^n)^{-1}, \text{Subst}[\text{Int}[x^{(m+n-1)}]/(-1+x^2/a^2)^{((n+1)/2)}, x], x, a*\text{Csc}[e + f*x], x] /; \text{FreeQ}[\{a, e, f, m\}, x] \&\& \text{IntegerQ}[(n+1)/2] \&\& !(\text{IntegerQ}[(m+1)/2] \&\& \text{LtQ}[0, m, n])$

#### Rule 270

$\text{Int}[(c_.)*(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{Exp and Integrand}[(c*x)^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, m, n\}, x] \&\& \text{IGtQ}[p, 0]$

#### Rubi steps



$$\begin{aligned} \int \cos^5(e + fx)(b \csc(e + fx))^n dx &= -\frac{b^5 \operatorname{Subst}\left(\int x^{-6+n} \left(-1 + \frac{x^2}{b^2}\right)^2 dx, x, b \csc(e + fx)\right)}{f} \\ &= -\frac{b^5 \operatorname{Subst}\left(\int \left(x^{-6+n} - \frac{2x^{-4+n}}{b^2} + \frac{x^{-2+n}}{b^4}\right) dx, x, b \csc(e + fx)\right)}{f} \\ &= \frac{b^5 (b \csc(e + fx))^{-5+n}}{f(5-n)} - \frac{2b^3 (b \csc(e + fx))^{-3+n}}{f(3-n)} + \frac{b (b \csc(e + fx))^{-1+n}}{f(1-n)} \end{aligned}$$

**Mathematica [A]** time = 0.553882, size = 81, normalized size = 1.04

$$\frac{\sin^5(e + fx) \left( (n^2 - 8n + 15) \csc^4(e + fx) - 2(n^2 - 6n + 5) \csc^2(e + fx) + n^2 - 4n + 3 \right) (b \csc(e + fx))^n}{f(n-5)(n-3)(n-1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[e + f\*x]^5\*(b\*Csc[e + f\*x])^n,x]

[Out] -(((b\*Csc[e + f\*x])^n\*(3 - 4\*n + n^2 - 2\*(5 - 6\*n + n^2)\*Csc[e + f\*x]^2 + (15 - 8\*n + n^2)\*Csc[e + f\*x]^4)\*Sin[e + f\*x]^5)/(f\*(-5 + n)\*(-3 + n)\*(-1 + n)))

**Maple [F]** time = 1.199, size = 0, normalized size = 0.

$$\int (\cos(fx + e))^5 (b \csc(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f\*x+e)^5\*(b\*csc(f\*x+e))^n,x)

[Out] int(cos(f\*x+e)^5\*(b\*csc(f\*x+e))^n,x)

**Maxima [A]** time = 1.14297, size = 116, normalized size = 1.49

$$-\frac{\frac{b^n \sin(fx+e)^{-n} \sin(fx+e)^5}{n-5} - \frac{2b^n \sin(fx+e)^{-n} \sin(fx+e)^3}{n-3} + \frac{b^n \sin(fx+e)^{-n} \sin(fx+e)}{n-1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f\*x+e)^5\*(b\*csc(f\*x+e))^n,x, algorithm="maxima")

[Out]  $-(b^n \sin(fx + e)^{-n} \sin(fx + e)^5 / (n - 5) - 2b^n \sin(fx + e)^{-n} \sin(fx + e)^3 / (n - 3) + b^n \sin(fx + e)^{-n} \sin(fx + e) / (n - 1)) / f$

**Fricas [A]** time = 1.76295, size = 178, normalized size = 2.28

$$\frac{\left( (n^2 - 4n + 3) \cos(fx + e)^4 - 4(n - 1) \cos(fx + e)^2 + 8 \right) \left( \frac{b}{\sin(fx + e)} \right)^n \sin(fx + e)}{fn^3 - 9fn^2 + 23fn - 15f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f\*x+e)^5\*(b\*csc(f\*x+e))^n,x, algorithm="fricas")

[Out]  $-((n^2 - 4n + 3) \cos(fx + e)^4 - 4(n - 1) \cos(fx + e)^2 + 8) * (b / \sin(fx + e))^n \sin(fx + e) / (fn^3 - 9fn^2 + 23fn - 15f)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f\*x+e)\*\*5\*(b\*csc(f\*x+e))\*\*n,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \cos(fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^5, x)
```

### 3.290 $\int (b \csc(e + fx))^n \sec^6(e + fx) dx$

**Optimal.** Leaf size=72

$$\frac{b\sqrt{\cos^2(e + fx)} \sec(e + fx) (b \csc(e + fx))^{n-1} \text{Hypergeometric2F1}\left(\frac{7}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e + fx)\right)}{f(1-n)}$$

[Out] (b\*Sqrt[Cos[e + f\*x]^2]\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[7/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*Sec[e + f\*x])/(f\*(1 - n))

**Rubi [A]** time = 0.0798404, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2631, 2577}

$$\frac{b\sqrt{\cos^2(e + fx)} \sec(e + fx) (b \csc(e + fx))^{n-1} {}_2F_1\left(\frac{7}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Csc[e + f\*x])^n\*Sec[e + f\*x]^6,x]

[Out] (b\*Sqrt[Cos[e + f\*x]^2]\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[7/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*Sec[e + f\*x])/(f\*(1 - n))

#### Rule 2631

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^m]\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^n  
 \_, x\_Symbol] :> Dist[(a^2\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1)  
 \*(a\*Sin[e + f\*x])^(m - 1)\*(b\*Cos[e + f\*x])^(n + 1))/b^2, Int[1/((a\*Sin[e +  
 f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !S  
 implerQ[-m, -n]

#### Rule 2577

Int[(cos[(e\_.) + (f\_.)\*(x\_.)]\*(b\_.))^n]\*((a\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^m  
 \_, x\_Symbol] :> Simp[(b^(2\*IntPart[(n - 1)/2] + 1)\*(b\*Cos[e + f\*x])^(2\*Fra  
 cPart[(n - 1)/2])\*(a\*Sin[e + f\*x])^(m + 1)\*Hypergeometric2F1[(1 + m)/2, (1  
 - n)/2, (3 + m)/2, Sin[e + f\*x]^2)]/(a\*f\*(m + 1)\*(Cos[e + f\*x]^2)^FracPart[  
 (n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]

Rubi steps

$$\int (b \csc(e + fx))^n \sec^6(e + fx) dx = \left( b^2 (b \csc(e + fx))^{-1+n} (b \sin(e + fx))^{-1+n} \right) \int \sec^6(e + fx) (b \sin(e + fx))^{-n} dx$$

$$= \frac{b \sqrt{\cos^2(e + fx)} (b \csc(e + fx))^{-1+n} {}_2F_1\left(\frac{7}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right) \sec(e + fx)}{f(1-n)}$$

**Mathematica [A]** time = 0.576896, size = 77, normalized size = 1.07

$$\frac{\tan(e + fx) \sec^2(e + fx)^{-n/2} (b \csc(e + fx))^n \text{Hypergeometric2F1}\left(-\frac{n}{2} - 2, \frac{1}{2} - \frac{n}{2}, \frac{3}{2} - \frac{n}{2}, -\tan^2(e + fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Csc[e + f\*x])^n\*Sec[e + f\*x]^6,x]

[Out] ((b\*Csc[e + f\*x])^n\*Hypergeometric2F1[-2 - n/2, 1/2 - n/2, 3/2 - n/2, -Tan[e + f\*x]^2]\*Tan[e + f\*x])/(f\*(1 - n)\*(Sec[e + f\*x]^2)^(n/2))

**Maple [F]** time = 0.352, size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n (\sec(fx + e))^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*csc(f\*x+e))^n\*sec(f\*x+e)^6,x)

[Out] int((b\*csc(f\*x+e))^n\*sec(f\*x+e)^6,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \sec(fx + e)^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^6,x, algorithm="maxima")

[Out] integrate((b\*csc(f\*x + e))^n\*sec(f\*x + e)^6, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \csc (fx + e)\right)^n \sec (fx + e)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^6,x, algorithm="fricas")

[Out] integral((b\*csc(f\*x + e))^n\*sec(f\*x + e)^6, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^6,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc (fx + e))^n \sec (fx + e)^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^6,x, algorithm="giac")

[Out] integrate((b\*csc(f\*x + e))^n\*sec(f\*x + e)^6, x)

### 3.291 $\int (b \csc(e + fx))^n \sec^4(e + fx) dx$

**Optimal.** Leaf size=72

$$\frac{b\sqrt{\cos^2(e + fx)} \sec(e + fx) (b \csc(e + fx))^{n-1} \text{Hypergeometric2F1}\left(\frac{5}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e + fx)\right)}{f(1-n)}$$

[Out] (b\*Sqrt[Cos[e + f\*x]^2]\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[5/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*Sec[e + f\*x])/(f\*(1 - n))

**Rubi [A]** time = 0.0794841, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2631, 2577}

$$\frac{b\sqrt{\cos^2(e + fx)} \sec(e + fx) (b \csc(e + fx))^{n-1} {}_2F_1\left(\frac{5}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Csc[e + f\*x])^n\*Sec[e + f\*x]^4,x]

[Out] (b\*Sqrt[Cos[e + f\*x]^2]\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[5/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*Sec[e + f\*x])/(f\*(1 - n))

#### Rule 2631

Int[(csc[(e\_) + (f\_)\*(x\_)]\*(a\_))^(m\_)\*((b\_)\*sec[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Dist[(a^2\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1)\*(a\*Sin[e + f\*x])^(m - 1)\*(b\*Cos[e + f\*x])^(n + 1))/b^2, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !SimplerQ[-m, -n]

#### Rule 2577

Int[(cos[(e\_) + (f\_)\*(x\_)]\*(b\_))^(n\_)\*((a\_)\*sin[(e\_) + (f\_)\*(x\_)])^(m\_), x\_Symbol] :> Simp[(b^(2\*IntPart[(n - 1)/2] + 1)\*(b\*Cos[e + f\*x])^(2\*FracPart[(n - 1)/2])\*(a\*Sin[e + f\*x])^(m + 1)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f\*x]^2])/(a\*f\*(m + 1)\*(Cos[e + f\*x]^2)^FracPart[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]

Rubi steps

$$\int (b \csc(e + fx))^n \sec^4(e + fx) dx = \left( b^2 (b \csc(e + fx))^{-1+n} (b \sin(e + fx))^{-1+n} \right) \int \sec^4(e + fx) (b \sin(e + fx))^{-n} dx$$

$$= \frac{b \sqrt{\cos^2(e + fx)} (b \csc(e + fx))^{-1+n} {}_2F_1\left(\frac{5}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right) \sec(e + fx)}{f(1-n)}$$

**Mathematica [A]** time = 0.508711, size = 77, normalized size = 1.07

$$\frac{\tan(e + fx) \sec^2(e + fx)^{-n/2} (b \csc(e + fx))^n \text{Hypergeometric2F1}\left(-\frac{n}{2} - 1, \frac{1}{2} - \frac{n}{2}, \frac{3}{2} - \frac{n}{2}, -\tan^2(e + fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Csc[e + f\*x])^n\*Sec[e + f\*x]^4,x]

[Out] ((b\*Csc[e + f\*x])^n\*Hypergeometric2F1[-1 - n/2, 1/2 - n/2, 3/2 - n/2, -Tan[e + f\*x]^2]\*Tan[e + f\*x])/(f\*(1 - n)\*(Sec[e + f\*x]^2)^(n/2))

**Maple [F]** time = 0.351, size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n (\sec(fx + e))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*csc(f\*x+e))^n\*sec(f\*x+e)^4,x)

[Out] int((b\*csc(f\*x+e))^n\*sec(f\*x+e)^4,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \sec(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^4,x, algorithm="maxima")

[Out] integrate((b\*csc(f\*x + e))^n\*sec(f\*x + e)^4, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \csc(fx + e)\right)^n \sec(fx + e)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^4,x, algorithm="fricas")

[Out] integral((b\*csc(f\*x + e))^n\*sec(f\*x + e)^4, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)\*\*4,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \sec(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^4,x, algorithm="giac")

[Out] integrate((b\*csc(f\*x + e))^n\*sec(f\*x + e)^4, x)

### 3.292 $\int (b \csc(e + fx))^n \sec^2(e + fx) dx$

**Optimal.** Leaf size=72

$$\frac{b\sqrt{\cos^2(e + fx)} \sec(e + fx) (b \csc(e + fx))^{n-1} \text{Hypergeometric2F1}\left(\frac{3}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e + fx)\right)}{f(1-n)}$$

[Out] (b\*Sqrt[Cos[e + f\*x]^2]\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[3/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*Sec[e + f\*x])/(f\*(1 - n))

**Rubi [A]** time = 0.0782317, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2631, 2577}

$$\frac{b\sqrt{\cos^2(e + fx)} \sec(e + fx) (b \csc(e + fx))^{n-1} {}_2F_1\left(\frac{3}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Csc[e + f\*x])^n\*Sec[e + f\*x]^2,x]

[Out] (b\*Sqrt[Cos[e + f\*x]^2]\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[3/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*Sec[e + f\*x])/(f\*(1 - n))

#### Rule 2631

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^m]\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^n  
 \_, x\_Symbol] :> Dist[(a^2\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1)  
 \*(a\*Sin[e + f\*x])^(m - 1)\*(b\*Cos[e + f\*x])^(n + 1))/b^2, Int[1/((a\*Sin[e +  
 f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !S  
 implerQ[-m, -n]

#### Rule 2577

Int[(cos[(e\_.) + (f\_.)\*(x\_.)]\*(b\_.))^n]\*((a\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^m  
 \_, x\_Symbol] :> Simp[(b^(2\*IntPart[(n - 1)/2] + 1)\*(b\*Cos[e + f\*x])^(2\*Fra  
 cPart[(n - 1)/2])\*(a\*Sin[e + f\*x])^(m + 1)\*Hypergeometric2F1[(1 + m)/2, (1  
 - n)/2, (3 + m)/2, Sin[e + f\*x]^2])/(a\*f\*(m + 1)\*(Cos[e + f\*x]^2)^FracPart[  
 (n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]

Rubi steps

$$\int (b \csc(e + fx))^n \sec^2(e + fx) dx = \left( b^2 (b \csc(e + fx))^{-1+n} (b \sin(e + fx))^{-1+n} \right) \int \sec^2(e + fx) (b \sin(e + fx))^{-n} dx$$

$$= \frac{b \sqrt{\cos^2(e + fx)} (b \csc(e + fx))^{-1+n} {}_2F_1\left(\frac{3}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right) \sec(e + fx)}{f(1-n)}$$

**Mathematica [A]** time = 0.442976, size = 75, normalized size = 1.04

$$\frac{\tan(e + fx) \sec^2(e + fx)^{-n/2} (b \csc(e + fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2} - \frac{n}{2}, -\frac{n}{2}, \frac{3}{2} - \frac{n}{2}, -\tan^2(e + fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Csc[e + f\*x])^n\*Sec[e + f\*x]^2,x]

[Out] ((b\*Csc[e + f\*x])^n\*Hypergeometric2F1[1/2 - n/2, -n/2, 3/2 - n/2, -Tan[e + f\*x]^2]\*Tan[e + f\*x])/(f\*(1 - n)\*(Sec[e + f\*x]^2)^(n/2))

**Maple [F]** time = 0.323, size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n (\sec(fx + e))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*csc(f\*x+e))^n\*sec(f\*x+e)^2,x)

[Out] int((b\*csc(f\*x+e))^n\*sec(f\*x+e)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \sec(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^2,x, algorithm="maxima")

[Out] integrate((b\*csc(f\*x + e))^n\*sec(f\*x + e)^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \csc (fx + e)\right)^n \sec (fx + e)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^2,x, algorithm="fricas")

[Out] integral((b\*csc(f\*x + e))^n\*sec(f\*x + e)^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc (e + fx))^n \sec^2 (e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))\*\*n\*sec(f\*x+e)\*\*2,x)

[Out] Integral((b\*csc(e + f\*x))\*\*n\*sec(e + f\*x)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc (fx + e))^n \sec (fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*sec(f\*x+e)^2,x, algorithm="giac")

[Out] integrate((b\*csc(f\*x + e))^n\*sec(f\*x + e)^2, x)

### 3.293 $\int (b \csc(e + fx))^n dx$

**Optimal.** Leaf size=72

$$\frac{b \cos(e + fx)(b \csc(e + fx))^{n-1} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e + fx)\right)}{f(1-n)\sqrt{\cos^2(e + fx)}}$$

[Out] (b\*Cos[e + f\*x]\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2])/(f\*(1 - n)\*Sqrt[Cos[e + f\*x]^2])

**Rubi [A]** time = 0.030244, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {3772, 2643}

$$\frac{b \cos(e + fx)(b \csc(e + fx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right)}{f(1-n)\sqrt{\cos^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Csc[e + f\*x])^n, x]

[Out] (b\*Cos[e + f\*x]\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2])/(f\*(1 - n)\*Sqrt[Cos[e + f\*x]^2])

#### Rule 3772

Int[(csc[(c\_.) + (d\_.)\*(x\_)])\*(b\_.)]^(n\_), x\_Symbol] :> Simp[(b\*Csc[c + d\*x])^(n - 1)\*((Sin[c + d\*x]/b)^(n - 1)\*Int[1/((Sin[c + d\*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

#### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

#### Rubi steps

$$\int (b \csc(e + fx))^n dx = (b \csc(e + fx))^n \int \left(\frac{\sin(e + fx)}{b}\right)^{-n} dx$$

$$= \frac{\cos(e + fx)(b \csc(e + fx))^n {}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right) \sin(e + fx)}{f(1-n)\sqrt{\cos^2(e + fx)}}$$

**Mathematica [A]** time = 0.10017, size = 65, normalized size = 0.9

$$\frac{\sin(e + fx) \cos(e + fx) \sin^2(e + fx)^{\frac{n-1}{2}} (b \csc(e + fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n+1}{2}, \frac{3}{2}, \cos^2(e + fx)\right)}{f}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Csc[e + f\*x])^n,x]

[Out] -((Cos[e + f\*x]\*(b\*Csc[e + f\*x])^n\*Hypergeometric2F1[1/2, (1 + n)/2, 3/2, Cos[e + f\*x]^2]\*Sin[e + f\*x]\*(Sin[e + f\*x]^2)^((-1 + n)/2))/f)

**Maple [F]** time = 0.471, size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*csc(f\*x+e))^n,x)

[Out] int((b\*csc(f\*x+e))^n,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \csc (fx + e)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc (e + fx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n,x)
```

```
[Out] Integral((b*csc(e + f*x))**n, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc (fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n, x)
```

### 3.294 $\int \cos^2(e + fx)(b \csc(e + fx))^n dx$

**Optimal.** Leaf size=72

$$\frac{b \cos(e + fx)(b \csc(e + fx))^{n-1} \text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e + fx)\right)}{f(1-n)\sqrt{\cos^2(e + fx)}}$$

[Out] (b\*Cos[e + f\*x]\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[-1/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2])/(f\*(1 - n)\*Sqrt[Cos[e + f\*x]^2])

**Rubi [A]** time = 0.0796031, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2631, 2577}

$$\frac{b \cos(e + fx)(b \csc(e + fx))^{n-1} {}_2F_1\left(-\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right)}{f(1-n)\sqrt{\cos^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[e + f\*x]^2\*(b\*Csc[e + f\*x])^n,x]

[Out] (b\*Cos[e + f\*x]\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[-1/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2])/(f\*(1 - n)\*Sqrt[Cos[e + f\*x]^2])

#### Rule 2631

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:> Dist[(a^2*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1))*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1))/b^2, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x], x]
/; FreeQ[{a, b, e, f, m, n}, x] && !SimplerQ[-m, -n]
```

#### Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol]
:> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*FracPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[(n - 1)/2]), x]
/; FreeQ[{a, b, e, f, m, n}, x]
```



Rubi steps

$$\int \cos^2(e + fx)(b \csc(e + fx))^n dx = (b^2(b \csc(e + fx))^{-1+n}(b \sin(e + fx))^{-1+n}) \int \cos^2(e + fx)(b \sin(e + fx))^{-n} dx$$

$$= \frac{b \cos(e + fx)(b \csc(e + fx))^{-1+n} {}_2F_1\left(-\frac{1}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right)}{f(1-n)\sqrt{\cos^2(e + fx)}}$$

**Mathematica [B]** time = 0.456792, size = 165, normalized size = 2.29

$$\frac{2 \tan\left(\frac{1}{2}(e + fx)\right) \sec^2\left(\frac{1}{2}(e + fx)\right)^{-n} (b \csc(e + fx))^n \left(\text{Hypergeometric2F1}\left(1 - n, \frac{1}{2} - \frac{n}{2}, \frac{3}{2} - \frac{n}{2}, -\tan^2\left(\frac{1}{2}(e + fx)\right)\right)\right)}{}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[e + f\*x]^2\*(b\*Csc[e + f\*x])^n,x]

[Out] (-2\*(b\*Csc[e + f\*x])^n\*(Hypergeometric2F1[1 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f\*x)/2]^2] - 4\*Hypergeometric2F1[2 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f\*x)/2]^2] + 4\*Hypergeometric2F1[3 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f\*x)/2]^2])\*Tan[(e + f\*x)/2])/(f\*(-1 + n)\*(Sec[(e + f\*x)/2]^2)^n)

**Maple [F]** time = 0.946, size = 0, normalized size = 0.

$$\int (\cos(fx + e))^2 (b \csc(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f\*x+e)^2\*(b\*csc(f\*x+e))^n,x)

[Out] int(cos(f\*x+e)^2\*(b\*csc(f\*x+e))^n,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \cos(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f\*x+e)^2\*(b\*csc(f\*x+e))^n,x, algorithm="maxima")

[Out] integrate((b\*csc(f\*x + e))^n\*cos(f\*x + e)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \csc (fx + e)\right)^n \cos (fx + e)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f\*x+e)^2\*(b\*csc(f\*x+e))^n,x, algorithm="fricas")

[Out] integral((b\*csc(f\*x + e))^n\*cos(f\*x + e)^2, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc (e + fx))^n \cos^2 (e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f\*x+e)\*\*2\*(b\*csc(f\*x+e))\*\*n,x)

[Out] Integral((b\*csc(e + f\*x))\*\*n\*cos(e + f\*x)\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc (fx + e))^n \cos (fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f\*x+e)^2\*(b\*csc(f\*x+e))^n,x, algorithm="giac")

[Out] integrate((b\*csc(f\*x + e))^n\*cos(f\*x + e)^2, x)

### 3.295 $\int \cos^4(e + fx)(b \csc(e + fx))^n dx$

**Optimal.** Leaf size=72

$$\frac{b \cos(e + fx)(b \csc(e + fx))^{n-1} \text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e + fx)\right)}{f(1-n)\sqrt{\cos^2(e + fx)}}$$

[Out] (b\*Cos[e + f\*x]\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[-3/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2])/(f\*(1 - n)\*Sqrt[Cos[e + f\*x]^2])

**Rubi [A]** time = 0.0763823, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2631, 2577}

$$\frac{b \cos(e + fx)(b \csc(e + fx))^{n-1} {}_2F_1\left(-\frac{3}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right)}{f(1-n)\sqrt{\cos^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[e + f\*x]^4\*(b\*Csc[e + f\*x])^n,x]

[Out] (b\*Cos[e + f\*x]\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[-3/2, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2])/(f\*(1 - n)\*Sqrt[Cos[e + f\*x]^2])

#### Rule 2631

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Dist[(a^2\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1))\*(a\*Sin[e + f\*x])^(m - 1)\*(b\*Cos[e + f\*x])^(n + 1))/b^2, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !SimplerQ[-m, -n]

#### Rule 2577

Int[(cos[(e\_.) + (f\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*((a\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_.), x\_Symbol] :> Simp[(b^(2\*IntPart[(n - 1)/2] + 1)\*(b\*Cos[e + f\*x])^(2\*FracPart[(n - 1)/2])\*(a\*Sin[e + f\*x])^(m + 1)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f\*x]^2])/(a\*f\*(m + 1)\*(Cos[e + f\*x]^2)^FracPart[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]

Rubi steps

$$\int \cos^4(e + fx)(b \csc(e + fx))^n dx = (b^2(b \csc(e + fx))^{-1+n}(b \sin(e + fx))^{-1+n}) \int \cos^4(e + fx)(b \sin(e + fx))^{-n} dx$$

$$= \frac{b \cos(e + fx)(b \csc(e + fx))^{-1+n} {}_2F_1\left(-\frac{3}{2}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right)}{f(1-n)\sqrt{\cos^2(e + fx)}}$$

**Mathematica [B]** time = 0.619548, size = 246, normalized size = 3.42

$$\frac{2 \tan\left(\frac{1}{2}(e + fx)\right) \sec^2\left(\frac{1}{2}(e + fx)\right)^{-n} (b \csc(e + fx))^n \left(\text{Hypergeometric2F1}\left(1 - n, \frac{1}{2} - \frac{n}{2}, \frac{3}{2} - \frac{n}{2}, -\tan^2\left(\frac{1}{2}(e + fx)\right)\right)\right)}{}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[e + f\*x]^4\*(b\*Csc[e + f\*x])^n,x]

[Out] (-2\*(b\*Csc[e + f\*x])^n\*(Hypergeometric2F1[1 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f\*x)/2]^2] - 8\*(Hypergeometric2F1[2 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f\*x)/2]^2] - 3\*Hypergeometric2F1[3 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f\*x)/2]^2] + 4\*Hypergeometric2F1[4 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f\*x)/2]^2] - 2\*Hypergeometric2F1[5 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f\*x)/2]^2]))\*Tan[(e + f\*x)/2])/(f\*(-1 + n)\*(Sec[(e + f\*x)/2]^2)^n)

**Maple [F]** time = 1.065, size = 0, normalized size = 0.

$$\int (\cos(fx + e))^4 (b \csc(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f\*x+e)^4\*(b\*csc(f\*x+e))^n,x)

[Out] int(cos(f\*x+e)^4\*(b\*csc(f\*x+e))^n,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \cos(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(f*x+e)^4*(b*csc(f*x+e))^n,x, algorithm="maxima")`

[Out] `integrate((b*csc(f*x + e))^n*cos(f*x + e)^4, x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \csc(fx + e)\right)^n \cos(fx + e)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(f*x+e)^4*(b*csc(f*x+e))^n,x, algorithm="fricas")`

[Out] `integral((b*csc(f*x + e))^n*cos(f*x + e)^4, x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(f*x+e)**4*(b*csc(f*x+e))**n,x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \cos(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(f*x+e)^4*(b*csc(f*x+e))^n,x, algorithm="giac")`

[Out] `integrate((b*csc(f*x + e))^n*cos(f*x + e)^4, x)`

### 3.296 $\int (b \csc(e + fx))^n (c \sec(e + fx))^{3/2} dx$

**Optimal.** Leaf size=81

$$\frac{b \cos^2(e + fx)^{5/4} (c \sec(e + fx))^{5/2} (b \csc(e + fx))^{n-1} \text{Hypergeometric2F1}\left(\frac{5}{4}, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e + fx)\right)}{cf(1-n)}$$

[Out] (b\*(Cos[e + f\*x]^2)^(5/4)\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[5/4, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*(c\*Sec[e + f\*x])^(5/2))/(c\*f\*(1 - n))

**Rubi [A]** time = 0.114924, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {2631, 2577}

$$\frac{b \cos^2(e + fx)^{5/4} (c \sec(e + fx))^{5/2} (b \csc(e + fx))^{n-1} {}_2F_1\left(\frac{5}{4}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right)}{cf(1-n)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Csc[e + f\*x])^n\*(c\*Sec[e + f\*x])^(3/2),x]

[Out] (b\*(Cos[e + f\*x]^2)^(5/4)\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[5/4, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*(c\*Sec[e + f\*x])^(5/2))/(c\*f\*(1 - n))

#### Rule 2631

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(a\_.))^(m\_.)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Dist[(a^2\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1))\*(a\*Sin[e + f\*x])^(m - 1)\*(b\*Cos[e + f\*x])^(n + 1))/b^2, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !SimplerQ[-m, -n]

#### Rule 2577

Int[(cos[(e\_.) + (f\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*((a\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_.), x\_Symbol] :> Simp[(b^(2\*IntPart[(n - 1)/2] + 1)\*(b\*Cos[e + f\*x])^(2\*FracPart[(n - 1)/2])\*(a\*Sin[e + f\*x])^(m + 1)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f\*x]^2])/(a\*f\*(m + 1)\*(Cos[e + f\*x]^2)^FracPart[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]

Rubi steps

$$\int (b \csc(e + fx))^n (c \sec(e + fx))^{3/2} dx = \frac{(b^2(c \cos(e + fx))^{5/2}(b \csc(e + fx))^{-1+n}(c \sec(e + fx))^{5/2}(b \sin(e + fx))^{-1+n})}{c^2} \int$$

$$= \frac{b \cos^2(e + fx)^{5/4}(b \csc(e + fx))^{-1+n} {}_2F_1\left(\frac{5}{4}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right) (c \sec(e + fx))}{cf(1-n)}$$

**Mathematica [A]** time = 1.82734, size = 92, normalized size = 1.14

$$\frac{2 \cot(e + fx)(c \sec(e + fx))^{3/2} (-\tan^2(e + fx))^{\frac{n+1}{2}} (b \csc(e + fx))^n \text{Hypergeometric2F1}\left(\frac{n+1}{2}, \frac{1}{4}(2n+3), \frac{1}{4}(2n+7), \sec^2(e + fx)\right)}{f(2n+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Csc[e + f\*x])^n\*(c\*Sec[e + f\*x])^(3/2), x]

[Out] (2\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^n\*Hypergeometric2F1[(1 + n)/2, (3 + 2\*n)/4, (7 + 2\*n)/4, Sec[e + f\*x]^2]\*(c\*Sec[e + f\*x])^(3/2)\*(-Tan[e + f\*x]^2)^((1 + n)/2))/(f\*(3 + 2\*n))

**Maple [F]** time = 0.159, size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n (c \sec(fx + e))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*csc(f\*x+e))^n\*(c\*sec(f\*x+e))^(3/2), x)

[Out] int((b\*csc(f\*x+e))^n\*(c\*sec(f\*x+e))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec(fx + e))^{\frac{3}{2}} (b \csc(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*(c\*sec(f\*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((c\*sec(f\*x + e))^(3/2)\*(b\*csc(f\*x + e))^n, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{c \sec(fx + e)} (b \csc(fx + e))^n c \sec(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*(c\*sec(f\*x+e))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c\*sec(f\*x + e))\*(b\*csc(f\*x + e))^n\*c\*sec(f\*x + e), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))\*\*n\*(c\*sec(f\*x+e))\*\*(3/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (c \sec(fx + e))^{\frac{3}{2}} (b \csc(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n\*(c\*sec(f\*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((c\*sec(f\*x + e))^(3/2)\*(b\*csc(f\*x + e))^n, x)



### 3.297 $\int (b \csc(e + fx))^n \sqrt{c \sec(e + fx)} dx$

**Optimal.** Leaf size=81

$$\frac{b \cos^2(e + fx)^{3/4} (c \sec(e + fx))^{3/2} (b \csc(e + fx))^{n-1} \text{Hypergeometric2F1}\left(\frac{3}{4}, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e + fx)\right)}{cf(1-n)}$$

[Out] (b\*(Cos[e + f\*x]^2)^(3/4)\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[3/4, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*(c\*Sec[e + f\*x])^(3/2))/(c\*f\*(1 - n))

**Rubi [A]** time = 0.0971722, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {2631, 2577}

$$\frac{b \cos^2(e + fx)^{3/4} (c \sec(e + fx))^{3/2} (b \csc(e + fx))^{n-1} {}_2F_1\left(\frac{3}{4}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right)}{cf(1-n)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Csc[e + f\*x])^n\*Sqrt[c\*Sec[e + f\*x]],x]

[Out] (b\*(Cos[e + f\*x]^2)^(3/4)\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[3/4, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*(c\*Sec[e + f\*x])^(3/2))/(c\*f\*(1 - n))

#### Rule 2631

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[(a^2*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1))/b^2, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !SimplerQ[-m, -n]
```

#### Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*FracPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (b \csc(e + fx))^n \sqrt{c \sec(e + fx)} dx = \frac{(b^2(c \cos(e + fx))^{3/2}(b \csc(e + fx))^{-1+n}(c \sec(e + fx))^{3/2}(b \sin(e + fx))^{-1+n}) \int \frac{(b \csc(e + fx))^{n+1}}{\sqrt{c \sec(e + fx)}} dx}{c^2}$$

$$= \frac{b \cos^2(e + fx)^{3/4}(b \csc(e + fx))^{-1+n} {}_2F_1\left(\frac{3}{4}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right) (c \sec(e + fx))^{3/2}}{cf(1-n)}$$

**Mathematica [A]** time = 1.61025, size = 90, normalized size = 1.11

$$\frac{2 \cot(e + fx) \sqrt{c \sec(e + fx)} (-\tan^2(e + fx))^{\frac{n+1}{2}} (b \csc(e + fx))^n \text{Hypergeometric2F1}\left(\frac{n+1}{2}, \frac{1}{4}(2n+1), \frac{1}{4}(2n+5), \sec^2(e + fx)\right)}{2fn + f}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Csc[e + f\*x])^n\*Sqrt[c\*Sec[e + f\*x]],x]

[Out] (2\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^n\*Hypergeometric2F1[(1 + n)/2, (1 + 2\*n)/4, (5 + 2\*n)/4, Sec[e + f\*x]^2]\*Sqrt[c\*Sec[e + f\*x]]\*(-Tan[e + f\*x]^2)^((1 + n)/2))/(f + 2\*f\*n)

**Maple [F]** time = 0.152, size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \sqrt{c \sec(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*csc(f\*x+e))^n\*(c\*sec(f\*x+e))^(1/2),x)

[Out] int((b\*csc(f\*x+e))^n\*(c\*sec(f\*x+e))^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \sec(fx + e)} (b \csc(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{c \sec(fx + e)} (b \csc(fx + e))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \csc(e + fx))^n \sqrt{c \sec(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x)`

[Out] `Integral((b*csc(e + f*x))^n*sqrt(c*sec(e + f*x)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \sec(fx + e)} (b \csc(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n, x)`

$$3.298 \quad \int \frac{(b \csc(e+fx))^n}{\sqrt{c \sec(e+fx)}} dx$$

**Optimal.** Leaf size=81

$$\frac{b^4 \sqrt{\cos^2(e+fx)} \sqrt{c \sec(e+fx)} (b \csc(e+fx))^{n-1} \text{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e+fx)\right)}{cf(1-n)}$$

[Out] (b\*(Cos[e + f\*x]^2)^(1/4)\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[1/4, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*Sqrt[c\*Sec[e + f\*x]])/(c\*f\*(1 - n))

**Rubi [A]** time = 0.0958881, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {2631, 2577}

$$\frac{b^4 \sqrt{\cos^2(e+fx)} \sqrt{c \sec(e+fx)} (b \csc(e+fx))^{n-1} {}_2F_1\left(\frac{1}{4}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e+fx)\right)}{cf(1-n)}$$

Antiderivative was successfully verified.

[In] Int[(b\*Csc[e + f\*x])^n/Sqrt[c\*Sec[e + f\*x]],x]

[Out] (b\*(Cos[e + f\*x]^2)^(1/4)\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[1/4, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2]\*Sqrt[c\*Sec[e + f\*x]])/(c\*f\*(1 - n))

### Rule 2631

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Dist[(a^2\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1)\*(a\*Sin[e + f\*x])^(m - 1)\*(b\*Cos[e + f\*x])^(n + 1))/b^2, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !SimplerQ[-m, -n]

### Rule 2577

Int[(cos[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^(n\_)\*((a\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_), x\_Symbol] :> Simp[(b^(2\*IntPart[(n - 1)/2] + 1)\*(b\*Cos[e + f\*x])^(2\*FracPart[(n - 1)/2])\*(a\*Sin[e + f\*x])^(m + 1)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f\*x]^2])/(a\*f\*(m + 1)\*(Cos[e + f\*x]^2)^FracPart[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]

Rubi steps

$$\int \frac{(b \csc(e + fx))^n}{\sqrt{c \sec(e + fx)}} dx = \frac{(b^2 \sqrt{c \cos(e + fx)} (b \csc(e + fx))^{-1+n} \sqrt{c \sec(e + fx)} (b \sin(e + fx))^{-1+n}) \int \sqrt{c \cos(e + fx)} (b \sin(e + fx))^{-1+n} dx}{c^2}$$

$$= \frac{b^4 \sqrt{\cos^2(e + fx)} (b \csc(e + fx))^{-1+n} {}_2F_1\left(\frac{1}{4}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right) \sqrt{c \sec(e + fx)}}{cf(1-n)}$$

**Mathematica [C]** time = 3.10075, size = 326, normalized size = 4.02

$$\frac{4(n-3) \sin\left(\frac{1}{2}(e+fx)\right) \cos^3\left(\frac{1}{2}(e+fx)\right)}{f(n-1) \sqrt{c \sec(e+fx)} \left(2(3-2n) \sin^2\left(\frac{1}{2}(e+fx)\right) F_1\left(\frac{3}{2}-\frac{n}{2}; -\frac{1}{2}, \frac{5}{2}-n; \frac{5}{2}-\frac{n}{2}; \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(b\*Csc[e + f\*x])^n/Sqrt[c\*Sec[e + f\*x]],x]

[Out] (-4\*(-3 + n)\*AppellF1[1/2 - n/2, -1/2, 3/2 - n, 3/2 - n/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2]\*Cos[(e + f\*x)/2]^3\*(b\*Csc[e + f\*x])^n\*Sin[(e + f\*x)/2])/(f\*(-1 + n)\*Sqrt[c\*Sec[e + f\*x]]\*(2\*(-3 + n)\*AppellF1[1/2 - n/2, -1/2, 3/2 - n, 3/2 - n/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2]\*Cos[(e + f\*x)/2]^2 - AppellF1[3/2 - n/2, 1/2, 3/2 - n, 5/2 - n/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2]\*(-1 + Cos[e + f\*x]) + 2\*(3 - 2\*n)\*AppellF1[3/2 - n/2, -1/2, 5/2 - n, 5/2 - n/2, Tan[(e + f\*x)/2]^2, -Tan[(e + f\*x)/2]^2]\*Sin[(e + f\*x)/2]^2))

**Maple [F]** time = 0.144, size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n \frac{1}{\sqrt{c \sec(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*csc(f\*x+e))^n/(c\*sec(f\*x+e))^(1/2),x)

[Out] int((b\*csc(f\*x+e))^n/(c\*sec(f\*x+e))^(1/2),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \csc(fx + e))^n}{\sqrt{c \sec(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n/(c\*sec(f\*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((b\*csc(f\*x + e))^n/sqrt(c\*sec(f\*x + e)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c \sec(fx + e)} (b \csc(fx + e))^n}{c \sec(fx + e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n/(c\*sec(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c\*sec(f\*x + e))\*(b\*csc(f\*x + e))^n/(c\*sec(f\*x + e)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \csc(e + fx))^n}{\sqrt{c \sec(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))\*\*n/(c\*sec(f\*x+e))\*\*(1/2),x)

[Out] Integral((b\*csc(e + f\*x))\*\*n/sqrt(c\*sec(e + f\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \csc(fx + e))^n}{\sqrt{c \sec(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n/sqrt(c*sec(f*x + e)), x)
```

$$3.299 \quad \int \frac{(b \csc(e+fx))^n}{(c \sec(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=81

$$\frac{b(b \csc(e+fx))^{n-1} \text{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1-n}{2}, \frac{3-n}{2}, \sin^2(e+fx)\right)}{cf(1-n)\sqrt[4]{\cos^2(e+fx)}\sqrt{c \sec(e+fx)}}$$

[Out] (b\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[-1/4, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2])/(c\*f\*(1 - n)\*(Cos[e + f\*x]^2)^(1/4)\*Sqrt[c\*Sec[e + f\*x]])

**Rubi [A]** time = 0.107255, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {2631, 2577}

$$\frac{b(b \csc(e+fx))^{n-1} {}_2F_1\left(-\frac{1}{4}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e+fx)\right)}{cf(1-n)\sqrt[4]{\cos^2(e+fx)}\sqrt{c \sec(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(b\*Csc[e + f\*x])^n/(c\*Sec[e + f\*x])^(3/2),x]

[Out] (b\*(b\*Csc[e + f\*x])^(-1 + n)\*Hypergeometric2F1[-1/4, (1 - n)/2, (3 - n)/2, Sin[e + f\*x]^2])/(c\*f\*(1 - n)\*(Cos[e + f\*x]^2)^(1/4)\*Sqrt[c\*Sec[e + f\*x]])

### Rule 2631

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*((b\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[(a^2\*(a\*Csc[e + f\*x])^(m - 1)\*(b\*Sec[e + f\*x])^(n + 1)\*(a\*Sin[e + f\*x])^(m - 1)\*(b\*Cos[e + f\*x])^(n + 1))/b^2, Int[1/((a\*Sin[e + f\*x])^m\*(b\*Cos[e + f\*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !SimplerQ[-m, -n]

### Rule 2577

Int[(cos[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^(n\_)\*((a\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_), x\_Symbol] := Simp[(b^(2\*IntPart[(n - 1)/2] + 1)\*(b\*Cos[e + f\*x])^(2\*FracPart[(n - 1)/2])\*(a\*Sin[e + f\*x])^(m + 1)\*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f\*x]^2])/(a\*f\*(m + 1)\*(Cos[e + f\*x]^2)^FracPart[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]



Rubi steps

$$\int \frac{(b \csc(e + fx))^n}{(c \sec(e + fx))^{3/2}} dx = \frac{(b^2(b \csc(e + fx))^{-1+n}(b \sin(e + fx))^{-1+n}) \int (c \cos(e + fx))^{3/2} (b \sin(e + fx))^{-n} dx}{c^2 \sqrt{c \cos(e + fx)} \sqrt{c \sec(e + fx)}}$$

$$= \frac{b(b \csc(e + fx))^{-1+n} {}_2F_1\left(-\frac{1}{4}, \frac{1-n}{2}; \frac{3-n}{2}; \sin^2(e + fx)\right)}{cf(1-n) \sqrt[4]{\cos^2(e + fx)} \sqrt{c \sec(e + fx)}}$$

**Mathematica [A]** time = 1.00577, size = 115, normalized size = 1.42

$$\frac{2 \cos(2(e + fx)) \cot(e + fx) \sqrt{c \sec(e + fx)} (-\tan^2(e + fx))^{\frac{n+1}{2}} (b \csc(e + fx))^n \text{Hypergeometric2F1}\left(\frac{n+1}{2}, \frac{1}{4}(2n-3), \frac{n+1}{2}, -\tan^2(e + fx)\right)}{c^2 f(2n-3) (\sec^2(e + fx) - 2)}$$

Antiderivative was successfully verified.

[In] Integrate[(b\*Csc[e + f\*x])^n/(c\*Sec[e + f\*x])^(3/2), x]

[Out] (-2\*Cos[2\*(e + f\*x)]\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^n\*Hypergeometric2F1[(1 + n)/2, (-3 + 2\*n)/4, (1 + 2\*n)/4, Sec[e + f\*x]^2]\*Sqrt[c\*Sec[e + f\*x]]\*(-Tan[e + f\*x]^2)^((1 + n)/2))/(c^2\*f\*(-3 + 2\*n)\*(-2 + Sec[e + f\*x]^2))

**Maple [F]** time = 0.136, size = 0, normalized size = 0.

$$\int (b \csc(fx + e))^n (c \sec(fx + e))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*csc(f\*x+e))^n/(c\*sec(f\*x+e))^(3/2), x)

[Out] int((b\*csc(f\*x+e))^n/(c\*sec(f\*x+e))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \csc(fx + e))^n}{(c \sec(fx + e))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n/(c\*sec(f\*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((b\*csc(f\*x + e))^n/(c\*sec(f\*x + e))^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c \sec(fx + e)} (b \csc(fx + e))^n}{c^2 \sec(fx + e)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))^n/(c\*sec(f\*x+e))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c\*sec(f\*x + e))\*(b\*csc(f\*x + e))^n/(c^2\*sec(f\*x + e)^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \csc(e + fx))^n}{(c \sec(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*csc(f\*x+e))\*\*n/(c\*sec(f\*x+e))\*\*(3/2),x)

[Out] Integral((b\*csc(e + f\*x))\*\*n/(c\*sec(e + f\*x))\*\*(3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \csc(fx + e))^n}{(c \sec(fx + e))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n/(c*sec(f*x + e))^(3/2), x)
```



# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,

```

```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, Csch,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

## 4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
11 see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```



```
56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65 else #result contains complex but optimal is not
66     if debug then
67         print("result contains complex but optimal is not");
68     fi;
69     return "C";
70 end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do not
as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'+'') or type(expn,'*') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

```

```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157 ElementaryFunctionQ := proc(func)
158     member(func,[
159         exp,log,ln,
160         sin,cos,tan,cot,sec,csc,
161         arcsin,arccos,arctan,arccot,arcsec,arccsc,
162         sinh,cosh,tanh,coth,sech,csch,
163         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
164 end proc:
165
166 SpecialFunctionQ := proc(func)
167     member(func,[
168         erf,erfc,erfi,
169         FresnelS,FresnelC,
170         Ei,Ei,Li,Si,Ci,Shi,Chi,
171         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
172         EllipticF,EllipticE,EllipticPi])
173 end proc:
174
175 HypergeometricFunctionQ := proc(func)
176     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
177 end proc:
178
179 AppellFunctionQ := proc(func)
180     member(func,[AppellF1])
181 end proc:
182
183 # u is a sum or product. rest(u) returns all but the
184 # first term or factor of u.
185 rest := proc(u) local v;
186     if nops(u)=2 then
187         op(2,u)
188     else
189         apply(op(0,u),op(2..nops(u),u))
190     end if
191 end proc:
192
193 #leafcount(u) returns the number of nodes in u.
194 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```

```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result  = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result  = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

## 4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by

```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

```

```

48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

```



```

93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
          sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

```

```

133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183                 else:
184                     return "B"
185             else: #result contains complex but optimal is not
186                 return "C"
187         else: # result do not contain complex, this assumes optimal do not as
188             well
189                 if leaf_count_result <= 2*leaf_count_optimal:
190                     return "A"
191                 else:
192                     return "B"
193         else:
194             return "C"
```